CURRENT STATE OF RESEARCH ON MATHEMATICAL BELIEFS XVII

Proceedings of the MAVI-17 Conference
September 17-20, 2011, Bochum, Germany

Edited by Bettina Roesken and Michael Casper

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Bettina Roesken, Michael Casper
Ruhr Universität Bochum

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RELABILITY AND FACTORIAL VALIDITY OF STUDENTS' MATHEMATICS–BELIEF, REPRESENTATION AND PREFERENCE ON FUNCTION

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This study aims to develop and validate an instrument for determining student beliefs, and preferences for multiple representations of functions and establishes base-line data for future research. It examines the reliability and factorial validity of four scales Daily Learning Behaviours, Views about the usefulness of different mathematics Representation, Perceptions on the difficulty of mathematics representation and perception of teachers' behaviour. The data suggests that this instrument is an effective, reliable, and convenient means of measuring student views, attitudes and preferences on mathematical representation.

INTRODUCTION AND BACKGROUND

Nowadays a lot of attention is paid to the construction of learning environments that provide instruction tailored to students' needs. Harel & Dubinsky (1992) argue that students who can use multiple representations of a concept yield deeper and more flexible understandings than those who used fewer representations. The multiple representations have become more accessible, for functions in particular, through technological advancement.

Scaife & Rogers (1996) abstracted three central characteristics of representations which we consider as a useful analytic framework. These are *computational offloading* , *re-presentation* and *graphical constraining* .*Computational offloading* is the extent to which different external representations reduce the amount of cognitive effort required to solve equivalent problems. Larkin & Simon (1987) argue that representations that are informationally equal differ in their computational properties. For instance representations such as tables tend to make information explicit, emphasize empty cells that direct attention to unexplored alternatives, and allow quicker and more accurate readoff (e.g. Meyer, Shinar, & Leiser, 1997). *Re-representation* refers to how different external representations that have the same abstract structure, make problem-solving easier or more difficult. Zhang & Norman (1994) showed that solving the Towers of Hanoi problem was enhanced when representations externalised more information. Ainsworth (1999) emphasized that by utilizing external perceptual processes rather than cognitive operations, graphical representations will often be more effective. *Graphical constraining* describes the limits on the range of inferences that can be made about the represented concept. Stenning & Oberlander (1995) argue that text permits expression of ambiguity in a way that graphics cannot easily accommodate. It is this lack of expressiveness that makes diagrams more effective for solving determinate problems.

There is evidence that an active mental connection between different representations of the same information enhances understanding, acquisition and memorization of learning contents (Dekeyser, 2001). The interplay between different forms of representation leads to the construction of richer meanings for the concepts to be learned (Moreno, 1995). Ainsworth, Bibby and Wood (1998) distinguish three functions of multiple representations: Some multiple representations support different ideas and processes; some constrain interpretations and some promote a deeper understanding of the domain.

Adaptation of the representational system to the learners' preference therefore would deprive the learner of the benefit of multiple representations. Kirby (1993) found that presenting information by means of multiple representations has either collaborative or competitive effects on learning, depending on certain conditions. Recent research results corroborate these findings, for instance presenting visuals emphasizing relevant critical details is effective where arbitrarily adding visuals does not increase learning at all. Certain combinations of representations hamper the translation between these representations (Ainsworth, Bibby & Wood, 1998).

Students' preferences for representation have tone towards two schools of taught (Keller & Hirsh, 1998). One school attempts to determine students preferences by the representation used to solve a problem task. Dreyfus and Eisenberg (1982) on a study on students intuitions on function concepts presented in diagram, graph, and table settings, found out that students with high ability preferred the graphical settings throughout, whereas low ability students preferred the table settings. The other school of taught, is oriented more towards learning or cognitive styles. Brenner at al. (1997) compared experimental classes of students who were taught multiple representations in the content of a pre-algebra class to students receiving instruction primarily on solution strategies. Compared to the solution-oriented classes, experimental classes exhibited more skill with representations and enhanced problem-solving success.

According to Jaspers (1992) this 'preference' can be understood in two ways, it can be subjective: "a tendency of the learners to prefer one modality rather than another: If not externally controlled, and when given a choice between several presentation modalities, for some reason or other they choose what they are most fond of", or it can be objective: "based on greater effectiveness of one of the modalities, where this effectiveness differs between individuals" (p. 236). According to Moore and Scevak (1995) some support is given to the suggestion that subjective preference is related to a more successful use of the preferred mode, i.e. objective preference.

On student preferences are a specific aspect of beliefs. Op't Eynde et al.(2002,2003) defined students' mathematics-related beliefs " as the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematics learners, and about the mathematics class context. These beliefs determine in close interaction with each other and with students' prior knowledge their mathematical learning and problem-solving activities in class" (p. 4).

Op’t Eynde et al. (2002, 2003) developed a framework on students mathematics-related beliefs based on the analysis of the nature and the structure of beliefs and belief systems pointing to the social context, the self and the object in the world that the beliefs relate to, as constitutive for the developing and the functioning of these systems as follows: (1) Beliefs about mathematics education: (beliefs about mathematics as a subject, beliefs about mathematical learning and problem solving, and beliefs about mathematics teaching in general); (2) Beliefs about the self: (Self-efficacy beliefs, Control beliefs, Task-value beliefs, and Goal-orientation beliefs); (3) Beliefs about the social context: (beliefs about social norms in their own class, the role and the functioning of the teacher, and the role and the functioning of the students and (4) Beliefs about socio-mathematical norms in their own class.

With the aforementioned background, the focus is to develop scales that measure student’s views/beliefs and preferences about the use of different types of mathematics representations on functions.

RATIONALE FOR RESEARCH ON FUNCTIONS

The function concept ties algebra, trigonometry, and geometry together and it appears and reappears like a thread throughout school mathematics from grade 1 to grade 12. We focus on functions of variables with two unknowns because it is a central topic in mathematics that is relatively understudied (Romberg et al., 1993). For example, Romberg, at al. (1993, p.1) noted that “functions are among the most powerful and useful notions in all mathematics” and Yerushalmy & Schwartz (1993, p.41) state that “the function is the fundamental object of algebra and .... it ought to be present in a variety of representations in algebra teaching and learning from the onset.”

The concept of function is difficult for many students to understand, maybe because it is presented in abstract algebraic form rather than in a concrete, practical context. Romberg et al.(1993, p.2) argue that the “abstractness of the algebraic expressions and the variety of transformations is such expressions have proved difficult for many students to fathom.”

Mathematics educators mostly discuss the representations of functions into three categories- tables consisting of ordered pairs of values, graphs consisting of a pictorial presentation, and equations consisting of algebraic notation (Romberg et al., 1993). Kieran (1993, p. 230) notes that there is “a renewed emphasis on integrating these various representations of functions, such as graphical, algebraic, and tabular.”

METHODS

The main aim of this study was to describe and validate the development of a questionnaire for measuring student’s beliefs about the use of different types of mathematics representations on functions. The questionnaire consist of 84 Likert scale items group under four scales, ‘Daily Learning behaviour’ (DLB 24-items), ‘Views about the usefulness of different mathematics representation’(VUDMR 16-items), ‘perception on the difficulty of different mathematics representations’

(PDDMRS 24-items)' and 'Perception of teacher's behaviours' (PTB 20-items)' on functions of variable with two unknowns. The items in the questionnaire are presented in appendixes table 1,2,3,4 respectively.

A sample item: What mathematical method(s) do you usually use in solving task(s) involving functions?

1. Graphical Method	*5 4 3 2 1*
2. Numerical (equations) Method	*5 4 3 2 1*
3. Tabular Method	*5 4 3 2 1*
4. Trial and Error	*5 4 3 2 1*

Based on classification of dimensions by Op 't Eynde and his colleagues, we constructed items on beliefs about mathematics as a subject (Which mathematical method(s) do you think is helpful in efficiently solving math questions), beliefs about mathematical learning and problem solving (Do you think using different method(s) helps you figure out how to solve such questions?), Self-efficacy beliefs (I am very good in solving maths questions graphically), Control beliefs (Does the availability of any of these options affect your choice of method in solving the equations?), Task-value beliefs (Which of the following mathematical method(s) do you think is useful in other subjects) and beliefs about the role and the functioning of the teacher ("Which of these method(s) are you told or encouraged by your math teacher to use in solving math questions?").

PARTICIPANTS AND THE INSTRUMENT

Administration procedures were first explained to all students who were asked to complete the questionnaire and anonymity assured. Students responded to the survey during a normal class. Participants consisted of 120 students (58 female, 62 male). Age of the students ranged from 14 to 23, mean of 18, median age of 18 and s.d of 1.3. They were all in Grade 12 and were enrolled either in Business, Science, Arts, or Vocational Science classes. The Students were enrolled in varying mathematics classes; core mathematics and elective mathematics.

The questionnaire was developed for measuring students preferences concerning functions. All items present statement about the various ideas in solving functions. In response to the statement, students were asked to indicate one of the responses from 'Not at all (5)' to 'Always (1)'.

ANALYSIS

Exploratory factor analysis was conducted for each subscale using IBM Spss (ver 18) to determine the common factor structure of all the items. We used the maximum likelihood method of extraction , Oblique rotation (Oblimin) and a variety of criteria to determine the number of common factors to retain and analyze, including scree test, Kaiser's criterion, the percentage of common variance explained by each factor using the weighted, reduced correlation matrix, and the interpretability of the rotated factors. All factors above 0.60 were maintain because of the diversity of constructs being measured (Kline, 1999) as depicted in the appendix tables 1,2,3,4.

Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy and Bartlett's test of sphericity were conducted (Field, 2005) as well as reliability coefficient(Cronbach alpha) were computed. Kaiser-Meyer-Olkin measure of sampling adequacy and Bartlett's test of sphericity indicated that factor analysis is a good measure for the data for each of the scales and multicollinearity and singularity were found not to be a problem.

The daily *learning behaviour* (DLB) scale consisted of 24 items which assessed student's daily learning behaviours concerning function (α=0.77). Results of factor analysis yielded four factors with Cronbach alphas for factors F1= 0.80; F2 = 0.75; F3 = 0,78; and F4=0.51respectively (Table 1). The results indicated a considerable and significant contribution of each of the items in measuring daily learning behaviour. A look through the correlation matrix indicated that majority of the variables have values less than 0.05 and none of the correlation coefficient was greater than 0.9 with a determinant value of (0.0000875) which is greater than the necessary value of 0.00001 (Field, 2005). Therefore multicollinearity and singularity is not a problem for the DLB. To sum up, all the questions in DLB correlate fairly well and none of the correlation coefficients are particularly large; F4 was removed since the Cronbach alpha was less < 0.60. From statistics above adjusted version of the scale can provide us with an instrument to validly and reliably measure students' learning behaviours concerning functions.

The Views about the *usefulness* of different mathematics representations (VUDMR) consist of 16 items which assessed students' self-efficacy and views about the usefulness of different representations in functions. The Cronbach's alpha for the questionnaire was α=0.73, Results of factor analysis yield three factors (Table 2) with Cronbach's alpha for F1 = 0.76, F2 = 0.66, F3=0.67. Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy increased from 0.67 to 0.73 after deleting four items with very low communalities. With Bartlett's test of sphericity highly significant ($p<0.001$), we were confident that factor analysis is an appropriate measure for the data.

The Perceptions on the *difficulty* of different mathematics representations (PDDMR) was a 24 Likert-types items which present statements of perception on the difficulty of different mathematics representation in solving functions. Factor analysis yield five factors (Table 3) with reliability of the whole questionnaires Cronbach's alpha = 0.67. The Cronbach's alpha coefficient for each factor identified by the exploratory factor analysis were: F1 = 0.83; F2=0.80; F3= 0.80; F4=0.64; F5=0.48 suggest an appropriate fit of the model with the data after F5 was removed since the Cronbach alpha was less < 0.60.

The Perception of *teacher's behaviours* (PTB) is a 20 Likert-types items which present statements of perception of teachers' behaviour on different representation in respect to how it will affect student choice of any of the representations. Cronbach alpha of 0.76 for the whole questionnaire with four factors after exploratory factor analysis were FI= 0.76, F2=0.82, F3=0.80, and F4=0.77.

CONCLUSIONS AND SUGGESTIONS

Although the reliability of the whole 84 questionnaire was reasonable (0.90) with the subscale daily learning behaviour (DLB) ,Views about the usefulness of different mathematics representations (VUDMR) , Perceptions on the difficulty of different mathematics representations(PDDMR) and Perception of teacher's behaviours (PTB) having a Cronbach's alpha of 0.77, 0.73, 0.67 and 0.76 respectively. Item analysis to find out about the item validity indicted that there were items in the 84-items subscales, which either repeated each other or did not measure well-enough the related category. All factors above 0.60 were maintain because of the diversity of constructs being measured (Kline, 1999). Based on the results of this analysis, the scales on Daily learning Behaviour was refined into 19 items, views on usefulness of different mathematics representations 12-item scale, Perceptions on the difficulty of different mathematics representations into 19-items which were more comprehensive. All together, the factor analysis and the alpha's suggest that the factors for the scales of the models are reasonable representation of the data and that an adjusted version of the scales[1] can provide us with an instrument to validly and reliably measure students' views-belief systems on functions.

In the questionnaire, there were several items that reflected the belief dimensions suggested by Op 't Eynde and his colleagues (2002, 2003). In our analysis some factors (eg. DLB-F2, PDDMR-F4) consist of two representations. Further studies will be required on why graphical and tabular representation and well as graphical and numerical representations were mostly within the same component.

Of the different scales to measure beliefs related to representational preferences, we found that students' Perception of teacher's behaviours was most consistent with respect to the different representations. It is therefore evidence for the relevance of students' beliefs about the role and the functioning of their own teacher on preference for representation. In Daily learning behaviours and in Perceptions on the difficulty of different mathematics representations, the preferences formed nice components, but the numerical or equation method became blended with items from graphical and tabular approaches. In the usefulness dimension for mathematics representations the tabular preferences form a very weak component in the analysis.

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[1] *Adjusting the scales in accordance with the factors found and leaving or reframing all items that load on more than one factor or have a loading of less than 0.40 (Fabrigar, et al., 1999; Field, 2005)*

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APPENDIX. Tables for factor loadings

Items	*Factors*	
What maths method you usually use in solving maths task(s) involving functions-Trial and Error	*F1*	*0.89*
Which of these method(s) do you try to copy from your teacher to solve similar questions- Trial and Error		*0.78*
Which of these methods shown in your textbooks do you try to use to solve other similar maths question-Trial and Error		*0.68*
Which of these different methods for solving function questions that your teacher shows on the board during class do you pay attention to?-Trial and Error		*0.62*
Which of the following mathematical method(s) do you think is useful in other subjects-Trial and Error		*0.41*
Does the availability of any of these options affect your choice of method in solving the question? -Trial and Error		*0.40*
Eigenvalue		*3.36*
Which of the following mathematical method(s) do you think is useful in other subjects -Numerical(equations) Method	*F2*	*0.67*
Which of these methods shown in your textbooks do you try to use to solve other similar maths question -Numerical (equations) Method		*0.66*
Which of these different methods for solving function questions that your teacher shows on the board during class do you pay attention to?- Numerical (equations) Method		*0.62*
What maths method you usually use in solving maths task(s) involving functions -Numerical(equations) Method		*0.57*
Which of these different methods for solving function questions that your teacher shows on the board during class do you pay attention to?-Graphical Method		*0.54*
Which of the following mathematical method(s) do you think is useful in other subjects -Graphical Method		*0.48*
Which of these method(s) do you try to copy from your teacher to solve similar questions -Numerical(equations)		*0.44*
Which of the following mathematical method(s) do you think is useful in other subjects -Tabular Method		*0.32*
Does the availability of any of these options affect your choice of method in solving the question? Graphical Method		*0.30*
Eigenvalue		*2.87*
Which of these methods shown in your textbooks do you try to use to solve other similar maths question -Tabular Method	*F3*	*-0.80*
Which of these method(s) do you try to copy from your teacher to solve similar questions -Tabular Method		*-0.74*
Which of these different methods for solving function questions that your teacher shows on the board during class do you pay attention to?-Tabular Method		*-0.66*
What maths method you usually use in solving maths task(s) involving functions -Tabular Method		*0.52*
Eigenvalue		*1.46*
Cumulative percentage of explained variance(%)		*40.46*

Table 1: Factor loadings of the four factors against the items associated with participants views on Daily Learning Behaviors.

Items	*Factors*	
Do you think it is good to use different methods in solving math questions involving functions?-Numerical Method	*F1*	*0.77*
If i have to explain how I have solved a math question involving functions, I prefer to use Numerical(equations) Method		*0.66*
Which math method(s) do you think is helpful in efficiently solving math questions involving functions?-Numerical Method		*0.56*
Do you think it is good to use different methods in solving math questions involving functions?-Numerical Method		*0.55*
Do you think using different method(s) helps you figure out how to solve such questions?-Graphical Method		*0.44*
Eigenvalue		*2.72*
Do you think it is good to use different methods in solving math questions involving functions?-Trial and Error	*F2*	*0.68*
Do you think using different method(s) helps you figure out how to solve such questions?-Trial and Error		*0.64*
If i have to explain how i have solved a math question, i prefer to use Trial and Error		*0.57*
Which mathematical method(s) do you think is helpful in efficiently solving math questions-Trial and Error		*0.41*
Eigenvalue		*1.55*
If i have to explain how i have solved a math question, i prefer to use Graphical Method	*F3*	*0.69*
Which mathematical method(s) do you think is helpful in efficiently solving math questions-Graphical Method		*0.69*
Do you think it is good to use different methods in solving math questions involving functions?-Graphical method		*0.49*
Eigenvalue		*0.90*
Cumulative percentage of explained variance (%)		*43.04*

Table 2: Factor Loadings of the three factors against the items associated with students' views about the usefulness of different mathematics representations.

Items	*Factors*	
I can easily explain how I've solved a question Graphical	*F1*	*0.80*
I am very good in solving maths questions Graphically		*0.77*
How easy is it for you in using the following different methods in solving math problems?-Graphical Method		*0.67*
Which of these method(s) are helpful in solving different kinds of math questions?-Graphical Method		*0.65*
Eigenvalue		*3.55*
I can easily explain how I've solved a question using Trial and Error	*F2*	*0.81*
I am very good in solving maths questions using Trial and Error		*0.71*
Which of these method(s) are helpful in solving different kinds of math questions Trial and Error		*0.70*
How easy is it for you in using the following different methods in solving math problems?-Trial and Error		*0.64*
Eigenvalue		*2.16*
I am very good in solving maths questions Numerically Method	*F3*	*-0.82*
Which of these method(s) are helpful in solving different kinds of math questions Numerical(equations) Method		*-0.81*
I can easily explain how i've solved a question Numerically(equations)		*-0.65*
How easy is it for you in using the following different methods in solving math problems?-Numerical(equations) Method		*-0.41*
Eigenvalue		*1.70*
How difficult is it for you to construct any of these methods by yourself for solving math questions?-Numerical Method	*F4*	*0.65*
How troublesome is it for you to use these different methods in solving math questions?-Numerical Method		*0.64*
How difficult is it for you to use these different methods in solving math questions Graphical Method		*0.51*
How difficult is it for you to construct any of these methods by yourself for solving math questions?- Tabular Method		*0.44*
How troublesome is it for you to use these different methods in solving math questions ?Tabular Method		*0.40*
How troublesome is it for you to use these different methods in solving math questions ? Graphical Method		*0.37*
Eigenvalue		*1.43*
Cumulative percentage of explained variance (%)		*46.57*

Table 3: Factor Loadings of the five factors against items associated with students Perceptions on the difficulty of different mathematics representations

Items	Factor			
	1	2	3	4
Which of these different method(s) do you think your math teachers uses to efficiently solve math questions?-Numerical Method	0.82			
Which of these method(s) are you told or encouraged by your math teacher to use in solving math questions?-Numerical Method	0.81			
Does your math teacher teach your class how to use these different methods in solving math questions?- Numerical Method	0.62			
Does your math teacher teach your class how to use these different methods in solving math questions?- Graphically	0.42			
How do these different method(s) used by your math teacher affect your choice of method?-Numerical Method	0.31			
How often your math teacher use these different method(s) in explaining how to solve math questions?-Trial and Error		0.80		
Which of these method(s) are you told or encouraged by your math teacher to use in solving math questions?-Trial and Error		0.74		
DBTRIERR Which of these different method(s) do you think your math teachers uses to efficiently solve math questions?-Trial and Error		0.69		
Does your math teacher teach your class how to use these different methods in solving math questions?- Trial and Error		0.64		
How do these different method(s) used by your math teacher affect your choice of method?-Trial and Error		0.47		
Which of these different method(s) do you think your math teachers uses to efficiently solve math questions?-Graphical Method			0.79	
How do these different method(s) used by your math teacher affect your choice of method?-Graphical Method			0.65	
How often your math teacher use these different method(s) in explaining how to solve math questions?-Graphical Method			0.61	
How often your math teacher use these different method(s) in explaining how to solve math questions?-Numerical Method			0.53	
Which of these method(s) are you told or encouraged by your math teacher to use in solving math questions?-Graphical Method			0.34	
Which of these different method(s) do you think your math teachers uses to efficiently solve math questions?-Tabular Method				0.90
How often your math teacher use these different method(s) in explaining how to solve math questions?-Tabular Method				0.61
Which of these method(s) are you told or encouraged by your math teacher to use in solving math questions?-Tabular Method				0.50
How do these different method(s) used by your math teacher affect your choice of method?-Tabular Method				0.49
Does your math teacher teach your class how to use these different methods in solving math questions?- Tabular Method				0.42
Eigenvalues	2.64	3.96	2.12	1.13
Percentage of variance explained	13.21	19.80	10.61	5.66
Cumulative percentages of explained variance	13.21	33.01	43.62	49.28

Table 4: Factor Loadings of the four factors against items on students Perception of their teachers' behaviour on different representations

DROP-OUT UNDERGRADUATE STUDENTS IN MATHEMATICS: AN EXPLORATORY STUDY

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A systematic, longitudinal survey on the population of students enrolling in the undergraduate course in mathematics at the University of Torino focuses on the possible causes of dropout. The comparison of students' careers with the results of a mathematical knowledge assessment test administered at the beginning of the first year of courses, supplemented by interviews with drop-out students, points toward the need of measuring not only knowledge, but also students' beliefs and motivations. We present evidence supporting this claim and we outline actions in progress in this direction.

INTRODUCTION

A longitudinal study on the undergraduate students in mathematics at the University of Torino has the purpose of monitoring their career, throughout all the university years (Andrà, 2009). In principle, students who enroll in an undergraduate course in mathematics are "good" in math and have a good relationship with mathematics; thus, one could expect that they should have no problem in doing mathematics at undergraduate level. Nevertheless, about one third of undergraduate mathematics students drop out at the end of the first academic year. The present paper proposes a work in progress on these students. For this population we shall consider the data emerging from the previous high-school career, data describing their situation at the very beginning of the university courses (TARM, a test aimed at assessing the mathematical knowledge), and the academic career up to the degree (marks in examinations). This general picture allows us to frame the specific state of leftovers. We will show that such drop-out students were not necessarily those displaying a significant lack of knowledge at the beginning of university studies: statistically relevant exceptions can be observed. One is led to conclude that measuring mathematical ability is not enough for understanding the dropout process, and other interpretative tools are necessary. Beliefs and other affective factors, such as motiva-tion, may have a key role in determining difficulties and reaction to difficulties.

BACKGROUND

The issue of transition from secondary school to university

The issue of transition is widely discussed in the literature. Gueudet (2008) provides a detailed overview of mathematics education studies concerning the transition that, according to the author, may be organized into two streams: observation and analysis of students' difficulties; discussion of teaching interventions aimed at fostering the transition. As regards the first stream, Gueudet notes that researchers may adopt different perspectives, focus on different aspects of the transition issue and, consequently, draw different conclusions, in terms of didactical actions. Researchers may focus: on the different thinking modes that are required at university, as evidenced by all the studies on Advanced Mathematical Thinking (Tall, 1991); on the different organization of knowledge and on the intrinsic complexity of the new contents to be learnt (see for instance Robert, 1998); on the different processes and activities that are at issue, proof for one (Moore, 1994); on the different didactical contract (Bosch *et al.*, 2004) and, more generally, on institutional issues, such as university courses organization (Hoyles *et al.*, 2001). A common feature of these studies is the focus on the differences between secondary school and university: in terms of content, organization, teaching methods and so on. Generally, difficulty in the transition is read in terms of a difficulty for students to adapt to the new context.

Clark & Lovric (2008) propose to look at the transition as a modern-day rite of passage, which encompasses a sort of shock. The authors suggest that transition should be smooth, and communication between the two institutions (school and university) should be improved.

An emerging perspective: students' affect

The crucial role of affect in mathematics learning is evidenced by a large amount of studies, that are well known to the audience of MAVI. Many of these studies focus on the situation of undergraduate students. For instance, Furinghetti & Morselli (2009) discuss the intertwining of affective and cognitive factors in the proving processes of university students in mathematics. The authors underline that also fourth year students in mathematics, who, in principle, should have a good relationship with the discipline, may have beliefs about themselves and about mathematics that can hinder the proving process and affect the process of causal attribution (Weiner, 1972; 1980) in front of difficulties. For sake of analysis, we recall that for Weiner attributions and perceptions of success/failure are categorized along three main dimensions: locus (internal vs. external), stability (stable vs. unstable), controllability (controllable vs. uncontrollable) of the causal agent.

Some studies explicitly deal with affective factors in the *transition* issue. For instance, Daskalogianni & Simpson (2001) discuss the concept of "beliefs overhang": some beliefs, developed during schooldays, are carried forward in university, and this fact may cause difficulties. The study points out the crucial role of beliefs (about mathematics) in determining university success or failure.

In our opinion, affective factors are useful interpretative lenses for understanding the phenomenon of drop out. In order to tackle the phenomenon, we distinguish: (1) the impact with the University world at the very beginning of the undergraduate studies, (2) the unsuccessful outcomes of the examinations, and (3) the justification students provide for such failures. A priori, we hypothesize that affective factors may intervene in three ways:

- Beliefs about mathematics, about oneself and about university may affect the way university courses are lived.

- Mathematical self-confidence and other beliefs may affect the performance during the university examinations.
- Beliefs may affect the process of causal attribution (Weiner, 1972; 1980).

Beliefs are a crucial yet problematic concept in mathematics education research (Leder, Pehkonen & Toerner, 2002). Research concerns both the theoretical construct and the development and refinement of instruments to measure them. Roesken et al. (2011) present a questionnaire aimed at studying students' views of themselves as learners of mathematics. This study has for us many sources of interest. First of all, it discusses from a theoretical point of view the concept of "view of mathematics" and the related concept of "beliefs about mathematics". The authors state they choose the term "view of mathematics" because it captures the structural properties of the affect-cognition interplay, while the concept of belief is more on the cognitive part. In other words, the authors say that "students' beliefs, wants and feelings are part of their view of mathematics". Secondly, the authors discuss a questionnaire aimed at studying the dimensions of students' views of themselves as learners of mathematics.

METHODOLOGY

The aim of this paper is threefold: understanding the situation of dropped-out students; setting up new observational tools for the next generations of mathematics undergraduate students; suggesting interventions during the first year for the future. The study draws from data collected at the University of Torino (Italy). Our investigation involves:

- data from students' previous career (diploma grades and type)
- the performance during the non-selective test for the assessment of minimum requisites (TARM) they took when enrolling in the undergraduate course
- students' marks and credits (obtained at University examinations).

As regards TARM, it is a multiple-choice test designed by a group of experts at a national level. It is administered to all enrolling students in scientific-type undergraduate courses and its items were calibrated according to the Item Response Theory models (the Rasch model, in particular). It is made of 25 items designed to measure a mathematical ability on the footsteps of the OCSE-PISA test. Moreover, TARM is in line with Clark's & Lovric's (2008) recommendation on *smoothing the transition*, since it is administered also at secondary school students and it is also a means to involve secondary school teachers into the reflection on transition issues.

Data have been collected since the academic year 2001/02 and the survey involves the entire population of enrolled students in the Mathematics course at the University of Torino. As said in the introduction, the aforementioned quantitative data are integrated with the analysis of phone interviews to drop-out students; the purpose of the interviews is to know their choices after leaving the undergraduate studies and to outline the main reasons of drop-out. Phone interviews, carried out according to the flowchart shown in figure 1, addressed all the students that did not continue their mathematics studies (in Italy it is necessary to enroll each academic year by paying an academic fee: all students who do not pay it, are considered as dropped-out). In other words, students who still enroll without passing the examinations are not taken into account in this study. Percentages of this kind of students, however, are pretty low (it is around 17%): nevertheless, almost one half of students takes the degree at least with one academic year of delay, and about one third of students drop out.

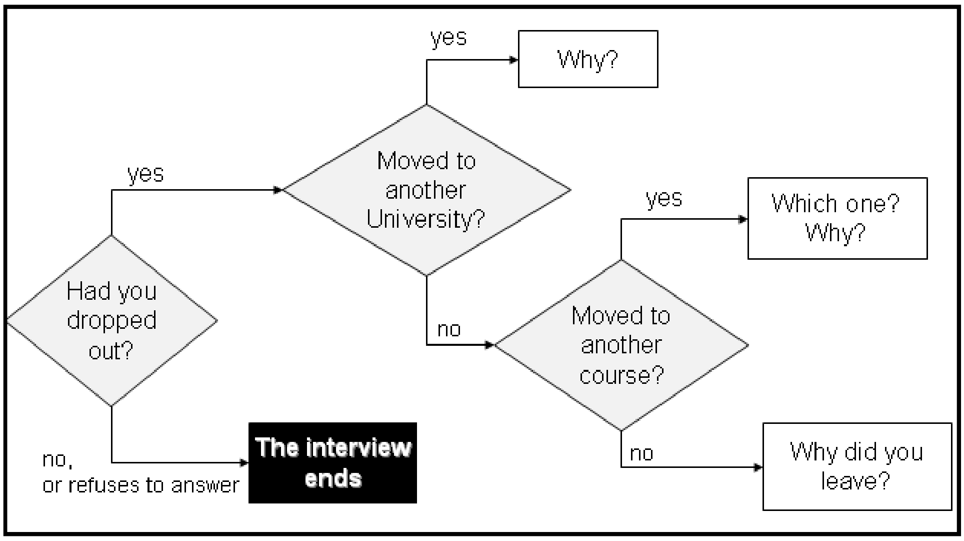

Figure 1: the questions of the phone interview.

DATA ANALYSIS

Diploma and TARM

When enrolling at University, students had taken a diploma graduation. Three diploma types are interesting in our survey: the first one is the scientific diploma type, which is taken by the majority (51%) of students who enroll in the undergraduate course in mathematics each year; the second one is the humanistic type, who is taken by a minority of students (6%); the third type is a technical diploma (20%). The first two diploma types are generally taken by students who intend to go on with their studies after high school, while the technical diploma enables students to search for a job after high school. Investigating the possible causes of drop-out, the diploma type is an interesting variable, since only the scientific diploma has the mathematics undergraduate course as a natural prosecution.

Focusing solely on the diploma grades, conversely, does not allow us to draw any conclusion about students' mathematical background. However, we can infer that students who enroll in the mathematics undergraduate course are 'good' students in general, since the mode of the diploma grades distribution is 100 and the majority is above 82 (diploma grades in Italy range from 60 to 100).

When students enroll in the mathematics undergraduate course, they are administered a non-selective assessment test: TARM. At the end of TARM, students are given three alternative messages, according to their score: (1) **Compulsory tutorial:** students that have a score within the first quartile are told they have to attend a tutorial course in basic mathematics (before the beginning of the first year); (2) **Suggested tutorial:** scores within the second quartile correspond to the message that students are advised to the same tutorial course, but it is not compulsory; (3) **No need for tutorial:** scores above the median implies no tutorial course (but students can choose whether to attend it or not). The thresholds on the TARM scores are set up each year in order to have 25% of students with the 'compulsory tutorial' message (first quartile of TARM scores), 25% in the 'suggested tutorial' group (second quartile), and 50% with 'no need for tutorial' (above the median). The first group of students must pass another test after attending the tutorial course (attending the tutorial is not compulsory); there is no extra testing for other students.

First semester examinations

Another data we have at disposal concerns the examinations after the first semester and/or the first year. We collected data from students who got the Bachelor degree on time (we call them *successful* students). For these students we compute the average grades and the minimum number of examinations they passed each semester. We found out that such *successful* students passed at least 3 out of 4 examinations during the first semester (they correspond to the 45% of the entire population), while students that take the degree one year later passed at most 2 examinations (20% of the population, *middle* students). Students who did not take the degree passed no or one examination at the end of the same period (35% of the population, *low-achievement* students). The situation that is outlined at the end of the first semester seems to remain the same throughout the whole 3-years course: a percentage below 5% of students moves from one group to another one (namely, successful, middle, or low-achievement). The majority of shifts is observed in the middle group. The majority of students in the low-achievement group drop out at the end of the first academic year (25-30% of the population drops out).

Crossing data

Let us now look at the relationship between the three groups referring to the academic career (successful, middle and low-achievement students) and TARM quartiles. Figure 2 shows the histograms of students' credits with respect to each TARM quartile. 7 credits are got for each examination. Students in the first quartile seem to have two opposite behaviors: one third of them do not get any credits (no examination passed), while the other two thirds seem to perform as their classmates in the third quartile: they pass three or all the examinations. Data from the second quartile show that the majority of students that do not pass any examination do not belong to the ones who had to attend the tutorial course. For students with a TARM score below the median it seems that the message told to them at the end of TARM (compulsory versus suggested tutorial) affects their initial behavior. Our interpretation is that students seem to do their best in order to fill an 'achievement gap' if the tutorial course is compulsory. Conversely, if it is only suggested, students tend to underestimate their real lack of knowledge.

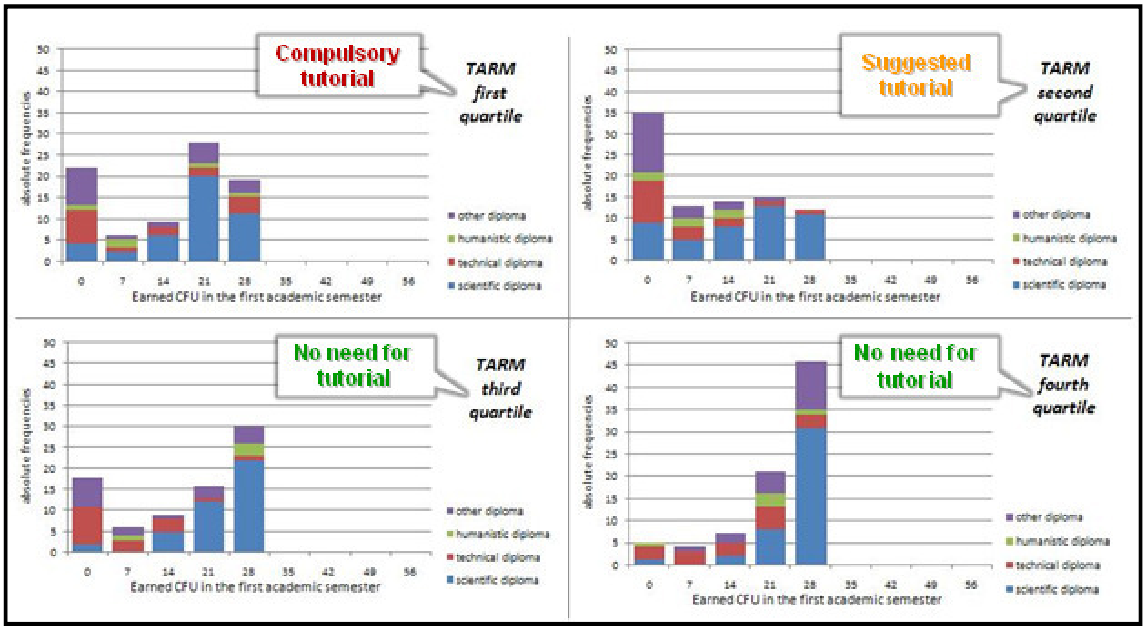

Figure 2: The histograms of credits (CFU) earned at the end of the first semester. The diploma type is reported. Students are split with respect to TARM quartiles.

The distribution of the averages of the grading in the first semester again brings to light the similarity between the first and the third quartile. Once more, we observe that in the second quartile there is the majority of students with difficulties. The situation remains almost the same throughout the six semesters in which the undergraduate course in split. The main difference is that the students of the second quartile drop out at the end of the first academic year. Hence, frequencies in the corresponding histogram decrease.

Drop-out students

Let us now have a closer look at drop-out students. As outlined in our background, the phenomenon of drop-out may be addressed in three ways: (1) the impact with the University world at the very beginning of undergraduate studies, (2) the unsuccessful examinations outcomes, and (3) the justification students provide for the choice of not continuing the university studies in mathematics at the University of Torino.

Table 1 shows that above the TARM median the percentages of *successful* students is significantly greater than the number of drop-out. In the first quartile, almost the same number of students either drops out or takes the degree on time. In the second quartile, a *large majority* of students drop out at the end of the first year. For these students, the TARM message was that a tutorial course is suggested. However, data at disposal reveal that the majority passed less than two examinations in the first semester (the majority has no examinations passed), and the grades were not so high.

Career TARM	DROP OUT *later than the 2nd year*	DROP OUT *at the end of the 2nd year*	DROP OUT *at the end of the first year*	GRADUATE *on time*	GRADUATE *with 1-year delay*	GRADUATE *with 2-years delay*
I quartile	3.7%	2.8%	10.6%	9.8%	4.1%	2.3%
II quartile	0.9%	3.2%	15.7%	3.2%	1.4%	0.9%

III quartile	1.4%	1.8%	3.2%	10.6%	3.2%	0%
IV quartile	0%	0.9%	1.8%	17.0%	1.4%	0%

Table 1: percentages of leftover students, compared to successful students, with respect to TARM quartiles.

From table 1 it is possible to quantify the dropping-out phenomenon with respect to the aforementioned aspects (1) and (2). The peculiarity of such a situation is that students who dropped out were not, in principle, the students who had lack of knowledge at the beginning of the academic year. Thus, their situation should not be considered as a mere matter of lack of knowledge. Our hypothesis is that also non-cognitive factors, such as affective factors, may intervene. Qualitative data help us to frame the situation. The first questions we investigate are the following:

Where do dropping out students go? Do they enrol in another undergraduate course, or do they search for a job?

How do dropping out students describe their experience? Is it perceived as a failure? Which is, in the students' interpretation, the cause of drop out?

In order to answer these questions, we set up a phone questionnaire (figure 1). It was administered to 250 leftovers, 80% of them accepted to be interviewed. Only 0.05% of the 200 participants declared that they moved to another University because he was unsatisfied with the organization of the University of Torino. The huge majority of students is split into two groups: 40% changed the undergraduate course, and 60% of them went out the University world and search for a job. With respect to the first group, students' exact wordings have been classified into some categories:

a) Accumulated delay in passing the examinations (16 students)
b) Studies revealed to be more difficult than expected (14 students)
c) Studies revealed to be different, in content, from expected (14)
d) The choice of enrollment was not enough weighted (13)
e) I enrolled only to please my parents (1)
f) Courses were not interesting (6)
g) I was more motivated towards other studies (33)

Groups (a-b) address the *perceived difficulty*. Groups (c-e) involve an issue of orientation: students declared that either the undergraduate course differs from their expectations or their choice was not attentively considered. Students belonging to groups (c-g), in general, attribute their withdrawal to a *mismatch* between their experience and their expectations, but differ in the perception of the *responsibility* of that mismatch.

The second group of students went out the University world. The categories are:

a) Need for a job (26)
b) I accepted a job offer (25)
c) I interrupted my studies to search for a job (31)
d) Degree is not useful in the job market (2)

e) Mathematics does not offer interesting job opportunities (2)
f) Personal facts (32)

It is interesting to distinguish between those who leave university because of a job offer, and those that leave university and, afterwards, look for a job. In the first case, external motivations (need for money etc.) may have an influence, in the second case, the relationship with the academic courses seems crucial. A minority of students relate their choice with the pointlessness of a degree in mathematics in the working world. We may note that this could be a sort of defense mechanism: *I decided to leave university, and I tell to myself that I'm right*.

The aforementioned categories may be read in terms of causal attribution. All the reasons may be seen as stable and uncontrollable, while it is less easy to label the internal or external attributions. For instance, the delay in passing the examinations may be seen as internal or external, it depends on the causes for the delay. We may say that the delay is a first-order causal attribution, and that further investigation is needed to shed light on the perceived causes for such a delay. We also note that attributions such as difficulty of the courses and lack of interest for the courses are linked to beliefs about self as mathematical learners, about mathematics, about university, that are worth to be investigated further.

DISCUSSION AND FURTHER DEVELOPMENTS

In the present paper we presented the context of our study on undergraduate students' careers and shed light on the special situation of drop-out students. The analysis of quantitative data on school background and mathematical prerequisites, combined with the analysis of academic career, shows that dropping out is not a mere matter of lack of knowledge. In order to deepen the phenomenon of drop out, we also performed phone interviews. We hypothesize that affective factors may have a crucial role. At present, the analysis of phone interviews gives some information on the declared reasons for drop out. For instance, the issue of intrinsic/extrinsic motivation emerged, as well as the issue of expectation (as regards the content and difficulty of courses).

In order to shed light on this point, it would be necessary to investigate the nature and role of affective factors at the beginning of university, during the first examinations and throughout the whole career, with a special attention to the moments of interpretation of difficulties and perceived failures. The first point is that, as outlined in our background, affective factors should be addressed also before the moment of abandoning the university. Among the suitable means for monitoring them at the beginning of the university career, we explore the opportunity of inserting new items in the TARM questionnaire. As a first attempt in this direction, in collaboration with Laura Nota (University of Padova) the 2010/11 TARM questionnarie has been enlarged to include a set of items from the *Career Adapt-Abilities Inventory* (Savickas *et al.*, 2009), from the *Perceived Responsibility Scale* (Zimmerman & Kitsantas, 2005) and from the *Source of School Mathematics Self-Efficacy Scale* by

(Usher & Pajares, 2009). Only the latter subset of items, however, focused on mathematical self-beliefs (the analysis of the results of this questionnarie is currently in progress: data on the first-year undergraduate careers for these students should be available by september 2011). Studies on the view of mathematics and self-beliefs of mathematics learners (Hannula *et al.*, 2005; Roesken *et al.*, 2011) are a helpful reference in order to write more specific items.

Phone interviews to drop-out students could as well be sharpened by inserting into the structure of the interview more questions concerning motivation, expectations and difficulties. Theme interviews, as suggested by Pehkonen (2010), could provide deeper insight into the situation of drop-out students. When analyzing phone interviews, a methodological point to be addressed is the crucial difference between declared and perceived causes for difficulty (see the aforementioned defense mechanisms).

Other investigative tools could be useful. Narrative accounts of the first experiences at university could provide a wide range of information. Indeed, affective dimensions can fully emerge only through the use of qualitative tools, but one has to cope with the problem of applying such tools to a large population of students.

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UNDERSTANDING THE IMPACT OF A PROBLEM SOLVING COURSE ON PRE-SERVICE TEACHERS' BELIEFS

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There are a range of different programs which aim at changing pre-service- and in-service teachers' beliefs about mathematics and about mathematics teaching towards a process-orientation. Reflexive problem solving courses which involve journal writing are one possibility for reaching this goal. The research project 'Mathematics teachers as researchers' is looking into the effects of such a course. Interviews with participants were conducted in order to get a deeper insight into individual belief systems and their change and to reveal which role the different components such as problem solving and journal writing play in this change.

INTRODUCTION

Several programs for in-service and pre-service teachers aim at initiating a belief change on mathematics and on mathematics teaching. These efforts relate to a static view on mathematics and a concentration on rote learning that have been reported in research (Pehkonen & Törner, 1999). A reflexive problem solving course which includes journal writing is one possibility to improve teachers' competences, to change their beliefs about mathematics and about mathematics teaching and to let them reflect teaching models in school. The research program 'Mathematics teachers as researchers'(Bernack, Holzäpfel, Leuders & Renkl, 2011a, Bernack, Holzäpfel, Leuders & Renkl, 2011b)[1] is looking into the effects of such a course. The participants are German students who study to become primary or secondary school teachers. The course conception includes work on open-ended problems, their documentation through journal writing, and reflection. At the end of the course (running for one semester) four students have been interviewed to identify in detail the exemplary processes which they pass through and to get a thorough insight in their belief system. This paper focuses on the reported subjective beliefs and on the role that different components of the course such as like problem solving or journal writing have in initiating a belief change.

BELIEF CHANGE

General remarks

Beliefs on Mathematics and on Mathematics teaching of prospective teachers develop primarily during their own school days (Thompson, 1992). Then they are shaped by their experiences as student (ibid.). Belief systems are dynamic in nature and undergoing change. This change occurs by one's experience (Furinghetti & Pehkonen, 2002, Thompson, 1992). In order to determine to which extent something can have an impact on beliefs it is worth it to have a look on Green's (1971) definition of belief systems:

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[1] University of Education Freiburg, University of Freiburg, Germany; funded by the Federal Ministry of Education, Research Project number: 01JH0913

> We may, therefore, identify three dimensions of belief systems. First there is the quasi-logical relation between beliefs. They are primary or derivative. Secondly, there are relations between beliefs having to do with their spatial order or their psychological strength. They are central or peripheral. But there is a third dimension. Beliefs are held in clusters, as it were, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs. Each of these characteristics of belief systems has to do not with the content of our beliefs, but with the way we hold them. (Green, 1971, pp. 47–48)

Green remarks that the disposition to change certain beliefs is related to the psychological strength of beliefs. Those that are located peripherally are easier to change than those located centrally (Green, 1971; Furinghetti & Pehkonen, 2002). Pajares (1992) argues in the same way though he differentiates between early and late acquired Beliefs assuming that the first ones are more robust while the latter are more vulnerable.

Taking into account the characteristic 'cluster', Green asserts that clusters – being in conflict – can coexist (Green, 1971). However, the extent to which somebody links these clusters indicates to which extent they are potentially open to change (Grigutsch, Raatz, & Törner, 1998).

The relationship between beliefs and teaching practice must be considered as complex and it cannot be seen as a cause-and-effect relationship. Grant, Hiebert & Wearne (1998) describe how beliefs act as a kind of filter by which teachers observe and evaluate their own and other teacher's teaching. Hence it stands to reason to initiate a change in pre-service teachers' beliefs already during their university studies.

The question why exactly it is so difficult for teachers to assimilate their schemes and to internalize new ideas remains unanswered (Thompson, 1992). For that reason an important goal of research is to detect what happens cognitively when beliefs are changing and to identify those factors and circumstances that promote a belief change (Grigutsch et al., 1998). The next sections will therefore describe the requirements that have been theoretically and empirically found.

Sources and origins of beliefs

Furinghetti & Pehkonen (2002) and Ambrose (2004) describe different sources for beliefs in their papers. First, beliefs can be formed and changed through emotion-packed experiences which occur during learning, interacting or expressing goals and desires. Second, they can be culturally transmitted or they can be transmitted by others, especially by authorities. In this case people often adopt them unreflected. Third, individuals can reflect their beliefs and become aware of previously hidden beliefs. Finally, individuals "can have experiences or reflections that help them to connect beliefs to one another and, thus, to develop more elaborated attitudes." (Ambrose, 2004, p. 96)

Belief change in teacher programs

Several studies and teacher programs report effected belief changes, mainly towards mathematics as a process and towards a constructivist view on mathematics learning. They mostly can be characterized by giving the participants the possibility to experience to be a learner themselves by involving problem solving tasks, open ended problems or new mathematical content (Berger, 2005; Borasi, Fonzi, Smith, & Rose, 1999; Chapman, 1999; DeBellis & Rosenstein, 2004; Liljedahl, Rolka, & Rösken, 2007; Lloyd & Frykholm, 2000; Yusof & Tall, 1999). Some programs include teacher work with students to get involved in children's' mathematical thinking (Ambrose, 2004; Chapman, 1999). Some programs take one step further and include the development and implementation of lesson plans, often supported by teacher trainers (Borasi et al., 1999; DeBellis & Rosenstein, 2004; Lloyd & Frykholm, 2000). Most of the programs aim at making the participants aware of their beliefs as well as providing emotion-packed experiences to them. In almost all programs reflection is an important component. These reflections can be about the problem solving process, about the participants' beliefs or about the mathematical thinking of children. Especially the documentation of the problem solving process and of the reflections (e.g. through journal writing) has often been reported (Berger, 2005; Borasi et al., 1999; DeBellis & Rosenstein, 2004; Liljedahl et al., 2007; Lloyd & Frykholm, 2000). Additionally, different forms of communication such as group work and group discussions are usually implemented in such programs reporting belief change (ibid.).

DESIGN OF THE STUDY 2010

Reflexive problem solving courses

We base our course concept on the described teacher programs, especially on DeBellis & Rosenstein (2004) and on Berger (2005). The research project is embedded in a university course entitled 'Mathematical Thinking'. The students are working individually on open ended problems. They can easily start with working on the problem because the required prior knowledge for the problems is low. Moreover there are different paths of discovery due to the open-endedness of the problem. Students do not get any problem-solving related support or directions by the instructor. They can experience themselves as independent and self-regulated problem solvers. Students are required to keep records of the whole problem solving process including all thoughts, emotions, reflections and preliminary ideas by means of a notebook which is called "research journal". To the beginning and at the end the participants reflect on Mathematical Thinking by means of concept maps and on their beliefs by means of written reflections. In an experimental intervention design involving a quantitative pre-post study on beliefs and belief change (not reported in this article, cf. Bernack et al 2011b) some course participants get the opportunity to discuss their solutions and strategies in groups at selected points of time.

As one example for problems used in the course the problem ‘Step-Numbers’ is presented here (cf. Mason, Burton & Stacey 1991):

Problem 3: Which numbers can you write as the sum of consecutive natural numbers (e.g. 12 = 3+4+5)? Can you tell which numbers can be written in which different ways?

When you have worked on the problem to your satisfaction, ask some questions, e.g. “What happens if…?” or vary the problem.

On such a problem students usually work for about 3-5 hours. At the end of the course, four students were interviewed to detect which components of the intervention, such as journal writing and reflection, initiate a belief change.

Research questions

The goal of the interviews is to give answers to research questions which cannot be addressed by the collection and analysis of quantitative data. The focus of this paper is on the question in which way the particular components such as journal writing influence the students’ belief change.

METHODOLOGICAL ISSUES

Interview

The interviews are conducted in the form of guided interviews which begin with a more narrative part and then move to closed questions in the end. By using this method topics that are especially relevant to the students could be detected without influence by the interviewer. With the help of directed questions the topics of the interview can be better controlled and the individual answers can be compared between the interviews.

The method of analysis follows Schulz (Schulz, 2010) who developed the ‘Thematic Sequential Analysis’ which combines qualitative content analyses (cf. Mayring, 2000) with the documentary method (cf. Bohnsack, 2010). In the beginning one develops categories based on theoretical and inductive considerations. The interviews are then divided into text passages. Now these text passages are classified by the developed categories. Afterwards one writes descriptive summaries of the passages, one for each category per person. Only after this step the researcher starts hypothesising. Finally one compares and contrasts the single cases (Schulz, 2010).

The interviews took place at the end of the course. They were conducted with two male and two female participants. Each of the interviews took approximately 45 minutes. Two of them were chosen for being analysed and presented in this article as they offer a higher density in their statements and as they contrast very well.

Tom is 26 years old, male and studies to become a primary teacher (currently in his third year). Anna is aged 25 year, female, in the middle of her fourth year of a program to become a primary school teacher.

SOME RESULTS

In order to classify the text passages of the interviews, the categories were mainly created according to the theoretical considerations above. Additionally the characteristics of the course were taken into account. The result of this mostly deductive process is as follows: Quality of beliefs, experience with problem solving, reflection, partner discussion, documentation in the research journal, influence of authorities, future mathematics teaching. First I will compare in which way some of the components of the course influence the students. For this reason, the presented results mainly focus on the text passages dealing with the quality of beliefs, experience with problem solving, reflection, partner discussion and documentation in the research journal. Afterwards I will give a summarized overview of each person's belief system integrating all text passages.

Impact of some components of the intervention: Experience with problem solving

In the text passages analysed in the following the two students were talking about how they were working on the open ended problems and how they were feeling about it. Tom only refers to the type of problems and describes that he proceeded like a detective, investigating patterns and structures. He is also talking about the characteristics of the problems such as the possibility of different approaches. He describes this type of tasks as something new, but nevertheless seems to feel familiar with the process of problem solving which does not cause him any problems.

Contrary to Tom, Anna describes to have initially shied away from working on such open problems. So far, mathematical tasks for her always had a unique and 'correct' approach. Now she has to look for an appropriate approach and for the core of the problem by herself. She describes it as a drastic experience which she went through.

Interviewer: What's now different to you?

Anna: That, what I experienced now with 25 years, to sit in front of such a task which is in fact exciting and which offers so many possibilities, and to think "Shit!" what will I do now? And I found that terrible at the beginning, I was rather afraid of such a task and I think the reason is that I didn't know something like this before.[2]

This student does not seem to be familiar at all with this kind of problems. Her unfamiliarity leads to intense emotions she experiences like being afraid not to be able to work on the problem. Her intense experience and emotions help her to recognize that mathematics can be 'done differently' than according to her current conceptualizations and that mathematics can involve individually chosen and individually differentiated problem solving processes.

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[2] All italic interview passages in the article are translations of the author.

This leads to the conclusion that the choice of problems plays a decisive role in initiating a belief change. Those who already feel competent in problem solving get a deeper insight into the discovery process. Additionally they can reflect the characteristics of the problems. Those who still feel afraid and less competent get the possibility to overcome their fears and they get a first insight into mathematics as a process. It is such an emotion-packed experience for them that it seems to initiate a restructuring of their beliefs. Thus, the interviews confirm the relevancy of emotions (or ‘emotional disturbances’) acting as a driver to initiate belief change as identified in the reported literature above.

Impact of some components of the intervention: Documentation in the research journal

The differences between the two students described above also appear in the passages on the documentation in the research journal. Tom, when asked for the role of writing the research journal, mentions his difficulties to write down his problem solving process. As one advantage he explains that some ideas and approaches would have gone lost without writing them down. Additionally he could better structure his own thoughts. For him, the journal seems to be more helpful for an outside observer who wants to reconstruct the problem solving process. Asked if he had written emotions in his journal he denies.

Interviewer: Did you write down emotions, thoughts… or merely your work on the problem?

Tom: So, I didn’t write emotions in the research journal. That depends on the personality, I think. […] I’m a rather rational person, so I wouldn’t tell a journal in a course my emotions even if it’s about maths.

Anna on the contrary assigns a significant role to the documentation. As a result of the writing she experienced the course as something individual. With the help of the research journal she could overcome her difficulties and was no longer emotionally blocked. She positively evaluated having the possibility to document her emotions and reflections. She even tells that she could not have gone through the problem solving process at all without writing everything down because of being totally blocked. In this case she had not experienced mathematics as something individual and dynamic.

The two contrasting cases exemplify the different importance that can be assigned to the documentation. On the one hand the documentation in the research journal can put a focus on the problem solving process itself and in this way intensify the experience with an open ended problem. On the other hand, it can – for some students – principally make it possible to start and to go on with these problems. For this group the research journal plays a motivational role. Consequently, it can be a pre-condition for changing beliefs through problem solving.

Impact of some components of the intervention: Partner discussion

Tom mentions the partner discussion when he is asked for a formative experience during the course. He complains that the other ones in his group wanted to find a formula whereas he wanted to talk about the context, the solution and the approach. The partner discussion helped him to proceed and to gain an insight into the differences of the approaches.

Anna mentions the partner discussion as a formative experience as well. As well as in the case of journal writing she talks about her inhibitions. During the partner discussion she recognized that she was not the only one being blocked by the openness of the problem. Additionally, she got input from the group discussion to proceed with the problem

The statements of these two students identify undirected partner discussions to perform a mainly motivational function but with no direct impact on the participants' beliefs. They were not talking about beliefs in this case but about the motivation they got to go on.

Impact of some components of the intervention: Reflection by means of concept maps and writing tasks

Tom seems not to be sure if he should evaluate these two reflection forms as relevant to him. Rather, he sees the benefit to outside observers who can get an insight into his beliefs. But he also approves that the reflections can help making aware of the individual belief change – a function that he evaluates positively. Personally, however, he could not see a belief change concerning mathematical thinking: in his opinion, the only thing that changed was the task format. He also prefers to reflect on his beliefs while talking to someone other.

Anna explains that her beliefs would not have changed to the same extent, at least not consciously, without the reflection. It is important to her to make explicit documentations so that any belief changes can become aware to her (this was also described by Tom).

Accordingly, the reflection helps the participants to make aware their belief change or their personal belief system and its 'evolution' over the course. This result matches the statements found in the literature reported above (Ambrose 2004).

The students' Beliefs structure and their change through the intervention

Both students draw a detailed picture of their belief systems. Tom supports open ended problems and the possibility of different approaches. It is also important to him that pupils pass through the problem solving process. Looking at his goals for his future teaching and based on his statements about his experience with problem solving one would assign a view of 'mathematics as a process' to him. However, in different passages he refers to the task format and not to mathematics itself. In the following statement he describes this difference as following:

Tom: In my opinion, one can vary tasks so that there are more open facets, but the science itself isn't really open. Surely some findings are rejected what has been shown in the lecture, […], but that doesn't mean that the science itself is dynamic.

Tom: Nevertheless, only by choosing these tasks and by undergoing different processes, it's still my opinion that the science itself is static. So, I' m trough the tasks - thankful I don't want to say – I found this interesting and meaningful.

Consequently one could interpret that Tom holds a view on mathematics itself in one central cluster. His beliefs on the task format are held in another central cluster. His beliefs to the task format are based on experiences in several courses during his studies but they were as well shaped and further developed through the problem solving course.

Anna, who had substantial initial problems with the task format, recognized that mathematics can be a process and that there is an individual part in it. But she is separating her personal experiences while doing mathematics from mathematics as science. Mathematics as a science is still relatively static to her. After being explicitly asked about the relationship of those two different views she admits not having reflected them in a linked way. According to Green (1971) one could interpret that beliefs on the science are held centrally whereas to view mathematics and doing mathematics as a process and something individual is new to her. This aspect is held peripherally. One recognizes also that it is not that central to her as to Tom. Anna has problems to transfer this view and the concept of working on problems to her teaching practice whereas Tom can explain in a detailed way how he imagines working in school with such problems.

Hence, both student participants experienced a belief change through the course work. The belief change through the course has also been shown by the quantitative results as well (cf. Bernack et al. 2011b). Whereas Tom's beliefs were reshaped, Anna's beliefs were restructured.

DISCUSSION

The interviews reported here can only exemplify what happened to the whole student group during the course but they corroborate the quantitative results showing a belief change and give a more detailed insight into how this process came about. In particular, the interviews provide a detailed insight into how the different components of the course interact and which role the different components play – in general and for different student groups with varying prior beliefs.

The experience as a problem solver seems to be central while shaping or changing beliefs. The documentation of the process can intensify the experience and it can be a precondition to pass through this process - especially for weak and doubtful students. The partner discussion mostly plays a motivational role whereas the reflection helps getting aware of one's own beliefs because it supports the process of shaping or changing beliefs. In relation to the literature reporting sources for a belief change in a more general way one should take more precisely the role of these sources into account.

The interviews also revealed that it is difficult for the students to transfer their own experience with open ended problems to mathematics in general. Such a transfer problem can be linked to the cluster structure of beliefs and should give reason to support this transfer in such courses.

The initial findings and conclusions from the first interviews conducted and reported here should be revalidated and confirmed by further interviews conducted within a longitudinal design in which belief change can better be detected. Additionally, increasing the number of interviewees will help to get more extensively confirmed results and it should also include different types of participants. Such a design is planned for the following data collection within the research project.

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CONNECTING THE BELIEFS AND KNOWLEDGE OF PRESERVICE TEACHERS

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This paper reports pilot data from an instrument designed to measure the knowledge and beliefs of preservice primary teachers. The results of Rasch analysis showed that the participants found it easier to agree with belief statements that were consistent with the aims of the course, than to provide appropriate responses to content knowledge and pedagogical content knowledge items. It is suggested that pre-service teachers adopt the rhetoric of student-centred teaching without necessarily understanding the meaning or implications of belief statements that they readily endorse. Implications for initial teacher education courses including the potential role of beliefs in the development of pedagogical content knowledge are discussed.

INITIAL TEACER EDUCATION

Internationally, preservice teacher education programs vary widely in terms of prerequisites, relative emphases on mathematics content, mathematics pedagogy and general pedagogy and the extent and placement of practicum experiences (Tatto, Lerman, & Novotna, 2010). Consistent with trends noted by Tatto et al. (2010) in many countries, in Australia, calls for greater accountability for tertiary institutions have coincided with moves to national accreditation of teacher education programs and national standards for Graduate Teachers (Australian Institute for Teaching and School Leadership, [AITSL], 2011a; 2011b). In such an environment it is imperative that course developments are informed by evidence of what works. The project that forms the context of the study reported here aims to establish what constitutes effective practice in initial teacher education by gathering evidence of the relative impacts of various factors that influence program outcomes. The theoretical framework was adapted from that used in the international Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto, et al., 2008) and is shown in Figure 1.

The characteristics of future teachers encompasses the prior experiences, qualifications, attitudes and beliefs that preservice teachers bring to their teacher education programs, as well as their age, gender, socio-economic, and cultural background. We know that many primary preservice teachers bring negative and even fearful attitudes to mathematics to their university study (Beswick, 2006; G. Brown, 2009), and have weak mathematical knowledge (G. Brown, 2009; Mays, 2005). Tatto et al. (2010) also identified the characteristics of teacher educators as relevant to the outcomes of teacher education programs. Such things as their academic backgrounds, extent and recency of their teaching experience, employment arrangements, and their own knowledge and beliefs are all likely to be relevant.

Characteristics of programs in relation to the dimensions of variation among them noted by Tatto et al. (2010) as well as such things as the extent to which mathematics content and mathematics pedagogy are integrated, and the placement of practicum experiences in relation to mathematics units and the extent to which the two aspects are integrated are also likely to be relevant. In terms of program outcomes, the project is concerned with preservice teachers' mathematical content knowledge, pedagogical content knowledge (PCK) (as described by Shulman, 1987) and beliefs about the nature of mathematics, mathematics teaching and mathematics learning. These aspects and connections between them are explored further in the following section.

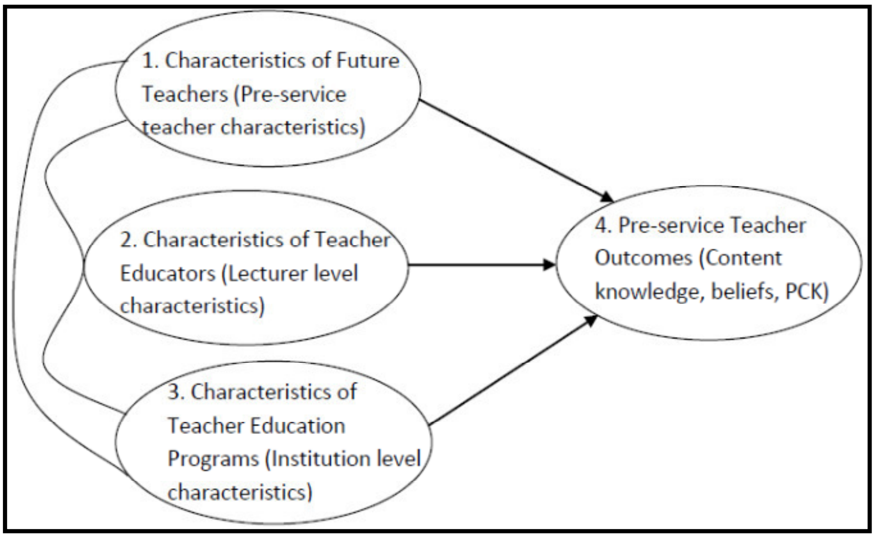

1. Characteristics of Future Teachers (Pre-service teacher characteristics)
2. Characteristics of Teacher Educators (Lecturer level characteristics)
3. Characteristics of Teacher Education Programs (Institution level characteristics)
4. Pre-service Teacher Outcomes (Content knowledge, beliefs, PCK)

Figure 1: Conceptual framework for improving the effectiveness of teacher preparation for mathematics teaching (adapted from Tatto et al., 2008, p. 15)

PRESERVICE TEACHERS' BELIEFS AND KNOWLEDGE

Many Australian pre-service primary teachers have difficulty with mathematical content at about Grade 8 curriculum level (Mays, 2005). Chick (2002) found, for example, that more than half of the subjects in her study could not correctly answer questions involving rates, and fraction and decimal equivalents. Internationally, it has been noted that even when pre-service teachers can competently use algorithms to perform calculations, conceptual understanding is often absent (Lubinski & Otto, 2004; Mewborn, 2001). The development of understanding of the mathematical content that they will be teaching is thus an important aim of pre-service teacher education. It is, however, just one aspect of the desired outcomes of initial teacher education. As illustrated in Figure 1, PCK evidenced by flexible access to and use of representations, examples, and explanations that make the subject comprehensible to students, is also an important part of the knowledge that graduating teachers should have. PCK has received considerable attention in recent years (Baker & Chick, 2006; Ball & Bass, 2000; Mewborn, 2001) and has been elaborated by Ball and colleagues in their conception of mathematics knowledge for teaching (Ball, Thames, & Phelps, 2008). Nevertheless only a few studies (e.g., Hill, Rowan, & Ball, 2005; J. M. Watson & Beswick, 2011) have linked teacher knowledge and student outcomes.

Teacher beliefs have similarly been considered an important area of research because of largely assumed links with practice and hence student learning. A. Watson and de Geest (2005) and Beswick (2007) described connections between quite broad but deeply held teacher beliefs, or principles, and the kinds of teaching practices employed and classroom environments created. At a much more detailed level Speer (2008) linked small collections of beliefs with quite specific practices of a college mathematics teacher. Beyond this parallel difficulty of linking with practice, it has been argued from a theoretical viewpoint that beliefs, defined as anything regarded as true, are a subset of knowledge (Beswick, 2011). Indeed the conceptualisation of knowledge used by Beswick, Callingham and Watson (2011) included beliefs and confidence in addition to Shulman's knowledge types and Ball et al.'s elaborations thereof. Their data suggested that this was reasonable at least for the middle school teachers involved in their study. In this study we were interested in exploring, (1) whether the various aspects of such an holistic view of teacher knowledge, operationalised by a particular instrument, together measure a single underlying construct, (2) how the development of pre-service teachers' abilities in relation to the various aspects of the this construct relate to one another, and (3) the relative difficulty that endorsement of a set of belief items presented for pre-service teachers.

T. Brown, McNamara, Hanley and Jones (1999) described how the transition from mathematics learner to mathematics teacher that begins with the commencement of initial teacher education requires a great deal of unlearning related to mathematical understandings and also to affective responses to the discipline. Many pre-service primary teachers enter their university study with powerful memories of confusion and struggle from their own experience of learning mathematics at school (T. Brown, et al., 1999) as well as the conceptual difficulties already discussed. T. Brown et al. (1999) suggested that lack of attention to the integration of cognitive and affective domains may be part of the reason that beginning teachers are often reported to teach as they were taught, and recalled McLeod's (1992) call for research incorporating both cognition and affect as a means to strengthen mathematics education research. The conception of knowledge adopted by Beswick et al. (2011) and examined in this study represents one approach to such integration.

RASCH MODELS

Rasch models use the interaction of persons responding to a test and the test items to estimate the abilities of persons in relation to items and conversely the difficulties of items in relation to persons. Person ability thus refers to the probability that a person will correctly answer items of a certain level of difficulty. Persons and items are compared in relation to a single underlying construct and placed on the same genuine interval scale in relation to that construct. The scale units are logits, the natural logarithm of the odds of success (Bond & Fox, 2007). In this study the partial credit model (Masters, 1982) was used.

The Rasch model has three requirements. Firstly, the underlying construct must be unidimensional. That is, the items work together to measure the same underlying construct. Unidimensionality means that Rasch measurement provides a means to investigate the extent to which different aspects of teacher knowledge can be thought of as working together to define an underlying construct. This is a major strength of Rasch modelling and was used by Beswick, Callingham and Watson (2011) to show that for the middle school teachers in their study various aspects of their knowledge, confidence and beliefs contributed to a single variable that they called teacher knowledge. The second requirement is that the underlying variable is measureable such that a higher value on the scale represents a greater quantity of the construct. Finally, the items contributing to the measure of the underlying construct must be independent of one another. That is, the probability of a person responding correctly to a particular item is independent of other items on the test and of the order of items.

THE STUDY

Participants

Fifty five preservice primary teachers, mostly approaching the end of their 4-year Bachelor of Education program, completed the online questionnaire. The preservice teachers were enrolled either in the second of two mathematics curriculum units in the program or in an elective unit designed for students wishing to strengthen their personal understanding of fundamental mathematics. Both units were run in the summer semester in fully online mode and approximately 10% of the preservice teachers enrolled were either catching up with the course because they had failed an earlier mathematics curriculum unit or had not reached an acceptable level of achievement on a mathematics competency test.

Instrument

The online survey included nine statements requiring respondents to indicate the extent of their agreement on 5-point Likert-type scales, along with a single item asking participants to rate their confidence to teach mathematics at the grade levels for which they would be qualified to teach on a similar scale. The belief items were drawn from those used in previous studies (e.g., Thompson, 1984; Van Zoest, Jones, & Thornton, 1994). The survey also included 13 multiple choice items designed to assess mathematics content knowledge (MCK) and a further 23 such items intended to measure the participants' PCK for teaching mathematics.

Data Analysis

In this pilot study the multiple choice content knowledge and PCK items were coded correct or incorrect. The belief items were coded 1-5 with '1' corresponding to strong disagreement and '5' to strong agreement with the relevant statement. The scale for the confidence item included five points from Not at all confident to Completely confident. Rasch analyses were conducted using Winsteps (Linacre, 2011), firstly with all 46 items and then with the 9 beliefs items and 1 confidence item only. The initial analysis allowed the extent to which the items worked together to measure a single underlying construct and, if so, to compare the relative difficulties of the various items and item types (MCK, PCK and beliefs). The analysis of the belief items and confidence item alone allowed a finer grained analysis of the belief items. The overall analysis provided the broader context within which the difficulty of the belief items could be considered.

The Rasch analysis was evaluated by a consideration of infit mean square values for both items ($INMSQ_I$) and persons ($INMSQ_{P)}$ (Bond & Fox, 2007). Generally accepted levels of fit lie between 0.77 and 1.3 logits (Keeves & Alagumalai, 1999) and have an ideal value of 1.0 logit. These statistics were available from the Winsteps output. The software also produces a Wright map of the variable, showing both items and persons on the same measurement scale. This output provides a visual picture of the relative difficulty of every item and relative ability of every person.

RESULTS AND DISCUSSION

Overall fit to the model

When all 46 items were analysed together, overall fit values were satisfactory for both persons and items ($MNSQ_P = 0.98$; $MNSQ_I = 0.96$) suggesting that the items from the three domains of beliefs, MCK, and PCK did provide a measure of a single unidimensional construct, Teacher Knowledge.

For the separate analysis of the nine belief items and single confidence item the overall fit values were also satisfactory ($MNSQ_P = 1.00$; $MNSQ_I = 1.00$), indicating that these items also provided a measure of Teacher Beliefs.

Individual item fit

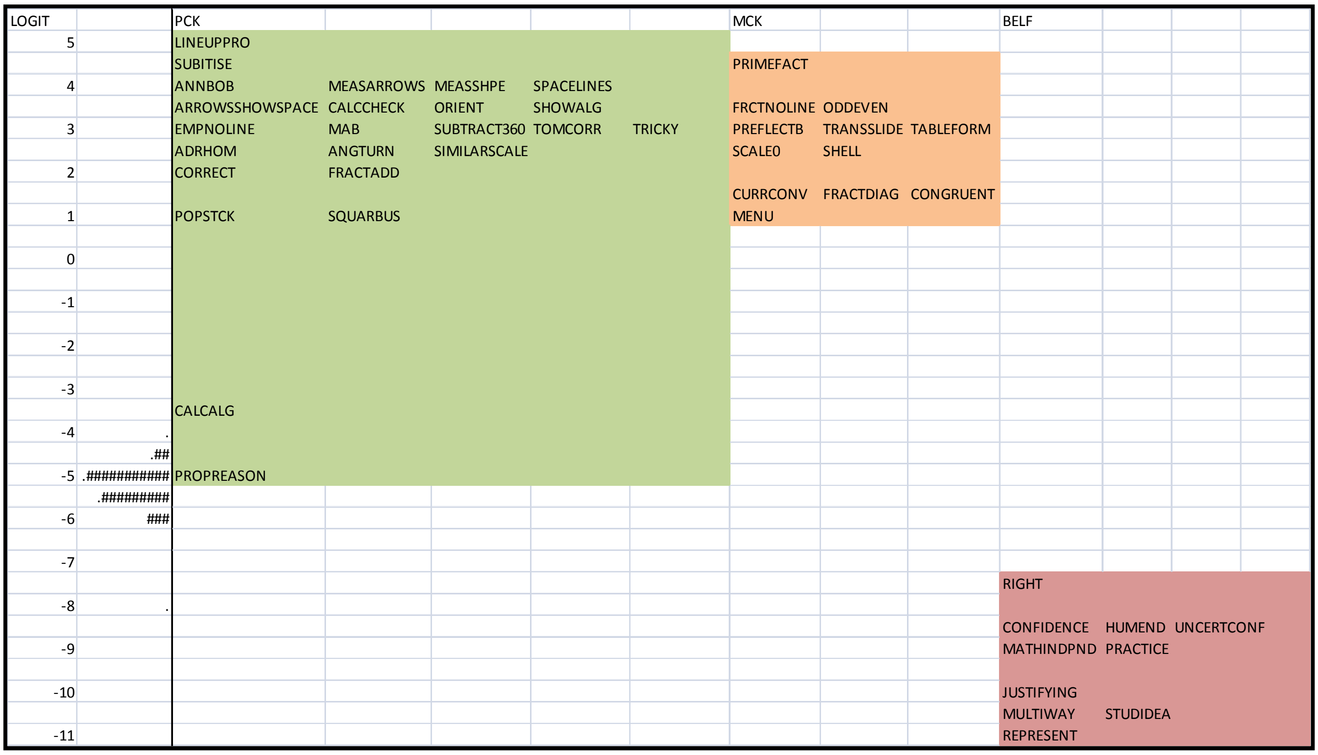
As an additional check on the coherence of the items, the fit of individual items was also considered. This test provides a more rigorous test of unidimensionality. For both the overall scale and the shorter beliefs scale, there was no item misfit confirming the presence of an overall construct of Teacher Knowledge, and a sub-scale of Teacher Beliefs.

The preservice teacher knowledge scale

Figure 2 shows the variable or Wright map for all items, separated according to item type (PCK, MCK, BELF). It is apparent that with the exception of one PCK item that related to proportional reasoning, the preservice teachers found the PCK and MCK items difficult. This is in contrast with the beliefs items. That is, it was much easier for the pre-service teachers to endorse the belief statements than to provide correct responses to the knowledge items.

The preservice teacher beliefs scale

The analysis of the belief items alone yielded the variable or Wright map shown in Figure 3. The scale shows the mean difficulty of the items. The pre-service teachers found it quite easy to agree that teachers must be able to provide a variety of representations of mathematical ideas (REPRESENT), should acknowledge that there are multiple ways of thinking about mathematics (MULTIWAY), and be receptive to students' ideas (STUDIDEA). The statement that justifying mathematical thinking is an important part of learning the subject (JUSTIFY) was also relatively easy to endorse. At the other end of the scale, the most difficult item to endorse was the statement that the procedures and methods used in mathematics guarantee right answers (RIGHT). Other relatively difficult items concerned confidence to teach mathematics (CONFIDENCE) and that mathematics is a beautiful and creative human endeavour (HUMEND). The next most difficult to endorse group of items comprised statements about mathematical ideas existing independently of human ability to discover them (MATHINDPND), periods of uncertainty and confusion being important for mathematics learning (UNCERTCONF), and the value for learning of practicing procedures and methods for performing mathematical tasks (PRACTICE).



LOGIT		PCK					MCK			BELF			
5		LINEUPPRO											
		SUBITISE					PRIMEFACT						
4		ANNBOB	MEASARROWS	MEASSHPE	SPACELINES								
		ARROWSSHOWSPACE	CALCCHECK	ORIENT	SHOWALG		FRCTNOLINE	ODDEVEN					
3		EMPNOLINE	MAB	SUBTRACT360	TOMCORR	TRICKY	PREFLECTB	TRANSSLIDE	TABLEFORM				
		ADRHOM	ANGTURN	SIMILARSCALE			SCALE0	SHELL					
2		CORRECT	FRACTADD										
							CURRCONV	FRACTDIAG	CONGRUENT				
1		POPSTCK	SQUARBUS				MENU						
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-7													
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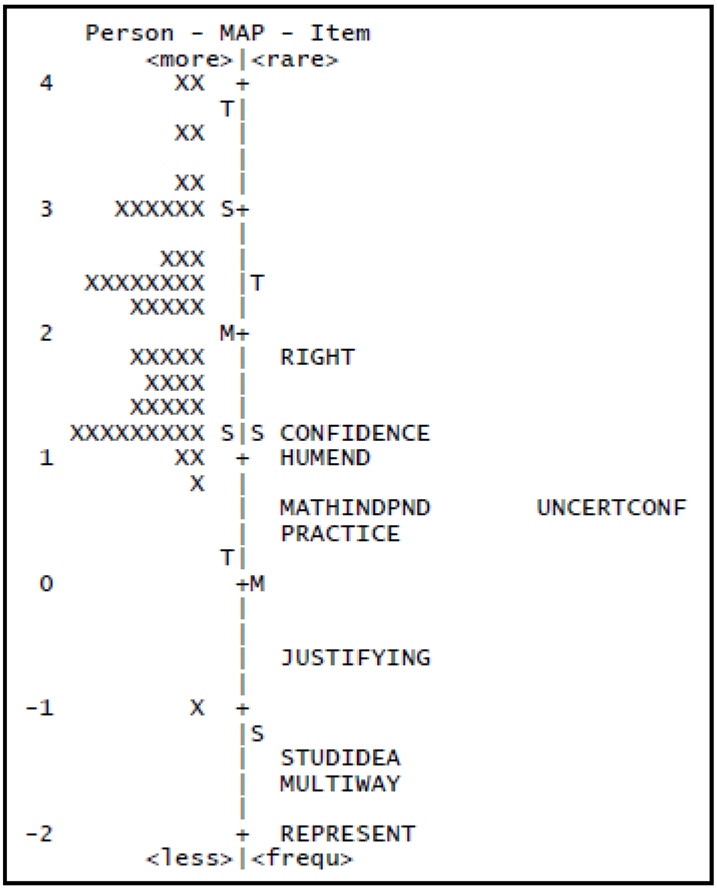
-8	.												
										CONFIDENCE	HUMEND	UNCERTCONF	
-9										MATHINDPND	PRACTICE		
-10										JUSTIFYING			
										MULTIWAY	STUDIDEA		
-11										REPRESENT			

Figure 2: Variable map separated according to item type. Note: Each # = 2 persons.

T. Brown et al. (1999) noted that a particularly strong theme among interviews with 20 pre-service primary teachers was that their own experiences of learning mathematics were dominated by a fear of failure and the need to get the right answer in spite of the mathematics being devoid of meaning. It seemed that the mathematics education pre-service teachers experienced as part of teacher education reduced their fear of failure with a major contributor being a reversal of the focus on the overriding importance of getting right answers. Although data collected in this study do not indicate whether a similar phenomenon occurred for these pre-service teachers, its existence would provide a plausible explanation for the difference in difficulty they experienced in endorsing the statement that mathematical procedures guarantee right answers (RIGHT) compared with items related to multiple ways of thinking (MULTIWAY), multiple representations (REPRESENT) and openness to student ideas (STUDIDEA). Rejection of the focus on a single correct answer and the emotional relief that would likely accompany it for many pre-service teachers, could lead to an attraction to statements that suggest a multiplicity of acceptable possibilities that is also emotive rather than having been rationally thought through.



```
    Person - MAP - Item
        <more>|<rare>
 4          XX  +
             T|
          XX  |
              |
          XX  |
 3      XXXXXX S+
              |
         XXX  |
    XXXXXXXX  |T
       XXXXX  |
 2           M+
       XXXXX  |  RIGHT
        XXXX  |
       XXXXX  |
   XXXXXXXXX S|S CONFIDENCE
 1          XX  +  HUMEND
           X  |
              |  MATHINDPND       UNCERTCONF
              |  PRACTICE
             T|
 0            +M
              |
              |
              |  JUSTIFYING
              |
-1         X  +
              |S
              |  STUDIDEA
              |  MULTIWAY
              |
-2            +  REPRESENT
        <less>|<frequ>
```

Figure 3: Variable map for beliefs items and confidence.

Similar reasoning could also explain why the idea that mathematical thinking should be justified (JUSTIFY) was more difficult to endorse than the group of items suggesting multiple ways. The statement may have connotations of pressure and being held to account that could conjure memories of unpleasant feelings experienced in mathematics classrooms such as T. Brown et al. (1999) reported from some pre-service teachers in their study. The value of uncertainty and confusion (UNCERTCONF) and practice (PRACTICE) to mathematics learning might also be relatively difficult to endorse for pre-service teachers with negative experiences of these aspects.

The items relating to the nature of mathematics as existing independently (MATINDPND) and being a beautiful creative human endeavour (HUMED) were both more difficult to endorse than most of the other belief items and reasonably similar in their degree of difficulty. These items are usually taken to represent Platonic and Problem solving views of mathematics and thus regarded as contradictory. The nature of the discipline is not directly addressed in the course and hence the most likely explanation is that the pre-service teachers had given little thought to these ideas.

Comparison of the two analyses raises the question of why the belief items were so much easier to endorse than the MCK and PCK items were to answer. It would seem to make the phenomenon of teachers teaching as they were taught in spite of teacher education that advocates different practices even more puzzling. How can this be when pre-service teachers so readily endorse progressive student-centred beliefs? These data, like those reported elsewhere, indicate that pre-service teachers struggle with MCK and PCK. Recognition of these difficulties has often lead to increased emphases on mathematical content and pedagogical skill particularly by regulatory bodies such as AITSL (2011a, 2011b) as well as in courses such as the one that provided the context of this study. Such efforts are likely to be useful but it is not yet apparent that they help beginning teachers to teach in accordance with the beliefs they endorse. An alternative explanation, raised by the analyses reported here but necessarily speculative, is that initial teacher education affords neither sufficient time nor opportunity for pre-service teachers to deeply understand the meaning and implications of belief statements that they find appealing at an emotional level. It could be that to impact practice the kinds of belief statements used in this study need to be unpicked and analysed so that stronger links with more cognitive aspects of pre-service teachers knowledge/belief systems can be formed resulting in their greater centrality. This in turn could provide a firmer foundation on which to build the sophisticated understandings of pedagogy (beyond endorsement of particular relatively superficial practices) that need to be integrated with their developing content understanding for PCK to develop.

CONCLUSION

This pilot study was necessarily constrained in scope and the issues it has raised require much more exploration. Interviews to be conducted in conjunction with the larger study may help to this end. Nevertheless, the use of Rasch measurement has shown that for these pre-service teachers an holistic conception of knowledge is valid and indeed has the capacity to highlight aspects of teacher knowledge that might not otherwise be apparent. In particular, it suggests that affective and cognitive aspects of teacher development are neither separate nor separable.

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ATTITUDE TOWARDS MATHEMATICS AND THE PLACE FOR MATHEMATICS IN STUDENTS' BELIEFS ABOUT LEARNING: RESULTS FROM A LARGE SAMPLE HUNGARIAN SURVEY

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INTRODUCTION

The aim of the current research was to provide data and insight about some components of the affective dimensions of mathematics learning within the framework of a bigger study that focused primarily on beliefs about learning in the fields of reading and foreign language learning, and on general epistemological beliefs. However, several items in one of the questionnaires administered pertained to mathematics; therefore some important and useful data could be provided for the mathematics education community.

The importance of the topic of investigating students' beliefs has been highlighted in several recent studies from different aspects. One important aspect is the role of general epistemological beliefs in school achievement as revealed among others by Schommer (1993), and Muis and Franco (2009). Hofer (2002) proposed a list of characteristics Students' beliefs about the source of knowledge and of the development and change in knowledge, and the system of their beliefs requires multidisciplinary and multidimensional considerations. From an educational point of view, Hofer (2002, p. 4.) provided a list of phenomena being in the focus of concern: "the definition of knowledge, how knowledge is constructed, how knowledge is evaluated, where knowledge resides, and how knowing occurs". In the field of mathematical beliefs, there is an agreement on the multidimensional characteristic of mathematical beliefs (De Corte, Op 't Eynde & Verschaffel, 2002), and in fact there have been several different factors found in empirical research (e.g., Andrews, Diego-Mantecon, Vankuš, Op 't Eynde & Conway, 2007). In a current investigation by Muis, Franco and Gierus (2011), the two-faceted nature of (graduate) students' beliefs in statistics has been empirically revealed.

A second important question to be investigated here is the role of attitudes towards school subjects since it may shape students' choice of professions, and have long-term effects on e.g. the new recruits to teaching mathematics. It has been revealed (see European Commission, 2007; Dieter & Törner, 2010) that there is a strong decreasing tendency in Europe in the number of mathematics, science and technology students in tertiary education. According to a large-scale Hungarian study (Csapó, 2000), mathematics – attitude measured on a five-point Likert scale – gradually becomes less and less popular during compulsory education from a meritorious 3.71 mean (SD = .99) in grade 5 to a mediocre 2.88 (SD = 1.05) in grade 11. In this study, no sex differences were found in the attitude towards mathematics.

A third rationale is whether mathematical knowledge items have a special status in children's belief about learning. Limón (2006, p. 23.) emphasize that "individuals may sustain different epistemological beliefs when [epistemological beliefs] are applied to different domains". The problem of domain-specificity of epistemological beliefs is to a great extent the problem of methodology. In a questionnaire where items from different content-domains are applied, there is an opportunity to empirically test different hypotheses about domain-specificity or domain-generality.

In the current investigation, questions of mathematical relevance in an omnibus study are shown and analyzed. The hypotheses listed below are arranged according to the sequence of questions in the questionnaire.

It was hypothesized that only some students would choose mathematics as their favorite school subject, all of whom have high marks in mathematics. On the contrary, many children would choose mathematics as their least preferred school subject, and this group presumably has varied math marks – not only students who struggle with mathematics dislike this subject. We also tested the differences in choosing mathematics as the most or least favorable subject with respect to students' gender.

According to the second hypothesis, students have strikingly expressed beliefs about the optimal age for learning different declarative and procedural knowledge items, including mathematical knowledge. The third hypothesis was that grade 7 students can make a difference in the durability of declarative and procedural knowledge component, e.g. they will know that people usually forget the definition of the rhombus in a relatively short time. Finally, three questions concern different aspects of choice of profession.

METHODS

The current investigation was part of a bigger project entitled "Students', teachers' and parents' beliefs about learning". In this research the so-called omnibus approach was used, i.e., subjects answered questions on different topics in a single session. One of the questionnaires administered contained questions of various types about different content domains including mathematics.

Questionnaire

One of the questionnaires in the investigation was entitled "Questionnaire about knowledge, learning, and school subjects" and contained 70 items of various types and content. Four groups of items pertained at least partly to mathematics. (1) Students were asked about their school marks in twelve subjects including mathematics and they were asked to name their favorite and their least preferred school subject. (2) There were different fields of knowledge given, and students had to decide which age is the most appropriate for learning them. (3) Ten items had to be judged as how long most people can keep them in memory. There were five pairs of items from five different fields of knowledge: music, mathematics, sports, geography and foreign language. In any of the fields, one item involved a mere factual knowledge element, and the other item concerned procedural activities. In mathematics, the factual knowledge item was the concept (definition) of rhombus, and the procedural knowledge item was the construction of triangles. (4) Some items elicited information on the choice of profession by enquiring about the degree of students' agreement with the following topics: students prefer school subjects that are taught by a rigorous teacher; one becomes a teacher of the subject he/she liked in the school very much; the 'favorite subject' play an important role in students' choice of profession; children watch films related to their 'favorite subject' in the television.

Sample

The whole sample of the project consisted of 605 grade 7 students representing the 13-14-year-old student population of Csongrád County (south-eastern part of Hungary). The sampling units were the schools; schools were randomly selected from a pool stratified by the size of the settlements where the schools were located. All grade 7 students of the selected schools have been tested. In the current investigation, the data of 570 students who filled in the "Questionnaire about knowledge, learning, and school subjects" were analyzed.

Data Analysis

Different groups of items required different statistical methods. (1) The open-ended questions concerning the favorite and the least preferred subjects were coded, and frequency values were computed. Correlations between school marks and school subject preference were also calculated. (2) Students marked their opinion about the most appropriate age for learning different types of knowledge in a matrix, hence enabling cluster analysis for knowledge items. (3) Similarly, the judgments of the length of keeping different knowledge items in memory were gathered in a matrix form. (4) The items about the choice of profession were judged on a five-point Likert scale.

RESULTS

Favorite and less preferred subjects

Table 1 shows the frequency values of students' answers concerning the open ended question: "Which is your favorite school subject?" As it was hypothesized, only 6.5% of the students named Mathematics as their favorite subject.

School subject	**Absolute frequency**	**Relative frequency (%)**
Biology	90	15.9
Chemistry	46	8.1
English as second language	44	7.8
Geography	13	2.3
German as second language	10	1.8
Grammar	9	1.6
History	50	8.8
Informatics	74	13.1
Literature	32	5.7
Mathematics	***37***	***6.5***
Music education	3	0.5
Physical education	101	17.9
Physics	20	3.5
Religious education	1	0.2
Technology	11	1.9
Visual arts	24	4.2

Table 1: Absolute and relative frequency values for the question “Which is your favorite school subject?” N = 565 (5 missing)

The most preferred subjects were Biology and P.E. Mathematics is in the middle in the rank of popularity. Table 2 shows the data about the least preferred school subjects.

School subject	Absolute frequency	Relative frequency (%)
Biology	13	2.3
Chemistry	34	6.0
English as second language	19	3.4
Geography	147	26.1
German as second language	14	2.5
Grammar	28	5.0
History	15	2.7
Informatics	16	2.8
Literature	13	2.3
Mathematics	***136***	***24.2***
Music education	15	2.7
Physical education	14	2.5
Physics	86	15.3
Ethics	1	0.2
Technology	7	1.2
Visual arts	5	0.9

Table 2: Absolute and relative frequency values for the question “Which is your least preferred school subject?” N = 563 (7 missing)

As for the least preferred subjects, three subjects have outstandingly high percentage of votes: Geography, Mathematics and Physics. As revealed in the Rocard-report (European Commission, 2007), there is a declining interest among young people throughout Europe for science and mathematics studies. The main reason is the methodology how these subjects are taught and not (only) the relative lack of success in school achievement. Table 3 shows how frequently students with different math marks named Mathematics as their favorite or least preferred subject.

School math mark	Relative frequency of naming Mathematics as favorite (%)	Relative frequency of naming Mathematics as least preferable (%)
1	0	8
2	3	33
3	3	33
4	32	22
5	62	4

Note:. In Hungarian schools students are assessed on a five-level scale, 5 indicating the best, and 1 indicating the worst achievement.

Table 3: Frequency distribution of naming Mathematics the favorite or the least preferred subject as the function of school math marks

As it was hypothesized those who named Mathematics as their favorite subject have usually good math marks. However, the frequency of naming Mathematics as the least preferable subject is relatively independent of math marks indicating possible instructional-methodological problems.

Looking at sex differences in choosing Mathematics as their favorite subject, 26 of the 37 students were boys, and 11 were girls. According to the binomial test, boys tend to choose Mathematics in a significantly higher rate than girls (p = .02). Nevertheless, there were 68 boys and 68 girls who named Mathematics as their least preferable subject, therefore in this respect no sex differences were found.

Appropriate age for learning

There were 10 different knowledge items listed, and students decided which age range is the most appropriate for the learning of each items listed in Appendix. The question was “When do you think it is easiest to learn the following things?” Students had to choose from five age intervals: kindergarten, grades 1-4, grades 5-8, middle school, and adulthood. One of the items was strongly related to mathematics: multiplication table 10x10. In accordance with their past experiences, 89% of the students chose the grade 1-4 interval, but 8% of them voted for the kindergarten years. There was no significant difference between these two groups in their math marks; t (536) = 1.16, p = .25. According to the results of cluster-analysis on all ten variables, the multiplication table item was connected to the big cluster of other items describing school-related knowledge, while an isolated cluster comprised of items learnt generally out of the school (car driving, airplane or sailboat driving).

Length of keeping different knowledge items in memory

Table 4 shows the arrangement of the question concerning the length of keeping different knowledge items in memory.

	In my opinion the majority of the people…			
	forget it within some days	*remember for some weeks*	*remember for some years*	*never forget it*
melody of a well-known folksong	18	51	139	357
writing music with ## key signature	70	129	284	81
construction of triangles	13	90	222	240
definition of rhombus	65	225	208	68
basketball shooting	4	27	91	440
place of the first Olympic games in the modern era	78	154	178	157
measuring distance on maps	36	86	145	294
capital of Italy	31	59	97	378
greeting in a foreign language	6	23	101	434
counting up to ten in a foreign language	9	17	131	411

Table 4: Frequency values for memory endurance items

Although the construction of triangles as a process during mathematics lessons involves several kinds of activities (e.g., according to the types of triangles), the optimism expressed in the results may point to the belief about the durability of procedural knowledge elements. On the contrary, the definition of rhombus as a mainly propositional knowledge component that consists of verbal propositional and visual imagery elements is thought to fall out of the memory within a much shorter period of time. Table 5 shows the two-table distribution of these two mathematical items.

Construction of triangles	Definition of rhombus				Total
	days	weeks	years	forever	
days	10	2		1	13
weeks	30	50	6	4	90
years	14	118	77	12	221
forever	11	54	124	50	239
Total	65	224	207	67	563

Table 5: Two-way table of the mathematical items in the memory endurance part of the questionnaire

Table 5 shows that there is significant connection between the two variables (contingency coefficient = .52, p < .001). It means that the two-dimensional distribution is significantly affected by the distributions of the two, theoretically independent variables. Those who thought that the definition of rhombus could be kept in memory for years of for ever (last two columns) considered the knowledge of construction of triangles even more durable. What is more, there were only 25 students out of 562 who considered the definition of rhombus to be kept for longer time than the procedural type of knowledge – the construction of triangles.

Items related to the choice of profession

At the end of the questionnaire there were three Likert scale items assessing the level of agreement on four statements related to the choice of profession. The first two questions explicitly mentioned connections between attitude towards school subjects and the teacher characteristics. The third question was expected to have a very high level of agreement.

	M	*SD*
Children usually prefer subject that are taught by a rigorous teacher.	1.89	1.11
One becomes teacher of the subject that he or she liked in school very much.	3.79	1.24
The ‘favorite subject’ plays a decisive role in students’ choice of profession.	4.42	0.88

Table 5: Descriptive statistics for items related to the choice of profession

As for the first question, the results show a very low level of agreement. Those students who named Mathematics as their favorite school subjects had significantly higher means in the second statement, Welch's d (49.71) = 3.38, p = .001.

DISCUSSION

In the current study, four hypotheses were examined. First, students' attitudes towards Mathematics as a school subject show the following pattern: Only 6.5% of the students named Mathematics as their favorite subject, and almost one fourth of them nominated Math as their least preferred subject. The latter group comprised of students from all achievement groups (as described by their school math marks), whereas those who favored Math belonged to the high achievers.

Secondly, the appropriate age for learning different knowledge components was measured throughout different domains. Students have clear and almost uniform belief about the appropriate age for learning the multiplication table, and this item belonged to the cluster of school-based knowledge items, whereas there was another remarkable cluster revealed about driving different vehicles.

Third, an interesting dichotomy of declarative and procedural knowledge appeared when asking students about the endurance of knowledge items. In the field of mathematics, the declarative element (definition of rhombus) was assumed to be less longer kept in memory than the procedural element (construction of triangles).

Fourth, the general question "who becomes a mathematics teacher" was examined by means of three statements. It has been revealed that the recruits to teaching mathematics will be probably those who favor mathematics as a school subject. Although Mathematics in Hungary is often considered as the subject of pure rigor, according to our data this image of the subject may hinder many students from choosing mathematics as the subject for the choice of profession.

The educational and practical relevance of the current study are summarized as follows. The high number of those children who chose mathematics as their least favorite school subject, and the fact that there are many students with good mathematics marks who dislike mathematics, has been revealed. Either by means of instructional methodological changes or by means of changes in the curriculum, we should aim for mathematics as being a more favorable school subject. It is clear that hardly anyone who disliked mathematics in the school will later search for the opportunity to learn higher mathematics in upper secondary and in the tertiary education.

Mathematics as a school subject currently possesses a distinguished place in the least favorable subjects. This characteristic is due to in great part the antipathy coming from high achievers. Nevertheless, high achievers do not regularly dislike school subject, so the unique place of mathematics in this respect is thought-provoking. It is imperative to develop and shape high achievers' attitude towards (and beliefs about) mathematics.

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Appendix

Ten knowledge items listed for the question „When do you think it is easiest to learn the following things?” [they were presented in alphabetical order in Hungarian]

car driving
biking
football
foreign language
basketball
counties of Hungary
airplane driving
computer use
10x10 multiplication table
sailboat driving

ETHIOPIAN PREPARATORY STUDENTS' PERCEPTIONS OF RELEVANCE OF PRIOR EXPERIENCES OF MATHEMATICS

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This paper reports results from a pilot study on preparatory students' perceptions of the relevance of mathematics. The analytical perspective used is cultural historical activity theory. Data are collected through interviews supported by classroom observation. Convenience and purposive sampling were used to select the school and students. Findings indicate that students perceive the relevance of prior experience of mathematics in different ways: as a conceptual relation, as an extension from previous level, as an increment of the level of difficulty, and as a reflection of the shift in identity. They also exhibit perceptions, which are characterized by trust for the curriculum and for the teacher. The languages and the contradictions; the school and the local communities; their role and the rules mediate their perceptions.

INTRODUCTION

Research indicates that students learn mathematics better when they find it relevant to their prior experiences (e.g. Heibert & Carpenter, 1992; Visnovska & Cobb, 2009), and their prior experience in learning mathematics affects their role in the classroom (Yackel & Cobb, 1996). Other studies show that the perceptions of students about the learning of mathematics affect their classroom practices and performances (e.g. Even & Tirosh, 2008, Moreno-Armela & Santos-Trigo, 2008, Nickson, 1992). The term perception is used to describe certain beliefs relating to personal competence (e.g. Nickson, 1992) and beliefs relating to classroom norms (e.g. Opt' Eynde, De Corte & Verschaffel, 2002). It is also used to describe the emergence of attitudes (e.g. Pepin, 2011). In this paper I use the term perception to refer to the meaning the students attach to the relevance of their mathematical experiences. Nickson (1992) used the term perception in explaining the meaning learners attach about the nature of mathematics. The definition of perception as well as the rationale for the choice of the term is discussed in Gebremichael, Goodchild & Nygaard (in press).

The students in this study are preparing for higher learning institutions (hence the school designation 'preparatory'). They attend this school following their successful performance in a national examination. Upon completion of the preparatory program they will sit for another national examination, and those who succeed in attaining the required grade will join the university. The textbook and the teachers' guide dictate what the teacher should do in the classroom. The teacher is evaluated for content coverage. This paper is based on data that was gathered for a pilot study. A preliminary analysis of this data is reported in Gebremichael et al. (in press). The purpose of this paper is to report a characterization of Ethiopian students' perception about the relevance their prior experiences of mathematics, which is intended to expose the perception in a particular context. The principal research question addressed in this paper is 'what are Ethiopian preparatory students' perceptions of the relevance of their prior experience of mathematics and how are these perceptions characterized'. The paper is structured in such a way that the theoretical perspective and methodology are discussed first, followed by the analysis presented within six themes that emerged through the analytic process. Finally, the discussion and conclusion are presented.

THEORETICAL PERSPECTIVE

Beliefs were considered as hidden variables (Leder, Törner and Pehkonen, 2002). Many researches have been done since then. At the moment, they are no more considered hidden variables in mathematics education (Goldin, Rösken & Törner, 2009). The lack of strength in theoretical foundation of affect research has been criticized by McLeod (1992). However, an agreed theory and model to study beliefs and affect remains an issue development (Hanulla, 2011). The theoretical perspective that guides this study is cultural historical activity theory (CHAT), which is one strand of sociocultural theory. Sociocultural theory is about the inherently social and cultural nature of human interaction, knowledge, and artefacts (Lerman, 2001). According to sociocultural theory the culture is a very rich store of things that shape the mind and the focus should be on the social individual (Cole & Engestrom, 1993).

The central issue in CHAT is that knowledge appropriation is a social process mediated by cultural tools such as language, with human activity as the unit of analysis (Roth & Lee, 2007). The model that can describe the situation is Engeström's expanded mediational triangle (Cole & Engestrom, 1993). It models activity within which human action is mediated by the tools, rules, division of labour, the community, and the contradiction between these elements and within them (Ibid). Students' learning of mathematics is mediated by tools and artifacts (textbook, television, language, etc; and their access to material resources and technology in their school as well as out of school practices); rules (curriculum, examination, grading, etc); the role she plays (whether the student is active or passive; ask & answer in the classroom); the community she is a part (the classroom community and the local community). The rules in the school activity system govern what the students and teachers should do (Jaworski & Goodchild, 2006). The textbooks are mediating artefacts which may or may not set learning experiences in a context that relates to students' prior experiences of learning mathematics. On the other hand, the textbooks and the lessons are in a language which the student uses at school only. Before the students were enrolled in lower secondary, they were using textbooks written in Amharic or other local vernaculars. The school language was also changed at this level. Contradiction might arise between the tools in the school activity system and the local activity system, which might lead to the development of certain perceptions. The students have been involved in the activity of learning mathematics long before they were enrolled in preparatory schools, and this activity has been there in the society long before the students come to experience it.

Roth (2007) used Engeström's model to analyze workplace activities, including mathematics. In particular he used this model to demonstrate the integrality of emotion, identity, and motivation to cognition, particularly to mathematical cognition. Roth (2007) is critical of the position that emotions are usually seen as negative factors that influence behavior. He argues that emotions are integral to mathematical activity (ibid). He considers emotion to be prevalent throughout practical action, while identity and motivation are byproducts of it. According to him emotions, identity and motivation mediate the learning of mathematics. In the process of learning the student develops identity, "who [one is] with respect to [oneself] and others" (Roth, Hwang, Goulart & Lee, 2005, p.3). As the students are enrolled into preparatory school they encounter a new mathematical experience, new environment. They come to this school from different schools and they are preparing for university studies. Thus, there is formation of new identity, resulting from negotiation of their old identity (Roth, 2007). According to Roth (2007) the student's motivation to undertake a certain action is directed towards achieving a particular emotional payoff.

METHOD AND METHODOLOGICAL REFLECTIONS

The research question and the theoretical framework lead to the development of a qualitative research methodology because it allows the investigation of the situation from the perspective of the participants. The intention is to understand students' perceptions of the connection between what they learnt before and at the moment. It is a case study design recognising that the knowledge gained is influenced by the peculiar culture of the setting. The data are collected through interview of students from a chosen school which has its own particular features although 'typical' of Ethiopian preparatory schools. Since the purpose is to see the relevance of mathematics through students' eyes it was chosen to conduct interviews that will enable one to hear their own account. Particularly focus group discussion was used where students were provided discussion points to expose their opinions, and probe them with further questions to enrich the data. Students of same sex from the same class were interviewed in the same group. This was intended to create a situation where the students feel secure among their own classmates, have lively discussion, and probe each other, as well as to engage with more informants in the time available. Classroom observation supported probing during the interviews, and exposed features of the mathematics classroom, which shows the context of the Ethiopian mathematics classrooms. I had taught for more than three years in the selected school and there are former colleagues who helped the selection of students. The topics and students' experiences might vary across gender, streams, grade levels, and achievement level. The department head selected 4 classrooms – 2 from social science and 2 from natural science (i.e. one from each of 11th and 12th grades) – where the mathematics teachers were homeroom teachers (the teachers having first line of responsibility for the students in a class) because they have better experience of the students and their academic standing. Then each teacher selected 3 female and 3 male students who were identified as low, medium, and high achievers. A total of 24 students were selected. Themes emerged from the analytic process, and each theme is analysed using CHAT as a framework for identifying key features.

DATA ANALYSIS

In this section the data analysis is presented as a descriptive account of students' perceptions of relevance, under six themes which are 'grounded' in the analysis.

Prior experience of mathematics is relevant because I trust the curriculum and the teacher

Some students' perception of relevance of prior knowledge is based on trust for the curriculum. Beza, a female 11th grade high achieving social science student, says, "I don't clearly remember but I think it is related". Fanaye's perception is based on trust for the teacher. She says, "The teacher tells us what we learnt before in relation to the topic, then we understand that they are related". She is a female 11th grade low achieving social science student. Her perception is mediated by her role as a student that she should listen to her teacher and accept what the teacher tells her. There is lack of power of using her agency to decide about the relationships between what she learnt and what she learns at the moment. The choice she is left with is trusting the teacher instead of proving for herself about relationships. On the other hand, Meseret, a female 11th grade medium achieving social science student, provides a specific example, she says, "We began [geometry] today; we were refreshing our memories of earlier grades. We expect to learn new things a bit later; this was how it went in logarithm". Her perception is mediated by the school curriculum as she is using her experience in learning logarithm. Her experience provided the agency to decide for herself about what is coming next. She had the choice of waiting for the teacher to tell the relationship, as Fanaye did. But, she exercised her agency to decide for herself about the learning of mathematics and the trend it follows based on her experience and knowledge.

Prior experience of mathematics is relevant because it extends from the previous

Some students perceive that their prior experience of mathematics is relevant to the preparatory mathematics as it expands from level to level. Alewia, a female 11th grade medium achieving natural science student, says, "it is the same but it expands from one level to the next". Whereas Debesh, a male, 11th grade, high achieving social science student considers specific examples, "not between what we learnt this year but with previous ones. Logarithm and exponential has become wider. Rational was in 9th now wider. Geometry also we learnt it before, and now with some additional concepts". He perceives that the relationship is not prominent between the concepts they are learning at this level but across grade levels. His perception is mediated by the curriculum as well as his role as a learner. Other subjects as well mediate students' perceptions. Ruth, a female 12th grade high achieving social science student, gives additional examples, "yes it is related, also in other subjects it expands as we go up. For example, limit was in sequence, now we are learning it as a topic by itself, and broadly. So, they are related." Asad, a male 12th grade medium achieving natural science student considers specific examples, “It is like branching; if we don’t know the previous we could be confused to understand the current”. Their perceptions are mediated by the school curriculum, which sets the topics in order, and Asad’s perception in particular is mediated by the artefacts in the community. The society also uses metaphors of branching, and stem and branches to explain relationships.

Prior experience of mathematics is relevant because some concepts are vital

Some students’ perceptions of relevance is characterized by consideration of some vital concepts. Netsanet, a female 12th grade medium achieving natural science student, says, “Those coming earlier are the basis for the next. Domain, etc. we learnt before and use them now”. Yirdaw, a male 12th grade high achieving social science student, says, “If we don’t know some of the concepts before we couldn’t learn the current”. Meada, a male 12th grade high achieving natural science student, says, “Derivative is derived from limit, and in limit we had rational function from 11th grade, so they are related”. Their perceptions are mediated by the school curriculum. Some students perceive that some of the topics in the present grade level are not interrelated. Fanaye, a female 11th grade low achieving social science student, says, “I don’t think the polynomial & rational are related to geometry. … [in geometry] we were able to answer his questions because we learnt it before”. Thus, for her some of the relationships are across grade levels. Her perception is mediated by the school community, the curriculum and her role as a learner.

Prior experience of mathematics is relevant because it has become more difficult

Some students’ perceptions of relevance are in terms of the level of difficulty. Fantu, a female 12th grade low achieving social science student, says, “Before it was little, easy, now it is becoming difficult as we go up in grade levels, and it has become wider.” Her perception is mediated by the school curriculum. Azenegash, a female 12th grade medium achieving social science student, relates it to her capability; she says, “As our capacity increases the subject is correspondingly developing …. It was in Amharic before. Now, with similar things but in English” [1]. Her perception of relevance is mediated by the school curriculum as well as her role. Her perception of relevance is mediated by the language as well as the school curriculum, which repeats the concepts in English which were thought in Amharic before. It was reported in Gebremichael, et al. (in press) that Debesh dislike word problems because they are difficult to understand and use difficult words. The local and the school activity systems use different languages, and offer her/him challenges. Contradiction arises between the two languages that s/he uses at school and in her/his daily life. We see that the students are struggling with understanding the mathematical language and concept as well as the English language. The contradiction between the tools might lead to considering mathematics as being far from real life. Whereas, Ibrahim’s perception is mediated by the local community; he says, “Our society tell us that it is difficult next”. He is a male 11th grade low achieving natural science student. It is also important to note that Ibrahim’s local community is exposed to the school curriculum.

The fact that mathematics is the gate keeper for joining the university and the attachment of academic success as a way of overcoming one's and family's problem might have contributed to the attention that mathematics is given by the local community (Gebremichael et al., in press).

Prior experience of mathematics is relevant because it reflects ones identity

Some students perceive that the relevance of prior experience of mathematics is reflected through the shift in their identities. Erikihun, a male 12th grade medium achieving social science student, says:

> I was not working well in grades 9 and 10. Thus, feel that I have missed some from those grade levels. For example, when I am learning geometry like sine, I see that I have missed something before, and I am missing it now again. Thus, they are related with previous grades.

His perception of relevance is mediated by the school curriculum, which insists that the student has to cover specific contents, and be evaluated for this coverage through examinations. His role in his classroom as a learner of mathematics also mediated his perception. He is paying attention to what he is learning and he is a medium achiever, now. Though there is a shift in his identity, he still couldn't perform well on areas which he perceives are related to what he has missed in previous grade levels. On the other hand, Meseret discusses about the prior teacher she had and her identity that she built in relation to mathematics. She says, "We had a mathematics teacher… He made us addicted to it". She is expressing her identity as member of a group of students in the classroom, whose identity in relation to mathematics had changed because of the teacher. Her perception is mediated by the school community, and the curriculum.

DISCUSSION

As long as there is life there is activity (Leont'ev, 1979). The learning of mathematics is an activity which has existed long before the students came into being. At the same time the students have been involved in the activity of learning mathematics long before they were exposed to the school mathematics. In this paper two activity systems, in which the students are involved, were identified: the local and the school activity systems. The school activity system is prominent. The characterizations of perceptions of relevance to prior knowledge have the dimensions mentioned as categories above: trust for the curriculum; extension from the previous; some concepts are vital; it has become more difficult, and it reflects one's identity. These dimensions are grounded in the data.

Rote memorization is a tradition in Ethiopian traditional schools [2]. The Ethiopian culture is such that the elder cares for the younger and the younger obeys to and follows the elder. At the same time, it encourages silence and a tendency that 'the elder is right'. The culture of the society which is embedded in the teachers and students practices, mediate the mathematics classroom culture (Bishop, 1988; P. Matos, personal communication, July 2010). The students rarely ask questions. The participant students expressed a tendency of not asking the teacher, but colleagues or senior students. The teachers give lecture starting with revising what they have taught in the previous lecture. They ask questions and decide whether the answer is right or wrong. The teacher gives the solution for the exercises, and the students copy it from the blackboard. The teachers rarely ask for further reasoning from the students, nor use students' responses for explanation. Sometimes wrong answers are not welcome. Cobb points out that teacher's authority is because of the assumption that the answer to a mathematical problem is right or wrong (in Heibert & Carpenter, 1992). Students rarely get the chance to express what they remember from previous lesson. Beza's trust for the curriculum that she doesn't remember but thinks that it is related, seems to be associated with this classroom culture. Trust has cultural roots in the society and school, and it is encouraged so as to create a conducive environment for the teaching-learning process (Lasky, 2005, Pace & Hemmings, 2007). Trust for the teacher and the curriculum that the participants expressed might be linked to the culture of the society; the classroom culture and the economic situation of the students and the country at large. It seems that the culture and the school rules are depriving the students of their agency which might have made trust an option. Other studies show that students' trust for the teacher prevails among students (e.g. Lasky, 2005; Pace & Hemmings, 2007). One of the factors for their trust is "how much the students perceive that their education will be of value to them" (Pace & Hemmings, 2007, p. 8). It is reported in Gebremichael et al. (in press) that mathematics has an exchange value. The students' perceptions of relevance expressed in the analysis by 'extension of the previous' and the presence of 'some concepts are vital' demonstrates the students use of their own agency to make their own reflections and interpretations about relationships between mathematical concepts they learnt before and at the preparatory level.

Language was mentioned as a mediating artifact. Before upper secondary, the students learn mathematics in Amharic or other local vernaculars. Setatic & Adler (2000) explain that teaching in a foreign language is a challenge. Moreover, the code-switching presents an obstacle in the teaching learning process (Ibid). As reported in Gebremichael, et al (in press) some students do not like word problems because of the difficulty in understanding the language. Thus, there is emotion associated with the language use. The students' perceptions of relevance of mathematics is mediated by the local community. Ibrahim's experience is exemplar. Among the local community education is valued as a means of improving one's life as well as family life and mathematics is a gate keeper in the joining of the university (Gebremichael, et al., in press).

In their perception of relevance, students tend to exhibit a sense of identity both in relation to future goal as well as prior knowledge (see Gebremichael, et al., in press). They also expressed emotion associated with their identity. This is in agreement with Roth (2007). In Erikihun's story, his identity has changed from being a student who never paid attention to mathematics to one who pays attention to it and recognized by his teacher as a medium achiever. Still he has got a feeling that he cannot cope with mathematical tasks relating to his prior experience. The mental representation of the concept of trigonometry is not with mathematical meaning only. It gives him a sense of identity. He is also anxious and has little confidence associated with this concept. It was reported in Gebremichael et al. (in press) that Beza became a high achiever because of her achievement in mathematics in the national examination. This gave her the lesson that working on mathematics pays off, and she was motivated to work on mathematics. Her identity is changed her prior experience is the basis for her identity. This identity is associated with motivation and confidence. Other studies show prior experience identity are associated with motivation and achievement (Mulat & Arcavi, 2009). As it is shown in Roth (2007) emotion, motivation and identity are integral to one another.

CONCLUSION

The students' understanding of the connections between prior experiences of mathematics and preparatory mathematics should not be seen as having one aspect only – having to do with conceptual relations. It has many dimensions as discussed above. The students' perceptions of relevance of prior knowledge are mediated by the structure: the curriculum, the community, the language, and the rules, which seem to be out of their control. The teacher has to use a textbook, and should cover a wide topic in an academic year and s/he is evaluated for that. This seems to hamper the teacher from using various modes of teaching that encourage students' participation and give a room for the student to exercise her/his agency. The school rules are found to be strong mediators of students' perceptions of relevance of prior experiences. In some cases, the kind of trust prevailing among the participants seems to be more of dependence and might not enhance students' use of their own agency and criticality. It requires attention and further investigation.

It is difficult to make conclusions about relationship between the different categories of students included in the sample and different characterizations of perceptions because of the small sample size. The availability of such trends can be checked by the analysis of the questionnaire data which is underway. The results in this study have implications for teacher education and teachers practices in the classroom, because it gives hint for further investigation. The teacher needs to get feedback from the students about how the teaching and learning is going, in particular how students relate the mathematics they are learning to themselves. Students' stories about their learning are important part of the diagnosis and solution to their difficulties in learning mathematics (di Martino, & Zan, 2010).

Note:

1. In the Ethiopian education system students learn primary education using Amharic, which is the national language of Ethiopia, or using the local vernacular, and secondary education is in English.

2. In both the Ethiopian Mosque and Ethiopian Orthodox Church there is a very long history of traditional education, before the introduction Western Education in Ethiopia a hundred years back. The traditional education still exists to date, which plays the role of exposing children to reading and counting. Many of students' parents or grandparents are products of these traditional schools.

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MATHEMATICAL BELIEFS AND COGNITIVE REFLECTION: DO THEY PREDICT ACADEMIC ACHIEVEMENT?

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INTRODUCTION

In the past decades, more and more scholars point to the key role of affective factors as constituting elements of the learning process, beside and in close interaction with metacognitive and cognitive factors. On a conceptual level, researchers try to capture the interrelated influence of (meta)cognitive and affective factors on mathematical learning and problem solving alongside a notion of “mathematical disposition” (De Corte, 2004; Schoenfeld, 2002). They highlight that students should acquire the mathematical disposition to become competent in mathematics, and that such a disposition would require the mastery of five categories of aptitude: (1) mathematical knowledge, (2) heuristic methods, (3) meta-knowledge, (4) self-regulatory skills, (5) positive beliefs about oneself in relation to mathematical learning and problem solving, and beliefs about mathematics and mathematical learning. They point out that much of the complexity of learning and teaching mathematics is due to the interconnection that the student must establish between these aptitudes. However, the difficulty lies in determining the operational elements that encourage this connection. In this article, we present a detailed discussion of two categories: beliefs about oneself in relation to mathematical learning, problem solving and mathematics; and self-regulatory skills (cognitive reflection).

In addition, another reason for considering these variables is the distinction between two types of cognitive processes in reasoning and judgement: those executed quickly without conscious deliberation, and the slower and more reflective ones (see, for instance, Evans, 2007; Evans & Over, 1996; García-Madruga, Gutiérrez, Carriedo, Luzón, & Vila, 2007; Kahneman & Frederick, 2002). Stanovich & West (2000) called these dual processes “System 1” and “System 2” processes, respectively. Processes belonging to "System 1" occur spontaneously and do not require much attention. They are also automatic and non-reflexive. However, solving mathematical problems requires an in-depth understanding of the problems. Self-regulatory implementation of specific strategies for resolution, that is, “System 2”, is probably necessary. In our research, we study beliefs about mathematics, the way it is learned, and its relationship to the metacognitive measure of System 2 analytical processes: Cognitive Reflection, as proposed by Frederick (2005). Finally, this research will provide two instruments. One will show that test scores of cognitive reflection and beliefs about mathematics, and beliefs regarding one's confidence in mathematical competence, are predictive of higher achievement levels in math. The second instrument provides future high school teachers the use of this study's results in their systematic assessment of this discipline.

Two hypotheses were considered in this research. First, we expect that individuals with higher scores on cognitive reflection will possess more positive beliefs about mathematics and learning. Second, we expect that both variables will predict academic performance in the mathematical levels of individuals.

The discussion in the article will proceed as follows: First, we highlight some theoretical issues underlying the study from the past up to the present. Then, we discuss the methodology and results. Finally, we summarize the conclusion and possible research topics for the future.

THEORETICAL FRAMEWORK OF THE STUDY

We present the theoretical framework underlying the study structured according to the following sections:

Belief systems about mathematics and learning

Many studies about beliefs in recent years might be emphasized; see for example: Leder, Pehkonen, & Toerner (2002) or Maass & Schloeglmann (2009). In this study, we consider an unifying framework for research on students' beliefs provided by Op't Eynde, De Corte, & Verschaffel (2002) that allows a better understanding of the interactions between different types of beliefs. These authors point to three basic components that constitute a belief system: beliefs regarding the social context, the self and the object. They then provide a Mathematics-Related Beliefs Questionnaire (MRBQ) framed according to this perspective. In previous studies (Gómez-Chacón, Op't Eynde & De Corte, 2006a, 2006b), we realized the need of operationalizing the MRBQ questionnaire for Spanish students.

The questionnaire was designed as part of a research project that aimed to analyze the relationship between several cognitive factors: working memory, reading comprehension, cognitive reflection and deductive reasoning, and mathematical belief systems. This led to the prioritization of four dimensions in the development of the questionnaire, three of which are covered by the previously mentioned studies (a, b and c). We also added another non-integrated dimension found in questionnaires such as the MRBQ (d). The dimensions are: a) students' beliefs about mathematics, b) students' beliefs about learning and solving math problems, c) students' beliefs about one's self (beliefs about the meaning of one's personal competence in mathematics, i.e., student confidence and the perception of their own ability), and d) an affective and behavioral dimension regarding their commitment to individual mathematical learning. Regarding this last dimension, we would like to remark that we have considered two aspects of engagement: affective and behavioral engagement in mathematical learning. Experts like Fredricks, Blumenfeld, & Paris (2004) provide a comprehensive overview of literature relating to school engagement in general. In our context (i.e., learning the discipline of mathematics), we refer only to the part of school engagement in the cognitive domain of mathematics. It is within this area where we decided to examine how students feel about the discipline (affective dimension of commitment) and how they behave when learning the subject (the commitment expressed in behavior).

In summary, we have tried to highlight the importance and appropriateness of assessment tools for investigating beliefs in mathematics teaching and learning. This complex structured system has both cognitive and affective implications for the individual that affects their behavior and mathematics performance.

Cognitive Reflection

For their proper solution, arithmetic problems often require a deep understanding of the problem. The Cognitive Reflection Test (CRT) used in this study is an adaptation of the test presented by Frederick (2005), which aside from the three problems he used in his original test, also includes two additional problems proposed and used by the author. The test tries to evaluate participant depth of reasoning in a simple task of mathematical reasoning, such as the following (problem 1): *A bat and a ball cost $1.10. The bat costs $1.00 more than the ball. How much does the ball cost?__cents.* Faced with this kind of mathematical problem, subjects tend to give an impulsive answer that comes quickly to mind: “10 cents”. However, this response is wrong as a bit of reflection will clearly show: the difference between $1.00 and 10 cents is only 90 cents, not $1.00 as the problem stipulates. The correct answer, “5 cents”, comes now easily to mind and hence demands that individuals act in a reflective way.

The problems of Frederick (2005) provide a simple measure of a very important metacognitive ability: cognitive reflection in solving problems. Thus, the test measures the aptitudes of individuals in controlling their behavior in a reflective way and not providing the first answer that comes "to mind". The author investigates the relationship of CRT alongside aspects of decision making, and he stresses that those individuals with lower cognitive reflection abilities tend to choose an incorrect first “intuitive” choice.

CRT test problems “appear” simple for participants, but they are actually quite difficult. In one study with a simple of 3,428 students from top U.S. universities, the overall percentage of correct answers was 41% (see Frederick, 2005). The reason for this difficulty lies in the fact that in order to respond correctly, participants must first inhibit their initial “intuitive” response in favor of a deeper and more thoughtful one.

According to Frederick (2005), the idea that CRT problems generate incorrect intuitive answers is mainly based on 3 facts: “First, among all the possible wrong answers people could give, the posited intuitive answers dominate. Second, even among those responding correctly, the wrong answer was often considered first, as is apparent from introspection, verbal reports and scribbles in the margin. Third, when asked to judge problem difficulty (by estimating the proportion of *other* respondents who would correctly solve them), respondents who missed the problems thought they were easier than the respondents who solved them" (Frederick, 2005, p. 27).

In explaining the actions of individuals in the Cognitive Reflection Test, Frederick (2005) puts forth the dual-process theory of reasoning and decision making. This distinguishes two types of cognitive processing: System 1 and System 2, as proposed by Stanovich & West (2000). According to the dual-process hypothesis, spontaneous responses both rapid and involving a superficial understanding of the problem are produced by System 1. On the contrary, the correct answers would involve the performance of System 2; they would also involve cognitive effort, motivation and concentration. Therefore, these would take longer to solve.

CONTEXT AND RESEARCH METHODOLOGY

This paper presents research conducted on 56 high school science students (mean age = 16.58, *SD* = 0.64), and had two objectives: a) to assess their belief systems in mathematics and their depth of reasoning, and b) to analyze the interrelationships between these two variables and how they might be predictive of mathematical performance.

Hence, we carry out two sets of analyses. First, we did a factorial analysis of the "CreeMat Questionnaire" to identify the major types of mathematical beliefs. Then, we used correlation and multiple regression to explore the extent to which the predictor variables under examination correlated with and accounted for the variance in mathematical performance.

Description of the questionnaires used

Mathematical beliefs questionnaire (CreeMat)

The beliefs questionnaire (CreeMat) is a Likert-style scale (Gómez-Chacón & García-Madruga, 2009). The purpose of this test is to evaluate systems of beliefs about mathematics in high school students. Its relevance lies in the predictive value this measure has in understanding the structure and nature of belief systems and academic achievement in mathematics. This questionnaire consists of 13 items covering different subscales or dimensions: affective and behavioral engagement in mathematical learning (items 1, 2, 3); confidence and beliefs regarding one's personal competence in mathematics (items 4, 6, 8, 12); mathematical beliefs (items 5, 9, 13), and beliefs about mathematical problem solving (7, 10, 11) (see Table 1). The distribution of the questionnaire score is in a score-sum format such that responses are distributed amongst five options that go from "strongly agree" to "strongly disagree". Items are scored from one to five.

Cognitive Reflection Test (RC)

We used an adapted version of Frederick's (2005) Cognitive Reflection Test with 5 items. Given that our participants were not university but high school students, we introduced some modifications to try to avoid a possible floor effect and to make the test easier. The task is presented as a "pencil and paper" test, which consists of a sheet of paper with five arithmetic problems such as the following (problem 4):

In a pond, there is a patch of lily-pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire pond, how long is it going to take for the patch to cover half the pond? a) 47 days; b) 24 days; c) Nobody knows.

As can be observed, contrary to Frederick's (2005) original test in which the participant task was that of recall (i.e, they had to write the correct conclusion on their own), in our task, the participants could choose amongst three alternatives: the correct alternative, the superficial alternative and the response "Nobody knows". We think that this modification will reduce intuitive superficial responses and increase reflective correct ones. After solving five problems, as an estimation of their difficulty each participant was asked about the percentage of peers who, in his or her view, was able to solve each problem correctly.

RESULTS

Belief systems about mathematics and learning

From the descriptive analysis of items from the CreeMat questionnaire (see Table 1), it should be noted that the group shows an acceptable level of confidence regarding their own personal competence (item 8); it is of average value and recognizes their effort in learning. However, the students reported that when asked to solve math problems, they often become nervous (item 6). This illustrates a low level of interest and enjoyment in developing their creativity in problem solving (item 3).

Table 1: Mean and standard deviation for the 13 items and total scoring of CreeMmat.

Item	Mean	*SD*
1. I work hard in Math (subject).	3.13	1.16
2. If I make mistakes, I work to correct them.	3.61	1.28
3. I like to invent new problems.	1.70	1.01
4. I learn mathematics quickly.	3.30	0.87
5. Mathematics allows us to understand the world we live in better.	3.41	0.95
6. When I am asked to solve math problems, I get nervous.	3.32	1.24
7. When I cannot solve a math problem quickly, I keep on trying.	3.77	1.08
8. I feel confident when I study or work on mathematics.	3.14	0.96
9. Everyone can learn mathematics.	4.18	1.05
10. I prefer challenging tasks in order to learn new things.	3.68	0.90
11. Mathematics should not place much importance on problem solving.	3.52	1.04
12. I feel happy when I solve math problems.	3.93	0.99
13. Math consists of concepts and procedures that we have to memorize.	3.21	1.14
Overall score of CreeMat	43.75	6.09

Students do not perceive mathematics as an inaccessible discipline (for example, the average score of item 9 -everyone can learn math- has a value of 4.18). Also, they do believe that math classes should stress the importance of solving mathematical problems (item 12). However, a medium-high percentage of students consider that mathematics is about concepts and procedures they have to memorize (item 13). We note the apparent discrepancy in their responses when asked about their beliefs regarding themselves. These indicate that their tendency is to memorize mathematical concepts and procedures. Whereas when responding to beliefs about mathematics itself, their response is more suited to social desirability.

We compare group means by student gender by outlining the significance of the differences (p <.002) in item 6 ("when asked to solve math problems I get nervous"). In general, boys show more confidence in themselves than do girls ($\overline{x}_{boys} = 3.72$ and $\overline{x}_{girls} = 3.00$). A factor analysis of the CreeMat questionnaire resulted in three factors (see Table 2). Factor 1: *Affective and behavioral engagement in the individual mathematical learning*. This factor is largely in line with the one we theoretically defined, but it adds a new aspect of behavior that refers to students who admit they memorize the math (item 13). Factor 2: *Mathematical confidence and beliefs in one's personal competence*. The items correspond to the size we theoretically specified. Factor 3: *Beliefs about mathematics and mathematical problem-solving*.

Table 2: Rotated Component Matrix[a] for the factorial analysis of CreeMat.

		Factor 1	Factor 2	Factor 3
Affective and behavioral engagement in individual mathematical learning (Factor 1 CreeMat)	Item 1	0.655		
	Item 2	0.832		
	Item 7	0.644		
	Item 13	- 0.581		
Mathematical confidence and beliefs about one's personal competence (Factor 2 CreeMat)	Item 3		0.598	
	Item 4		0.741	
	Item 5		0.492	
	Item 8		0.743	
Beliefs about mathematics and mathematical problem-solving (Factor 3 CreeMat)	Item 6			0.690
	Item 9			0.350
	Item 10			0.578
	Item 11			0.431
	Item 12			0.474

Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization.

[a] Rotation converged into 5 iterations.

In this study group, two dimensions of beliefs about mathematics and beliefs about mathematical problem-solving are grouped into a single factor, at variance with theoretical suppositions. By focusing on the descriptive group of data based on these factors, we can claim that our groups of students have low scores regarding the confidence of their own personal competence in learning mathematics (Factor 2). However, they show average scores regarding their beliefs about mathematics and mathematical problem solving (Factor 3), which may indicate a greater tendency in sustaining a dynamic and social perspective of mathematics. The correlations between the factors indicate that for these students, there is a closer relationship between factors 1º - 2º and 1º - 3º. That is, correlations between behavior and engagement in learning mathematics are related to their levels of confidence in their personal competence ($r = 0.321$, $p < .05$) and a dynamic perspective underpinning mathematics and problem solving ($r = 0.424$, $p < .01$).

Depth of students' reasoning

The CRT problems were difficult (see Table 3), but there was no floor effect in the correct responses for any of them. The overall percentage of correct responses was somewhat higher than the overall percentage of superficial responses. There were also a relevant overall percentage of “no conclusion” responses, although the variability among the diverse items in this kind of response was very high. As expected, participants underestimated the difficulty of the problems: the overall percentage of estimated correct responses was 69%, clearly higher than the overall percentage of correct responses (42%). The correlation between the estimation of difficulty and the percentage of superficial responses was .14 ($p = .15$). On the whole, the performance of participants was relatively high in comparison with the results presented by Frederick (2005) with university students; that is to say, more correct responses and fewer superficial ones. We think that modifications introduced in our version of the test –a multiple-choice task instead of a recall task– can explain these results.

Table 3: Percentages of responses in the Cognitive Reflection Test, as well as the estimation of difficulty, for each one of the five problems (correlations between superficial responses and estimation of difficulty appear in parentheses).

Item	Correct responses	Superficial responses	Other incorrect responses	Estimation of the difficulty
1	36	61	3	88 (.12)
2	57	43	0	72 (.33*)
3	21	25	54	53 (.13)
4	50	36	14	65 (.28*)
5	45	23	32	65 (.08)
Overall	42	38	21	69 (.14)

Interrelationships between systems of belief and depth of students' reasoning

and mathematical performance implications

Table 4 shows the correlations between different dimensions of belief, cognitive reflection and mathematical achievement of students. The performance of students on the Cognitive Reflection Test achieves small to medium significant correlations between, the total score of beliefs, factors 1 and 2 and academic performance.

Table 4: Inter-correlations between Creemat measures, Cognitive Reflection and Mathematical Achievement.

	CreeMat	Factor 1 CreeMat	Factor 2 CreeMat	Factor 3 CreeMat	Cognitive Reflection Test	Math Achievement
CreeMat	1	.79**	.62**	.74**	.34**	.47**
Factor 1 CreeMat		1	.32**	.42**	.25*	.53**
Factor 2 CreeMat			1	.11	.35**	.39**
Factor 3 CreeMat				1	.14	.16
Cognitive Reflection					1	.25*
Math Achievement						1

** $p < .01$ (1-tailed); * $p < .05$ (1-tailed)

Correlations between the total score on beliefs and Factors 1 and 2 in academic performance are medium and high. Regarding the first hypothesis, we find that there is a significant positive correlation between Creemat and Cognitive Reflection, confirming the hypothesis. However, this correlation is primarily from the first two Creemat factors (Factor 1: Affective and behavioral engagement in the individual mathematical learning, and Factor 2: Mathematical confidence and beliefs in one's personal competence). With Factor 3 (Beliefs about mathematics and mathematical problem-solving), the correlation remains positive but not significant.

Concerning the second hypothesis, we conducted a multiple linear regression analysis of mathematical achievement from the Creemat Factor 1, Creemat Factor 2 and Cognitive Reflection. This model explains 30% of the total variance in math performance (Adjusted $R^2 = .300$, $F(3.52) = 8.865$, $p < .0001$), highlighting the variables Factor 1 ($B = .443$, $Beta = .445$, $p = .001$) and Factor 2 ($B = .243$, $Beta = .227$, $p = .074$), while the cognitive reflection variable did not reach the level of sufficient significance ($B = .524$, $Beta = .054$, $p = .662$).

CONCLUSION

We would like to emphasize two aspects of our work. At first, we confirm the hypothesis that indicates that the scores on the cognitive reflection test, beliefs about mathematics, and beliefs about one's own competence are predictive of mathematical performance. Therefore, individuals with higher scores on cognitive reflection will possess more positive beliefs about themselves. The study of the third CreeMat factor, beliefs about mathematics and problem-solving, needs to be addressed in greater depth. The lack of significant relationships between cognitive reflection and academic achievement may be due to the influence of social desirability in responses to the items of this factor. Secondly, the in-depth analysis of the cognitive processes and contexts of mathematical learning needs to be considered in improving students' mathematical disposition, where the key is the interaction between student belief systems and the limits in how reflexive their mind is (conscious or unconscious psychological processes). It seems that this explanation needs to be extended in further experimental studies for us to have a deeper understanding of problem-solving processes.

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BELIEFS AND STRATEGIES OF IDENTITY IN MATHEMATICAL LEARNING

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In recent years, several researchers have highlighted the need for a better account of the interplay between the cultural, social and personal systems. This article focuses on some theoretical and methodological aspects which should be considered in lifelong education. The research was carried out with 23 students from a professional school. We will discuss the importance of social identity perspective consideration in having a step towards the understanding of the influence of affective dimension on the knowledge of mathematics. The results showed a number of meanings of beliefs (about mathematics, about oneself) used by students as identification strategies and the range of adults' motivation and resistance to learning mathematics.

INTRODUCTION AND INITIAL RESEARCH QUESTIONS

This article is motivated by the fact that beliefs do not exist in isolation but instead, they form networks or systems. They are also embedded in the affective and cognitive structures of individuals and woven by social expectations and shared belief systems. In the field of teaching and learning mathematics, it has been shown that belief structures have much to do with motivation, commitment, the use of instructional strategies, etc. (Goldin, Roesken, & Toerner, 2009).

Furthermore, in the last decade, several researchers have emphasized the need to consider the interplay among cultural, social and personal aspects (see for example Abreu (2002) with a summary of those research or Evans and Wedege (2004) with references to adult education). These studies have led us to questions related to the uniqueness of individual patterns of development in the reconstruction of cultural tools, as well as issues related to the social value of knowledge, changes in social structures, and personal and collective sense of identity.

In this article, motivated by previous statements, we explore the behavior of students, interactions that occur between systems of beliefs about mathematics, its learning and strategies that come into play to negotiate their social identity (affective structure, as part of their self-concept, with the value and emotional significance attached by their membership in a social group).

More specifically, we formulated the following issues: What kind of situation causes resistance or rejection in mathematical learning? How are students positioned in relation to their sense of belongingness to the social group? What kind of interaction can be found between systems of beliefs about mathematics, its learning and strategies that play a role in negotiating their social identity? How could the students' social identity be a reference in understanding the meaning of their behaviors and their emotional reactions in a Math class?

With these questions, we try to highlight the need to understand the formative role of social system, sociocultural interface and affective personal system on learning. Our discussion will focus on the following messages that are constantly received by the students about the meaning of knowing mathematics and the social significance of learning mathematics. Their self-concept as learners is related to their attitudes, their perspective of the world of mathematics, their belief system about mathematical learning and their social identity. The students' self-concept has a strong influence on their notion of mathematics and their reaction to it. Incorporating the social identity perspective stresses the need to consider the influence of symbolic social relationships.

The proposed research is patterned on the triangle of cognitive conditions, emotional and social skills in the learning process in adult education, particularly in social exclusion contexts. In order to illustrate this approach using data taken during two academic years, a group of 23 students (17 - 19 years of age) from a professional school for adults. These young people have the experience of academic failure and are at risk of social exclusion.

THEORETICAL FRAMEWORK. SOME GUIDING CONCEPTS: GLOBAL AFFECT AND STRATEGIES OF IDENTITY

At this point, we collect several definitions that are guiding concepts in the study. In addition to considering the notion of belief systems (Goldin, Roesken &Toerner, 2009), we have considered two notions: global affective and social identity dimensions.

Notion the global affective dimension

In order to incorporate the affect in a systematical way in the mathematical learning, we propose the consideration of two structures of affect: a local structure and a global one. In studies conducted by Goldin (2000) and also in our own research studies (Gómez-Chacón, 2000b), it is revealed that in order to understand the affective reactions of students towards mathematics, it is not enough to observe and know the stages in the process of emotional shifts or changes during problem solving ("*local affective dimension*"). It is also not enough to detect cognitive processes associated with positive or negative emotions. We need to contextualize their emotional reactions within the social reality which gives rise to them. The "*global affective dimension*" is understood as a result of the paths followed by the individual in the local affective dimension. These paths are established with the cognitive system and they contribute in the construction of the general structures of one's self-concept as well as beliefs about mathematics and the learning of mathematics.

Notion of social identity

This notion of social identity linked to membership in a group is based on Tajfel's (1978, 1981) intergroup theory. In this theory, the concept of *social identity* is restricted to "that part of an individual's self-concept which is derived from his knowledge of his membership in a social group or groups together with the value and emotional significance attached to that membership" (Tajfel, 1978: 63).

We have also based our study on the contributions of the interactionists who emphasize the identity construction processes and who conceive identities as *identity strategies* ('strategies identitaires') (Camilleri et al., 1990). These authors have coined the following working definition of identity strategies ("strategies identitaires"): "processes or procedures set into action (consciously or unconsciously) by an agent (individual or group) to reach one or more goals (explicitly stated or situated at an unconscious level); procedures elaborated in function of the interaction situation, that is in function of the different determinations (socio-historical, cultural, psychological) of this situation" (Camilleri et al., 1990: 24).

This approach stresses the importance of a symbolic communication system in social relationships. Identity is understood as a structured joining of elements which permits the individual to define himself/herself in a situation of interaction and to act as a social agent. Starting from identity would permit us to exactly determine the identity elements or identity markers revealing two categories in relation to the global affective dimension. One of these categories pertains to attributes which define an individual’s personal identity (what is unique in each human being coinciding with his personal profile). The other category is what defines his social identity (the status he shares with other members of a social group).

It would also permit us to perceive the identity structure as an organizing center that mobilizes the whole of each individual's affective reactions towards others, the situation of interaction where the identification strategies interlock and the consequences that generate the individual's position towards mathematics and the learning of the subject. It is in the interaction of the group where the identification processes are reinforced and where the individuals become persons others may identify with. It is in the group where negotiation of identity takes place.

RESEARCH CONTEXT AND METHODOLOGICAL ASPECTS

The research study was carried out with the group of 23 students from a professional school, aged between 17 and 19. The study group was followed during two academic years. These students live in a social exclusion situation and have experienced failure at school when they were children. All of them did not finish Elementary School. They often lack basic skills both in literacy and numeracy. They have poor basic mathematical skills or previous difficulties in learning mathematics. These students commonly identified mathematical abstraction and lack of relevance as contributing factors for their dislike of and failure in mathematics.

In designing the research, ethnographic techniques were combined with those of case studies, as well as a reflection on the action itself. There are several sources and procedures of data collection used: questionnaires, interviews of the students, semi-structured interviews of group debate, interviews of the workshop master, observations in the classroom, field notes, audio recordings, productions of the students' mathematical work, etc.

In the case studies, we primarily concentrated on the local affective dimension and simple scenarios in problem solving (methodological aspects were described in Gómez-Chacón (2000) and Gómez-Chacón & Figueral (2007)). To determine the origin of the subject's emotional reactions in the latter, we focus on global affective dimension and complex scenarios. We used the observation of a math class and a cabinet-making workshop. This is where beliefs and emotional reactions towards mathematics were spontaneously fostered in the interaction, followed by subsequent confirmation in the interview and the subject's evaluation of his own progress where they talked about their positive and negative recent and past experiences in learning mathematics. We read the data trying to answer the questions mentioned in the introduction and exploring inconsistencies and consistencies between them.

We took the point of view of the social interaction approach emerging in conversation analysis that defines identity as "the set of verbal practices through which persons assemble and display who they are while in the presence of, and in interaction, with others" (Hadden & Lester, 1978: 331).

To answer the research questions, global analysis and case study were used. Firstly, two pillars for the analysis have been established: a) the kind of members they are and how they position themselves as members of the group (the affective position they assume, their values, beliefs and attitudes); and b) how they negotiate their social identity. Secondly, a discussion of such cases in relation to the perspective of social identity and beliefs was presented.

The guidelines that have continued to operationalize these issues in the analysis are:

a) What kind of members they are and how they are positioned in relation to membership of the group: experience, information, position.

b) How to negotiate their social identity. We focus on the following items:

- Behaviors and situations where the students' identities are supposedly noticeable: in relation to the behaviors that are supposedly representative of them and in relation to the scenarios where they change their behaviors.
- Negotiation of identity by students: in relation to when to "negotiate"; the condition under which one identifies himself/herself; purpose and identity and management of inequality of a group that is marked with a negative social identity.
- The resources available to negotiate their identity: identification strategies.

RESULTS

The results are structured as follows:

Simple scenarios versus complex scenarios in the affective structures

When studying the cognitive-affective responses of students of mathematics in classroom interaction, we put our attention on situations of rejection in learning and lack of basic conditions that make learning possible. We detected similar and different elements.

1. Similarities: In the way they experience lack of confidence in their skill to deal with problem-solving; in their fear of reliving previous experiences of failure in school; in their lack of dedication to this task (processes etc.). They manifest resistance, fear and insecurity. They prefer exercises requiring direct application. They show lack of knowledge of the strategies and stages of problem solving. The same as in the stage of processing mathematical information, they are afraid of abandoning old ways which provide greater security. Their reasoning skill is linked to their hatred for the task and their own insecurity. In order to develop their ability to generalize mathematical objects, relationships and operations, they need cognitive support from the teacher.

2. Differences: in their insecurity in relationships and the interaction with their classmates depending on their position and experience in the social-group, in the auxiliary representations they use for mathematical reasoning and in their way of thinking. (Some of them use their own informal procedures or strategies acquired in the context of practice or in daily living). The different images of the workshop-center help them in understanding the problem structure.

Each of the situations causing resistance or rejection has different characteristics. Therefore, when trying to determine the sociocultural influence on the individual in the ways the information is appropriated and what gives shape to their structure of belief, we consider it justifiable to use the sociological concept of 'scenario' in designating these situations. It is what we have called 'more complex scenario' in the previous heading. By ‘scenario’, we refer to what makes a scene as organized as it is. Therefore, this is to talk about what takes place within a concrete context and a concrete time with some specific resources. Whenever these similar circumstances come into play, the persons involve will more or less act in the same way because they are predisposed to do so by their social and individual learning.

Different behaviors that common sense considers characteristic of youngsters in situation of social disadvantage who have failed in school were examined, as well as the factors causing them in classroom interaction. Alternatives offered for those behaviors were also considered. In our case, it has been shown that factors can be reduced to the following: *school adjustment, self-justification, demand for interdependence and for the response to messages or differentiation* (Table 1).

The data showed that reaction to classroom mathematics basically happening in *school adjustment scenario* will come out 'in a flash' in the following situations: when the activity evokes a true-to-life experience, when they are confronted with certain mathematical contents they had problems with, when the teacher himself/herself evokes past negative situations or when the concept of mathematics is not in agreement with their own mechanical way of doing things.

Table 1: Types of scenarios, student behaviors and their learning effects

SCENARIOS	BEHAVIORS	EFFECTS ON LEARNING
School adjustment	• The person evokes his school experience. • He shows aggressiveness and tries to avoid the fear of not being acknowledged. • Key moments of his school experience have marked him negatively. • He shows a more mechanical way of doing mathematics not needing the learning of so many strategies with unknown quantities.	- Opposition to teacher's authority. Student-teacher interactions or interactions with group of students. - Suspicions towards the teacher. - Lack of confidence. - Resistance to learning mathematical concepts. - Resistance to tasks demanding thinking, e.g. problem-solving.
Self-justification	• He shows his opinions, values, skills, preferences, and emotional reactions justifying the group he belongs to.	- Use of informal structures of problem-solving. - Valorisation of oral mathematics. - Appreciation of shared learning. - Resistance to develop and advance in mathematical knowledge and procedures. - Understanding teacher's task. - Defence mechanisms: mockery and boredom.
Demands for (Inter)dependence	• He relies on the strength of the group, and is afraid of new ways of doing things unknown in his milieu: "It has always been that way".	-Student-teacher communication. - Students-students and student-group communication - Classroom norms. - Classroom management. - Resistance to learning mathematical concepts. - Idea of curriculum.
Response to messages (Differentiation)	• He shows his different by his external appearance (clothes, hairdo, etc.). • He is different from those within the group of classmates. • In relation to appraisals and beliefs about learning mathematics in the context of practice: "this has nothing to do with mathematics".	-Student-teacher communication. - Gesture in classroom interaction. - Various interferences in the learning atmosphere. - Isolation, resistance to cooperative work. - Preferences and liking for a certain type of activity.

The self-justification scenario works differently according to the way they understand it. Those who understand the need for self-justification as something interior will always see themselves in that setting, trying out or justifying the quality of the persons and their way of doing things according to the norms of the context they live in and of the group they belong to. Those who understand it as a need to justify themselves with regard to others will try to make their actions appear as rightful by words and gestures. For example, they express their opposition to learning in ways well known to teachers, ways that are specific of school life and which constitute almost a ritual aspect of the vital structures of these young people such as: making noise with their chairs, jeering at any suggestions, moving all the time, changing their seats and not looking at the teacher nor at the blackboard when the teacher is explaining. Some of them show their disdain for learning by leaning their heads on the desk pretending to be asleep. Others turn their backs to the teacher while looking through the window or simply staring indifferently. An 'idea' of teacher and student-teacher communication emerges.

The *scenario of the demand for (inter)dependence* includes elements of school adjustment as well as self-justification because students who have this kind of reaction keep it as a tradition from their social group. They live within that tradition without thinking of the possibility of breaking away from it or not finding a good reason for doing so. The main factors causing it are their feelings of insignificance or isolation. The students continue to react in a negative way because "it has always been that way". They try to attract attention, wanting to be noticed even if it is for something negative. They are afraid of doing something that has never been done in their social context or milieu. They maintain an imaginative dialogue in competition with that of formal instruction: "I don't understand you", "What's the matter with you, fool?", "I'm fed up with it", "Can I go home?" The significance of academic success and the opportunities it offers for finding a job reflect these young people's culture. The value they attach to mental independence emerges. Tasks which demand a qualification in mathematics seem 'to offer little and ask too much'. They consider it a myth to believe that opportunities can be acquired by means of education. They affirm themselves by means of manual work.

With regard to the scenarios we have referred to as response to messages (or differentiation), it could be said that the youngsters in our case studies know that their group has a negative social identity marker. In these circumstances, the individual will fight to acquire a positive one, and in order to do so, he will identify several alternatives to solve these conflicts of identity. These alternatives, when taken extremely, would force the individual to move to another group with positive identity or to reinterpret his negative identity in the light of new values. In our study, these alternatives taken as opposite extremes appear in youngsters who try to solve their identity conflicts by leaving the group (which requires remaining in school and being successful) and in those who choose to reinterpret their social group values, attributing positive values to the group practices. These are extreme positions. Most of them will adopt intermediate ones.

We would like to make some observations. The first one is with regard to something we did not point out when talking about the different scenarios which give rise to the young people's behaviors best known by teachers and people in general. This belongs to the sociological concept of scenario. The scenario a student responds is not executed in a mechanical way, as a result of certain characteristics which are simply 'objective'. On the contrary, in order for a person to 'act on the stage' as a response to a situation, it is necessary for him to have spontaneously interpreted this situation in a context in which he is risking precisely what has made this situation part of that kind of scenario. In our case, for example, this would be the interpretation of a learning situation in which one is supposed to solve a problem with definite mathematical strategies and procedures as a setting where interdependence is at stake or the interpretation of a situation of classroom interaction giving way to fights, jokes, etc. in a self-justification setting. We may add that an individual's behavior may be situated in several scenarios at a time.

Our second observation refers to the concept of learning interruption or barrier. From the complex scenario perspective, this concept acquires broader nuances which demand on the part of the teacher a response and further reflection, as well as to take into account the elements that take place in the context of the classroom. At the same time, these elements are necessary conditions for the teaching-learning process. For example, the student-teacher communication, teacher's understanding of the task, the way to understand the students' interaction with their classmates and its implications on the construction of mathematical knowledge and social identity.

From the perspective of scenarios, classroom culture provides a set of criteria (which is actually 'unofficial' at the moment) as educational proposals directed to these young people and this would serve as criteria through which we can judge the kind of learning situation most relevant for the individual, the rules of the game which combine the above elements and create favorable conditions for learning.

Beliefs and strategies of identification

The students' beliefs (about the nature of mathematics, its learning, about themselves...) appear at some point in the results of the research, as proof of the knowledge of the position that the individual assumes towards this practice. In the social interaction involving mathematics that happens in the classroom or in the professional workshop, identity markers that involve beliefs about mathematics and about oneself appear and result into emotional reactions towards mathematics. We therefore ask ourselves what the relationship was between the learning of mathematics and the construction of social identity. We are interested in proving that this is a key reference which would help us in understanding the meaning of their behavior and emotional reactions. We look at the findings in order to answer the question whether their beliefs (about mathematics, about themselves) could be considered as strategies of identification which they themselves use or not. We saw indicators that allow us to confirm that in a group of 23 students, *beliefs about what mathematics is can be placed among the strategies of social visibility*. Job and the certificate or diplomas are considered elements of selection and social mobility. Knowledge of mathematical concept is seen as a means of achieving a goal such as social skills (communication, to be in charge of someone...). This justifies the fact that some young people reject mathematical activities which have their center of interest on cabinet-making (a particular practice with a social marker).

Beliefs that they manifest *concerning the learning of mathematics* could be interpreted *as strategies used to give relevance to their identity*. Examples are (1) opposition between learning mathematics and learning what is essential; (2) dislike towards study as an identifying element; (3) free time as an evasion of school time; and (4) the image they have formed about the workshop-center: 'a center for learning something' as opposed to studying.

As a group, they adopt *strategies of instrumentalization of their designated identity*. These are manifested in young people's view about success and failure and in an extended form, the preferences and importance attributed to manual work that appear in their interaction in the classroom: "To be outside", "It runs in the family", "The pleasure".

Among the *strategies of affirmation and defense mechanism,* we emphasize that we recognize that young people make of the use of mathematics in the academic and practical environment but not among the underprivileged socio-economic groups. As far as school is concerned, their lack of interest is due to the fact that in it, they perceive the difficulties to be insurmountable. As a result, they express their boredom as a defense mechanism and use mockery as a force to get into the system.

Strategies of differentiation and demanding their specific place are also observed when they emphasize their external appearance and underline this remark: "I don't want to change on the outside but from the inside", as a reference of differentiation put into words, when they indicate that their motivation depends on their 'mood' and when they demand that teachers modify their beliefs concerning young people like them. The *students' subjective knowledge of mathematics and its learning* and self-concept as learners are used as procedures. They (consciously or unconsciously) become part of a process in order to achieve a certain aim. This structured set of elements allows students to define their position in a situation of interaction and live as a key agent in the society.

According to this data, we think it is possible to establish a relationship between beliefs and the emotional reactions of students towards mathematics and the reactions they show in those strategies of identification. With the learning of mathematics, the social agents are requires modification in the identity attributed to them. *Students' beliefs concerning self-concept, mathematics and its learning* reveal the position of the group in the social structure and their respective individual positions in the group.

This factor has allowed us to detect variations among individuals and the features of identity that are predominantly present in everyday life. This also helps us in the processes of formalization and systematization of the behaviors that emerge in order to avoid conflict or moderate the situation of cultural disparity.

CONCLUSION

The data show the interrelationship between cultural, social and personal systems in the beliefs of students on mathematical thinking. We made a research on social identity of these students, the question concerning the meaning of mathematics and the learning process as far as they are concerned. This research suggests that it is possible to have new approaches (formulations) to the affective dimension in mathematics in lifelong education - at least for similar groups of people (people branded as having negative identity). In fact, the features of students' identity in their context are equivalent to a network of meanings in which they are relevant. It will be manifested in the learning of mathematics. These meanings seem to enlighten our search for a greater understanding of their global configuration of affective aspect, their way of knowing and reacting affectively to the learning of mathematics their way of constructing beliefs systems and their knowledge about this process.

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ENGINEERING STUDENTS AND THEIR LEARNING OF MATHEMATICS

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Students of engineering have to cope with advanced mathematics lectures even though they may not want to focus on this area of expertise. In the context of MP², a project based at Ruhr-Universität Bochum, their learning strategies were examined. The results allow interesting insights into the level and development of crucial skills, whose transfer to the teaching of mathematics in general suggests combining mathematical topics with the teaching of learning strategies.

INTRODUCTION

Learning mathematics in tertiary education has received much attention in recent years, focusing both on students who are studying mathematics as major or as service subject. For instance, Guzman, Hodgson, Robert, and Villani (1998) differentiate epistemological/cognitive, sociological/cultural and didactical obstacles students are confronted with when shifting from secondary to tertiary mathematics. Regarding the former category, Guzman et al. (1998) point out that “the mathematics is different not only because the topics are different, but more to the point because of an increased depth, both with respect to the technical abilities needed to manipulate the new objects and the conceptual understanding underlying them” (p. 752). The specificities of studying mathematics at an advanced level are sometimes referred to as an “abstraction shock” that students encounter since university mathematics adds a *formal* world to the mathematics encountered at school (Tall, 2004). Some authors elaborate on motivational aspects (cf. Hoyles, Newman, & Noss, 2001) or point out the significance of affective variables (cf. Liston & O’Donoghue, 2008) while discussing the secondary-tertiary transition. However, challenges occur at many levels and require specific learning strategies students may not have developed throughout school time (Rach & Heinze, 2011).

In Germany, the high numbers of students giving up studying mathematics before graduation is alarming. In this paper, our focus is on engineering students and their learning of mathematics. At Ruhr-Universität Bochum, the project MP^2 was implemented, which aims at supporting students in their first year studies of engineering. In particular, the project looks into what motivates students to pursue a challenging course, which learning strategies prove successful, which cognitive dispositions and beliefs seem advantageous – and how universities can support their students. The findings presented here are part of a larger study where motivation and beliefs are investigated in detail; here the focus is on (meta)cognitive learning strategies.

THEORETICAL BACKGROUND

We have known since the times of Piaget (Piaget, 1973) that cognitive development comes in phases even though the stages involved might not be discrete. There are different theories how these phases can be typified. “Procept” theory (Gray & Tall, 1991) stresses the duality of *process* and *concept* and its relevance for students’ understanding of mathematics. In particular, Gray and Tall (1994) “wanted to encompass the growing compressibility of knowledge characteristic of successful mathematicians. Here, not only is a single symbol viewed in a flexible way, but when the same object can be represented in different ways, these different ways are often seen as different names for the same object” (p. 120). Successful learners can change flexibly between process and concept, depending on what kind of tasks they face. Another popular theory is APOS (Dubinsky & McDonald, 2001) which distinguishes between action, process, objects and schemas and allows to describe and to evaluate learning theoretically.

Special attention has been paid to those factors that allow a teacher or lecturer to enhance learning visibly. Metacognition, knowledge about the learning process and how to modify it effectively (Flavell, 1979), plays an important role here, as well as affective variables (Liston & O’Donoghue, 2008), such as attitudes, motivation, beliefs, self-concept and approaches to learning. Where the transition to university mathematics is concerned, the difference to the mathematics taught at school and the necessity for a deeper understanding is stressed (Zucker, 1996; Engelbrecht, 2008). The learning policy followed and the individual learner types are the center of intensive research (Duffin & Simpson, 1993; Tall, 1997; Marais, 2000; Rach & Heinze, 2011; Rach & Klostermann, 2011).

The focus of this paper is on learning strategies as they represent a starting point where concise and specific measures can make a difference. Therefore the LIST questionnaire (Wild & Schiefele, 1994) for measuring learning strategies in academic studies was used. It was first compiled in the 1990s and has been modified and tested several times. It is not limited to a specific field but encompasses general items that can be applied to all kinds of subjects.

In the course of the complete study many questions about motivation, learning strategies and the efficiency of different measures are of interest. As a first step, the concentration is on three issues which represent an adequate starting point from which to evaluate the complete project:

- In what respect does a semester of university work encourage engineering students to modify their learning strategies?
- Do different interventions produce significant differences in the modulation of learning strategies? If so, how can they be described and accounted for?
- What conclusions can be drawn with respect to the teaching of mathematics, in particular to those who do not feature an inner bond with the subject?

METHODOLOGY

The LIST questionnaire

There are 77 questions in LIST (Wild & Schiefele, 1994) which fall into three main categories: cognitive strategies, metacognitive strategies, and resource-related strategies. As the latter are not the topic of this paper, we will concentrate on the first two. “Cognitive strategies” comprise the scales

- *organizing* ((re-)structuring the subject matter in charts, lists, groups or other arrangements, 8 items),
- *elaborating* (aiming at a deeper understanding, in particular relating new areas to those already known, 8 items),
- *critical checks* (scrutinizing statements and justifications, 8 items, see remark below), and
- *repeating* (aimed at memorizing facts, rules, and formulas, 7 items).

The “metacognitive strategies” fall into the three parts *planning*, *monitoring* and *regulating* and include 11 items altogether.

The questionnaire was adapted for our purposes in three respects: First, the original wording from 1994 did not account for the Internet, its digital resources, e-learning platforms and search engines. Today students do not bother to copy from the board during lectures, but rely on digital scripts – a technique that requires distinct skills and strategies. So whenever reference books, journals or lecture notes were mentioned in the questionnaire, their digital variants were added. A second change was necessary because of the special character of mathematics, the subject in the focus of our investigation. LIST originally contains the scale “Critical checks” which makes sense for the humanities but does not seem appropriate for first year mathematics. Therefore all items from this scale were eliminated. Third, a new starting item was added, to get in the mood and open up for the topic: “I study for my courses.” It was not used for evaluation.

A five-level Likert scale was applied to all items, consisting of *very rarely (1)*, *rarely (2)*, *sometimes(3)*, *often (4)* and *very often (5)* in order to take into consideration which learning strategies were employed how frequently. The LIST questionnaire was given out at the beginning (pre) and at the end (post) of the first semester.

MP² - basic assumptions and conceptual decisions

MP² is a project based at Ruhr-Universität Bochum in Germany. In order to prevent engineering students from unnecessarily failing to graduate, MP² offers support in first year mathematics. It was awarded a prize by “Stifterverband für die Deutsche Wissenschaft” for its innovative concept.

MP² is based on two assumptions (Dehling, Glasmachers, Härterich & Hellermann, 2010): First, that many students fail because of their lack of self-organization.

Second, that many students lose their motivation and willingness to apply themselves because they cannot see sufficient practical application of the subject matter. MP² consists of two parts, *Math/Plus* and *Math/Practice*, which aim to tackle the two problem areas respectively. *Math/Plus* encourages and assists learning strategies in the first semester. *Math/Practice* presents an option to apply first year mathematics directly to practical engineering problems in the course of the second semester.

Math/Plus was set up in spring 2010 and carried through in fall and winter 2010/2011, followed by evaluation. *Math/Practice* started half a year later. Both parts wills be repeated in the consequent academic year. The target group comprises first year students of engineering at Ruhr-Universität Bochum, all of whom were given the LIST questionnaire. About 1,000 of them read mathematics and obligatorily have to pass a math exam which in the past has proved a considerable obstacle.

Recruitment and organization

After the first weeks of lectures and a written mini test, students had to apply for participation in *Math/Plus*. The 180 students accepted were distributed randomly among three groups. The group most attended to was the "Supported Learning Group" (SLG) which consisted of three tutorial groups of 20 students each. Another 60 students were assigned to the "Self-Directed Group" (SDG) and the "Monitored Group" (MG) respectively. As these groups experienced different core and accompanying measures, the efficiency of the single measures could be evaluated separately. In particular, without the funding it would have been difficult to implement the SLG extra tutorials. MG serves only as a control group with preconditions comparable to the other two groups'.

Core and accompanying measures

Both SLG and SDG kept a daily learning diary based on Landmann and Schmitz (2007) every day. The diary contains parts "before learning" and "after learning" as well as questions about the mental state and motivation. It serves not only as a log for recording the time and learning strategies used, but also as an instrument of intervention as it aims at self-regulation (Schmitz & Wiese, 2006). The efficiency of the learning diary is the focus of another part of the evaluation of the project. Only the students in SLG met with a tutor every week for a preparatory tutorial. The meetings concentrated on a specific learning strategy, e.g. the work environment or note-taking (handwritten or digital), in combination with a mathematical problem. The students from SLG had access to a helpdesk where student assistants offered help and explanations, checked assignments and gave hints for further work. In addition to this, e-learning courses were open 24/7 for the students from SLG and SDG. They provided various test questions with immediate response, and there were filing options as well as a forum and wiki for discussion and communication.

Procedure

Although the LIST questionnaire has been used for decades and its value and internal reliability have been tested many times, it was necessary to prove the validity of our modified questionnaire. Therefore, Cronbach's α was calculated for all scales in the pre and post questionnaires in order to show that the items in each scale match up. In order to combine the different scores of items of a scale into one specific significant number, they underwent the linear transformation $2.5\cdot(10/n\cdot\Sigma x_i-10)$ where n is the number of items in a scale, and $x_1, \dots x_n$ are their scores. This takes into account that there are different numbers of items in particular scales and renders numbers from 0 to 100. The Kolmogorov-Smirnov test was used to show that these transformed numbers were normally distributed. The scores were then compared and subjected to Student's t-test which delivered results for interpretation.

Cronbach's α	pre	post
organizing	0.859	0.901
elaborating	0.808	0.833
repeating	0.794	0.808
metacognitive strategies	0.768	0.822

Table 1: Cronbach's α

N	pre	post
not MP²	200	107
SLG	24	10
SDG	28	21
MG	7	7

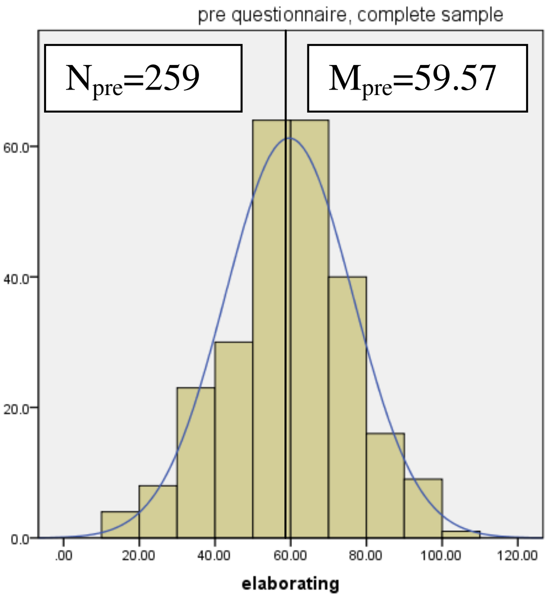
Table 2: Number of usable questionnaires

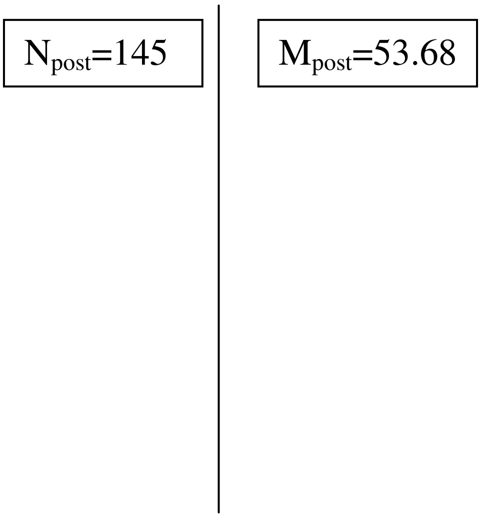
RESULTS

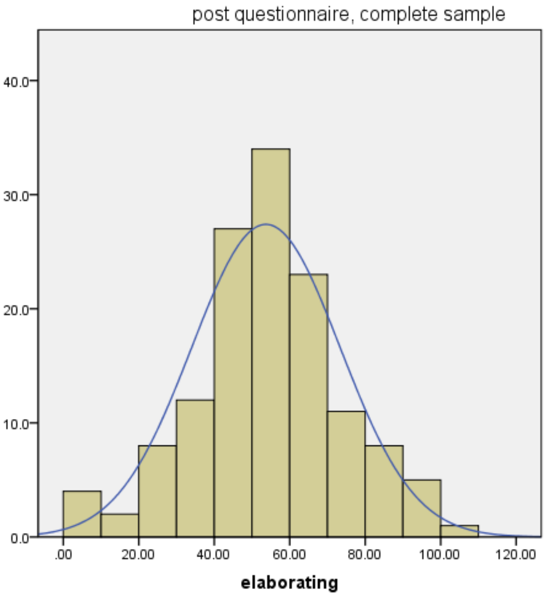
All Cronbach's α values were above 0.7, see table 1. As is usual in statistics, not all questionnaires sent out were filled in properly. The numbers of the usable questionnaires are given in table 2. Those with more than five empty spaces were deleted, questionnaires with up to five empty spaces were completed with the middle value 3 of the Likert scale. The low number of usable questionnaires from MG means that results from this group have to be interpreted very carefully, if at all.

pre ***post***	organizing	elaborating	repeating	metacognitive strategies
not MP²	M=47.91; SD=20.85 *M=46.52; SD=25.33*	M=60.02; SD=16.80 *M=53.74; SD=19.86*	M=43.88; SD=18.24 *M=41.22; SD=18.94*	M=54.70; SD=14.66 *M=52.44; SD=18.29*
SLG	M=52.73; SD=22.75 *M=53.44; SD=25.49*	M=52.60; SD=16.78 *M=49.38; SD=13.88*	M=46.28; SD=21.13 *M=27.50; SD=13.47*	M=57.01; SD=13.63 *M=53.63; SD=10.62*
SDG	M=49.44; SD=19.75 *M=41.19; SD=20.86*	M=62.16; SD=13.72 *M=51.42; SD=15.42*	M=47.36; SD=16.25 *M=35.71; SD=16.60*	M=58.77; SD=16.58 *M=45.25; SD=13.69*
MG	M=47.77; SD=18.98 *M=50.00; SD=25.52*	M=60.27; SD=26.62 *M=62.50; SD=28.53*	M=43.37; SD=23.37 *M=45.92; SD=24.87*	M=51.95; SD=10.12 *M=50.32; SD=14.10*
complete sample	M=48.52; SD=20.80 *M=46.78; SD=24.88*	**M=59.57; SD=16.86** ***M=53.68; SD=19.66***	**M=44.46; SD=18.39** ***M=40.00; SD=18.98***	**M=55.28; SD=14.69** ***M=51.83; SD=17.30***

Table 3: Statistics for (meta)cognitive learning strategies, distinguished by MP² groups







The Kolmogorov-Smirnov tests for normal distribution confirmed that the data was normally distributed. As an example the histograms for *elaborating* are given above, with their approximate normal distribution curves.

The most interesting part of our evaluation is doubtless the comparison of the scores of the pre and post questionnaires. Our data allows us to compare not only the pre and post questionnaires of the entirety, but also those of the three groups within MP². As expected, not all of these comparisons are statistically significant; therefore we will concentrate on those which are. However, table 3 gives a comprehensive overview. It is obvious that the scores for not MP² participants and the complete sample hardly differ. This is mainly due to the numbers, see table 2 above.

Student’s t-tests provided statistically reliable results about in which interval the scores differed with a certain probability. For *elaborating* (see histograms above), *repeating* and *metacognitive learning strategies* the tests showed that the lower scores in the post questionnaires (boldface in table 3) for the complete sample were significant, see table 4.

	Levene significance	variances	confidence interval (p=.95)
elaborating	.106	≠	[-9.81; -1.97]
repeating	.554	=	[-8.34; -.58]
metacognitive strategies	.289	≠	[-6.89; -.01]

Table 4: Negative confidence intervals produced by Student's t-test

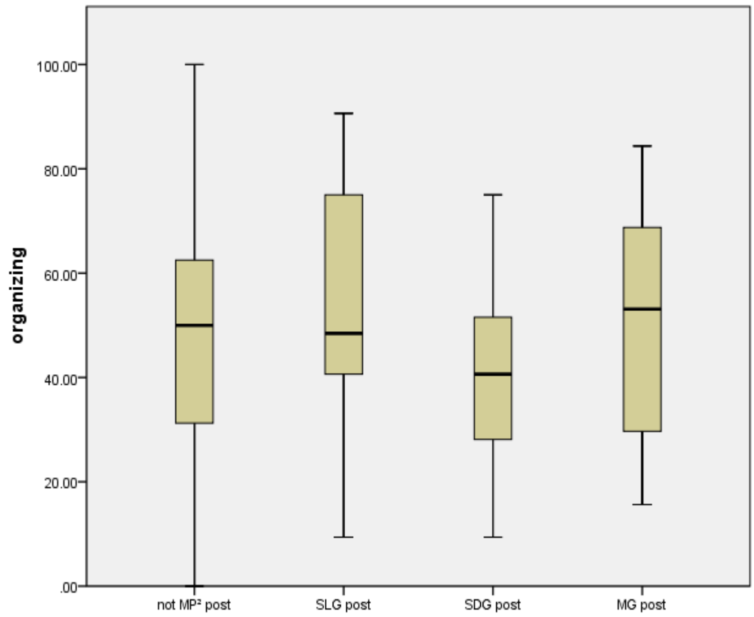
Taking into account that this is what the students said about themselves, there are several possible interpretations. One of them is that while the students considered their learning strategies (with respect to *elaborating*, *repeating* and *metacognitive strategies*) more or less adequate at the beginning of the semester, at the end of the semester they realized they had not worked hard enough. This interpretation makes sense if we take into account that university mathematics differs considerably from school mathematics (cf. Zucker, 1996; Engelbrecht, 2008). Another possible interpretation is that though the students intended to work seriously, they did not do so, for whatever reason. An answer to the problem which of these interpretations is correct may be found in the interviews done with some of the students. Those will be evaluated in the future; at first sight there is no indication that the students met learning obstacles other than inadequate learning strategies and the advanced level of university mathematics.

A more detailed evaluation is to follow, in this paper the focus is on three significant differences between the MP² groups, see table 5.

	Levene significance	variances	confidence interval (p=.95)
organizing SLG / SDG post	.457	≠	[-9.28; 33.77]
repeating SLG / not MP² post	.445	≠	[-23.77; -3.67]
metacognitive strategies SLG / SDG post	.277	≠	[-2.77; 19.54]

Table 5: Confidence intervals of mainly identical algebraic sign, produced by Student's t-test

The first is the very much higher score in *organizing* of the “Supported Learning Group” (SLG) than of the “Self-Directed Group” (SDG) at the end of the semester (*M=53.44; SD=25.49* to *M=41.19; SD=20.86*), see boxplot diagrams for more detailed information on the distribution, its median and quartiles. The main difference between these two groups is the fact that SLG met with a tutor once a week whereas SDG did not. Both groups had access to an e-learning course and filled in a learning diary. This scenario means the results can be regarded as due to the tutorials aimed at advancing learning strategies. The figures prove that the tutorial resulted in a significantly higher score in the learning strategy *organizing*, so the *Math/Plus* concept can be seen as a success in this respect.



The second result to be looked upon here is the very much lower score in *repeating* of SLG than of students not participating in MP^2 at the end of the semester (*M=27.50; SD=13.47* to *M=41.22; SD=18.94*). It may be that the MP^2 interventions resulted in a focus on more advanced learning strategies, somewhat neglecting *repeating*. It may also be that in retrospect the students of SLG thought their repeating strategies were inadequate when compared to what would have been necessary. Be that as it may, in this respect the participants in the central *Math/Plus* group scored disappointingly. This result will be taken as a hint how to improve *Math/Plus* in the future.

The third and last difference in scores is the higher score in *metacognitive strategies* of SLG than of SDG at the end of the semester (*M=53.63; SD=10.62* to *M=45.25; SD=13.69*). As metacognition is the first step on the way to modifying and regulating behavior, this result means that the personal tutoring that SLG experienced is a substantial aspect. Looking at the figures for the students not taking part in MP^2 which are similar to those of SLG, it may be concluded that an e-learning course and a learning diary *without* the support of a weekly tutorial and the personal contacts that go with it can result in increased frustration that finds its expression in the comparatively low score in *metacognitive strategies*. It must therefore be considered carefully if students are to be exposed to interventions without offering personal support.

CONCLUSIONS - RELEVANCE FOR THE TEACHING OF MATHEMATICS

Traditionally, mathematics is regarded as a difficult subject; therefore, the focus of teaching mathematics often lies on the subject matter. Recently the attention has shifted to learning strategies (Rach & Heinze, 2011) which makes sense as they are the universal tool when tackling new areas. In particular, the transition from school to university mathematics requires a different and more elaborated learning behavior than that generally promoted in schools. Reaching a higher level of expertise in mathematics needs knowledge and command of one's own cognitive processes.

Our results show that working on learning strategies in combination with personal and digital support systems can advance the cognitive strategies notably. It remains to be examined which interventions are the most effective; however, the personal component seems to be of importance.

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APPENDIX (LIST QUESTIONS)

1 Organizing:

1.1 I make charts, diagrams and graphics in order to have the subject matter in front of me in a structured form.
1.2 I compile short summaries of the most important contents as a mnemonic aid.
1.3 I go over my notes and structure the most important points.
1.4 I try to order the subject matter in a way that makes it easy for me to remember.
1.5 I compile a summary of the main ideas out of my notes, the script or other sources.
1.6 I underline the most important parts in my notes or in the texts.

1.7 For bigger amounts of subject matter I find an arrangement that mirrors the structure best.
1.8 I assemble important terms and definitions in my own lists.

2 Elaborating
2.1 I try to find connections to other subjects or courses.
2.2 I think of practical applications of new concepts.
2.3 I try to relate new terms or theories to terms or theories I already know.
2.4 I visualize new issues.
2.5 In my mind I try to connect newly learnt facts to what I already know.
2.6 I think of practical examples for certain curricular facts.
2.7 I relate what I am learning to my own experiences.
2.8 I wonder if the subject matter is relevant for my everyday life.

3 Repeating
3.1 I imprint the subject matter from the lecture on my memory by repeating it.
3.2 I read my notes several times in a row.
3.3 I learn key terms by heart in order to remember important facts better in the exam.
3.4 I commit a self-compiled compendium to memory.
3.5 I read a text and try to recite it at the end of each paragraph.
3.6 I commit rules, technical terms or formulas to memory.
3.7 I learn the subject matter by heart using scripts or other notes.

4 Metacognition: Planning
4.1 I try to consider beforehand which areas of certain topics I have to study and which I do not have to study.
4.2 I decide in advance how much subject matter I would like to work through in this session.
4.3 Before starting on an area of expertise, I reflect upon how to work most efficiently.
4.4 I plan in advance in which order I want to work through the subject matter.

5 Metacognition: Monitoring
5.1 I ask myself questions on the subject matter in order to make sure that I have understood everything correctly.
5.2 In order to find gaps in my knowledge I sum up the most important contents without using my notes.
5.3 I work on additional tasks in order to determine if I have truly understood the subject matter.
5.4 In order to check my own understanding I explain certain parts of the subject matter to a fellow student.

6 Metacognition: Regulating

6.1 If I am confronted with a difficult subject matter, I will adapt my learning technique to the higher demands.
6.2 If I do not understand everything I am reading, I will try to make a note of the gap in my knowledge and sift through the material again.
6.3 When an aspect seems confusing or unclear, I examine it again thoroughly.

INSTRUCTOR AND STUDENT VOICES FROM MATHEMATICS COURSES FOR ELEMENTARY TEACHERS

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This cross case study analyzes results from two qualitative studies of mathematics content courses for prospective elementary teachers. One study involved students in the U.S. completing the courses, and the other study involved instructors in Canada teaching the courses. Results were examined for converging themes, and salient commonalities were found. Two themes will be discussed here: the role of affect in student learning and the role of connections to the elementary classroom.

Pervasive concerns about the adequacy of the mathematical preparation of elementary teachers (Ball, Hill, & Bass, 2005; Rowland, Huckstep, & Thwaites, 2005) prompted many institutions of higher education to require specialized mathematics content courses for prospective teachers. These courses, referred to here as *Math for Teachers* (MFT) courses, aim to provide deep understandings of elementary mathematics concepts in order to develop prospective teachers' confidence and flexibility in teaching mathematics (Williams, 2008). MFT courses are most often taught in mathematics departments by mathematics faculty, an approach which is endorsed in recommendations by Greenberg and Walsh (2008).

RELEVANT RESEARCH

The two extant studies used in this cross case analysis were framed by prior research on mathematical knowledge for teaching (e.g. Ball & Bass, 2003) and instructor beliefs about mathematics (Ernest, 1989). Of relevance to both studies, and of particular interest in this paper, is the literature pertaining to: the role of affect and beliefs in prospective elementary teacher learning (e.g., Di Martino & Sabena, 2007); post-secondary mathematics content courses for prospective elementary teachers (e.g., Lubinski & Otto, 2004; Philipp, 2007); student perspectives on and characteristics of effective university mathematics instruction (e.g., Schulze & Tomal, 2006; Weinstein, 2004); and assessment of university mathematicians' facility with reform pedagogies (Wagner, Speer, & Rossa, 2007; Speer & Wagner, 2009. While there is an abundance of literature on the significant role of beliefs and affect in the pedagogical practices of teachers, far fewer studies are available on preservice teachers in mathematics content courses and those that were found were primarily limited to outcomes of reform-based interventions in one-semester courses. Scant studies have looked at characteristics of effective post-secondary mathematics instruction, even in traditional mathematics courses.

THEORETICAL PERSPECTIVE AND RESEARCH QUESTION

This cross-case analysis, as well as the two original studies, is grounded in phenomenological interpretation (Burch, 1990). Students and instructors individually reported their perspectives on the MFT courses. Through the interviews, participants reflected on and retrospectively identified the significant or memorable events from their MFT experiences. Through this process, they recovered and verbally re-enacted the meaningful components of their lived experiences.

For this cross-case study, we were interested in determining what intersections might exist between the perspectives of instructors of a MFT course and the perspectives of students in MFT courses. Specifically, we asked: *What convergent themes exist in instructor perspectives and student perspectives on mathematics content courses for prospective elementary teachers?*

SOURCE CASES

The two original studies on MFT courses that formed the basis of our cross-case comparison are briefly described here. The reader is referred to the original studies.

The *instructor-focused* study examined ten instructors' perspectives on a MFT course at several institutions in south western Canada (Oesterle and Liljedahl, 2009). The purpose of the study was to provide insights into the instructors' approaches in the course and how their beliefs impacted pedagogical decisions. Data from the semi-structured, individual interviews (1 hour in duration) were analyzed for emergent themes using constant comparative analysis. The emergent themes included: instructor identity, tensions, resources, student knowledge, student affect, orientation to mathematics, orientation to teaching, and classroom environment.

The *student-focused* study explored the perspectives of 12 elementary education majors (i.e., prospective elementary teachers) who had completed MFT courses at a university in the south eastern U.S. (Hart and Swars, 2009).The study was inspired by concerns over the poor success rates of elementary education majors enrolled in these courses. Data collection included semi-structured, individual interviews (~1 hour in duration). Constant comparative analysis was applied to the data, revealing three emergent themes: (1) domains of mismatch (2) affective reactions, and (3) classroom practices. The domains of mismatch included: mismatch with elementary classroom, mismatch in programmatic emphasis, and mismatch in mathematics content.

METHODS

Participants and Setting

The student-focused study involved 12 students (11 females and 1 male) from one urban university in the south eastern U.S. The researchers achieved saturation of the themes after ten interviews, i.e., no new themes were identified after the tenth interview. Two additional interviews were conducted to confirm. The students had completed three or four MFT courses. They were randomly selected from 4 cohorts of students in the elementary teacher preparation program with a combined size of 99 students, thus representing approximately 12% of the total population. Collectively they had taken 42 sections of MFT courses. At the time of the study, all of the students were in the last semester of the program and completing student teaching.

The instructor-focused study used theoretical sampling to capture perspectives of instructors from a variety of post-secondary institutions, having a range of experience teaching MFT courses. Saturation of themes occurred after ten interviews. As the scope of this paper precludes discussing results from all ten instructors, examples will be drawn from only two: Harriet and Bob (pseudonyms). These have been selected because their transcripts reflect the wide diversity in the instructor perspectives revealed in the original study and provide a sufficient basis for illustrating the common themes identified in this analysis. Harriet and Bob are both experienced instructors, having taught in mathematics departments for 22 and 13 years respectively. Harriet is relatively new to teaching the MFT course but had taught the course six times over three years, while Bob taught it nine times over nine years. Both have Master's degrees in mathematics but neither took mathematics education courses nor had formal teacher training. Harriet was initiated into teaching the MFT course by a colleague with a Master's degree in mathematics education who taught MFT courses for many years. Bob's first forays into teaching the course were guided by his institution's curriculum, the textbook, and informal discussions with colleagues.

Data Analysis

For analysis, Yin's (2009) method of cross case analysis was employed. A cross-case analysis was appropriate because the cases were independently studied by different researchers; further, the two case studies had similar research goals and methods. To conduct the cross-case analysis, the researchers created a matrix with the themes from the two studies, specifically examining the data for convergence. This analysis revealed four commonalities across the themes, two are highlighted in Figure 1 and discussed in this paper.

Student-focused Study	Instructor-focused Study
• ***affective reactions*** ←	→ ***student affect***
• **classroom practices**	• **instructor identity**
• **domains of mismatch**	• **resources**
elementary classroom ←	→ ***orientation to teaching***
programmatic emphasis	• **tensions**
mathematics content	• **orientation to mathematics**
	• **student knowledge**
	• **classroom environment**

Figure 1: Convergent themes from the two extant studies.

CROSS-CASE RESULTS

Theme I: Connections to Elementary Classroom – Student Perspectives

After experiencing many other courses in their teacher preparation program, the students were acutely aware of disconnections between their experiences in the mathematics content courses and other experiences in the program. The students frequently described an inability to position the mathematics content coursework within their growth as educators, which led to perceptions of lack of usefulness or relevance of the courses as evidenced by this statement: “I mean a lot of us were always questioning, you know, why we have to take these math courses . . . It’s not even necessary” Similarly, another student stated, “The reason why we were taking those courses was never brought to our attention. We had no clue why we were taking those classes, no clue . . . It seemed very unnecessary” Another student said:

> It [mathematics courses] had no connection to elementary schools. Anybody could take those courses. I don’t think we ever talked about kids. I seriously don’t think we ever talked about teaching or students or anything like that. I just don’t remember ever that connection.

Another student explained the lack of connection with elementary classrooms as:

> In our [elementary] classrooms right now, you know, they’re not graphing how many bagels and coffee people are going to eat and drink tomorrow. It was just not very logical for, you know, a kindergartner.

The following statements further support this sentiment:

> They’re [mathematics instructors] blind to what we are actually doing with our lives; [Elementary] Students were never even brought up . . . I mean, students or when you get your own classroom were never brought up”; and “We’re thinking we’re learning something about how to be teachers. But, in reality we’re learning how to get through their math courses.

These findings are not intended to suggest that there were no connections ever made to elementary classrooms throughout the 42 courses, but such connections were the exception in the interviews. The few comments indicating relevance related to modeling pedagogical methods that might be used in an elementary classroom or using materials appropriate for elementary students’ use. For example, one student commented:

> [The Number & Operation course instructor] was big on you know . . . we would do that in class, it wasn’t like, take this home and do this but we actually [did activities in class] and we did a lot of group work and that was really good too.

Ideas for changing the courses were proffered, including*:* “Get some teachers who were actually qualified in elementary [teaching] . . . They actually know what children are going through and that would help;” “They [mathematics instructors] could talk to elementary teachers;” and “Maybe have us students say, hey, this is what is going on in my [elementary] classroom.”

Connections to the Elementary Classroom – Instructor Perspectives

Harriet's descriptions of her goals and strategies for teaching the MFT course are permeated with comments related to mathematics for teaching knowledge (Ball & Bass, 2003) and how her students' learning relates to their future as teachers. When asked if there is anything that she teaches MFT students about fractions that she would not teach other students, she states:

> The fact that there are different models, there are different ways of picturing what's going on, and that they are appropriate for . . . what may work well for some situation, or for some [elementary school] student, may not work for some other one.

She also emphasizes connections between mathematical ideas both within and across grade levels. She explains:

> At all times I connect it [the course content], as far as I can, to what goes on at different levels. What you might do with a grade 1 class, how that connects to what they're going to see in, you know grade 4 or 5 or something like that, how that connects to what they might do in high school and how that connects to what I'm doing in Calculus. Because they've got to see how it's connected, and how we build bigger and bigger . . . understandings of sets of numbers, or calculations.

Harriet does not just pay lip-service to these ideas. She describes assignments that allow her students to build their mathematics-for-teaching knowledge, such as analysis of pupil errors and discussion of alternative solutions.

In contrast, Bob makes very little reference to mathematics-for-teaching knowledge. His emphasis is instead on developing a strong understanding of fundamental mathematics and communication skills. Varieties of algorithms and models form part of his course content, but he does not specifically address how they can be applied differently at various grade levels. Bob needed to be pressed by the interviewer to consider what aspects of the course content might be particularly relevant to prospective teachers as opposed to general learners of mathematics. Initially his comments revolve around his teaching methods, such as the use of group work and manipulatives, but he makes no reference to any special mathematics knowledge for teaching. Eventually he describes challenging his students to think about the kinds of questions that they will encounter as teachers:

> . . . What kinds of questions will you encounter? And why is it important that you to be able to communicate your ideas effectively . . . why should you understand this material to the most . . .fundamental and basic level, and understand all of the structure?

He adds:

> When you get some of these obtuse questions, that are seemingly . . . obtuse, you have to be able to appreciate it and be able to differentiate whether that's something that can lead you into a teachable moment

His response appears to be a justification for his goals of developing strong mathematics content knowledge and communication skills. For Bob, mastery of the subject content along with general pedagogical skills, seem to be sufficient for the teaching of mathematics—a traditional and prevalent point of view (Hill et al., 2007).

Theme II: Student Affect - Student Perspectives

A second theme across the student interviews was affective reactions to the MFT experiences. Many negative emotions were described, for example, they used words such as “emotional wreck,”, “so stressed,”, “very belittling,”, “discouraged,”, “terrified,”, “struggling,”, and “frustrating”. A student asserted, ‘I felt like I was just hanging on. Just trying to dig myself out of a hole, and I kept falling down.’

The students also portrayed the courses as having deleterious influences on their mathematics teaching efficacy beliefs and self-efficacy beliefs, which were often linked with the classroom practices of the instructors. Most often, descriptions of ineffective pedagogy were related to traditional approaches to instruction. The students mentioned a preponderance of “lecture,” “note-taking,” and “power point presentations”’ and asserted the “classes were not hands-on.” In describing how the courses impacted teaching efficacy beliefs, a student stated, “I felt less confident [about teaching mathematics] when I walked out of those classes because it’s just so much and it just seemed so unnecessary . . . It was just very discouraging.” In response to a question on how the courses prepared her to teach elementary mathematics, a student stated the courses made her, “Feel less prepared. Feeling more scared, definitely.” One student attributed this negative impact on her teaching efficacy to the attitude of the instructor of the course, “The attitude was if you don’t get this [math content], you won’t be able to teach it, basically.”

Students also commented on how the experiences in the courses influenced their self-efficacy beliefs in mathematics, as represented by this student’s statement:

> [I felt] terrified, struggling, especially in geometry. It was just, it was very frustrating because I didn’t get it. I didn’t understand why we’re doing what we were doing, how we were coming out with the answer . . .

Another student noted:

> Like geometry . . . I came out of there in tears. I felt very disappointed. I felt stupid. I felt alone. And, I know that I am an intelligent person, or I have the potential to learn something. If I don’t know it, I’m willing to give up my time and my efforts. But, I felt like my efforts didn’t matter.

Similarly, another student said:

> It (mathematics courses) made me feel so low in math. Even though I knew those math courses, I would never be teaching that stuff . . . It totally lowered my self-esteem in mathematics.

Student Affect – Instructor Perspectives

Both Bob and Harriet describe their students as suffering from mathematics anxiety and lacking confidence in their ability to do mathematics. However, there are considerable differences in their perspectives and pedagogical approaches to these negative affective states.

Harriet observes that her students: "are very anxious around problem solving. They are just terrified, most of them, of a problem they haven't seen before"' Her efforts to address this seem to be centered on changing their ideas of what the enterprise of mathematics is all about. She tries to convince them that "We're supposed to have fun with this" and tells her students that "You may never have seen it; you might not get all the way through it. But what I'm looking for is how far did you get, and how well can you explain what it is that you got," shifting the focus away from getting the right answer toward less threatening goals. By the end of the course she hopes her students have grown in confidence and also "They have more of a sense of play . . . I think they're more flexible. They think they're more flexible. They're not as scared . . . that someone will ask them a question that they can't answer."

Bob describes his students as believing that mathematics is arbitrary and incomprehensible: "So many things seem magical to them." He affirms that "It's not your standard sort of math group, it's one that has encountered some challenges along the way, and it hasn't always left them with a positive impression of mathematics." In his view, their confusion and anxiety is closely linked to their skills:

> In many cases, some of the very elementary arithmetic operations are in fact, confused in their minds and so when they hit upon things, in particular when you hit rational numbers, as an example, that's one place where students have a great deal of anxiety and they would demonstrate poor understanding of ideas.

More than once he describes the MFT course as a second start for these students. He attempts to reshape their beliefs and attitudes by providing them with opportunities to see the logical structure of mathematics. For Bob, the course "focuses on a very sound fundamental ability to appreciate it [mathematics], in a theoretical way, why things work, as opposed to technical aspects of how do you do mathematics." However, although he believes that improved skills will lead to increased appreciation and confidence, he confesses that the realities of the course conspire against this occurring. Early in the interview he expresses a wish that his MFT students develop a love of math, but when asked about whether this goal is accomplished, he admits: "in terms of the other goal, for love of math? Unfortunately, the course is so packed, that in some ways, I think they do get a little bit beaten by the end, and they're just tired." This statement illustrates Bob's realization that the volume of content covered in a limited time is at odds with his affective goals.

DISCUSSION

Although the two groups of participants in these studies were in different Geographic settings, both speak, from their own perspectives, about the experience of MFT courses. Notwithstanding the potential for differences between the two contexts, when juxtaposed the data reveals salient commonalities, whose analysis provides important insights into issues and concerns around creating experiences in MFT courses that best support elementary prospective teachers' learning of mathematics;

they also enrich our understandings of the realities of MFT classrooms, revealing both the affordances and the constraints.

The student voices emphatically call for the need for connecting the mathematics to the elementary classroom. Without this connection, the students were not able to find relevance in their learning. This need is recognized in the literature (Philipp, 2007). Ball and Bass (2003) also strongly advocate for this link:

> Practice in solving the mathematical problems they will face in their work would help teachers learn to use mathematics in the ways they will do so in practice, and is likely also to strengthen and deepen their understanding of the ideas. (p. 13)

The instructor-focused study reveals how differently instructors may perceive the need for incorporating these connections. Harriet is very aware that these links help to motivate her students, helping them to see why a deeper understanding of mathematics is required of them in this course as compared to their previous mathematics courses. For Bob, making these connections is not an explicit part of his course. One reason for this may be that as a mathematician his lack of experience in elementary classrooms limits his ability to do so. However, Harriet also lacks such experience. Another possibility is that Bob takes such connections for granted. His inability to identify content in his course that would be particularly relevant to future teachers of mathematics as opposed to general mathematics students reflects a lack of awareness of specialized mathematics knowledge-for-teaching. For Bob, subject content knowledge and pedagogical knowledge are distinct. He sees his role as supporting the development of the former.

With regard to the theme of affect, the student-focused study reports an alarming number of negative comments, indicating increases in students' anxiety and decreases in self-efficacy. Both instructors were acutely aware of the impact of affect and described their students as coming into the course with high mathematics anxiety and lack of confidence. However, their perceptions about the cause of the anxiety and strategies for addressing it were quite different. For Bob, the source is students' lack of fundamental skills. As a result, his solution is to help them see the logical structures of mathematics and develop these skills, though he acknowledges that the sheer volume of the material he must cover, in fact, adds to his students' stress. For Harriet the source is negative past experiences and a perception of mathematics as rigid. Her efforts focus on moving students away from the 'one right answer' view of mathematics, helping them develop more flexibility in approaching mathematical problems and to just have *fun*.

From the students we hear that traditional instructional methods of lecture, power point presentations, and drill and practice tended to elevate anxiety and decrease efficacy, while reform approaches such as small group work, hands-on learning, and opportunities to share and discuss were less stressful and increased efficacy. They also shared that the instructor having a caring manner, an approachable demeanour, and a perceived willingness to help supported their learning. These findings echo the study of Schulze and Tomal (2006) cited above. Regardless of their perceptions of the source of their students' anxieties, knowledge of this research could help inform instructor choices with respect to how to address concerns around student affect.

The voices of the students in the student-focused study lend support to concerns that mathematicians in mathematics departments may be unprepared to take on the task of preparing elementary teachers. The lack of connections of content with the elementary classroom and traditional teaching approaches seem to contribute to frustration and anxiety as well as decreased self-efficacy. However, the instructor-focused study shows that though lack of explicit connections to elementary learning may occur, this need not be so. The differences between Harriet and Bob in this regard may have been the result of the mentorship Harriet received, suggesting a potential means for supporting the mathematicians who teach these courses.

Another side of this issue is that mathematicians, at their best, have much to offer future teachers, even at the elementary school level (Hodgson, 2001; Williams, 2008). Jonker (in review) describes mathematicians in mathematics departments as 'stewards of their discipline,' 'passionate about mathematics', and 'eager to share their excitement with students and concerned about the place of mathematics in the world.' The challenge is to create opportunities for conversations between mathematics educators and mathematicians so that students in MFT courses are better prepared to teach mathematics to elementary children.

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A STORY OF THE MULTIPLICATION RING – ABOUT STUDENTS´ SELF-EFFICACY BELIEFS ON MATHEMATICS

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This article is about story writing as an open assignment and the 3rd grade students´ (N=53) self-efficacy beliefs on mathematics. The students wrote mathematics stories after a mathematics lesson of open-ended problems in autumn 2010. This study is a part of the three-years' research project where the aim is to develop a model for improving the level of mathematical understanding, skills and self-confidence of lower grades´ comprehensive school teachers and students when involved actively with open-ended problems. As results in this study the students express well their self-efficacy beliefs in the story writing assignment. There are rich and meaningful thoughts. Anxiety, stress, serenity and optimism can be identified in the students´ stories.

INTRODUCTION

In Finland we have a nine-year comprehensive school where all students learn mathematics in heterogeneous classes. The comprehensive school is divided into lower grades 1-6 and upper grades 7-9. Class size varies about 20 students, and therefore, teachers have difficulties in balancing between low-achievers and high-achievers, especially in upper grades 7-9. (See more in Pehkonen, Ahtee & Lavonen 2007). Teaching in schools is regulated with a national curriculum (FNBE 2004).

According to the three-years´ longitudinal study of Niemi and Metsämuuronen (2010) Finnish comprehensive school lower grade (grades 3-5) students (N= 4551) consider mathematics to be a useful subject. Girls and boys perform almost equally well. However girls have lower perceptions of their own skills than boys. The study emphasizes that it will be necessary to improve girls´ self-esteem in terms of their mathematics skills. The same study highlights also that more focus should be put on problems in solving production assignments, since students´ performance on production assignments is not so promising.

In one of the mathematics stories, collected for this study, a girl writes about a mathematics ring that solves all the multiplication problems. This amazing ring just shines right answers for multiplication problems. There are also other stories with mixed feelings about mathematics. What do the students write about mathematics in these stories? Will there be an answer to what is it that activates these students to achieve more or contrarily less in mathematics?

Here is a story about a mathematics ring.

> A girl and a boy studied in a small old village school. They liked mathematics although it was not their favourite subject. The girl´s name was Anni and the boy´s name was Juho.

> They were siblings. It was an early sunny morning and they were in a classroom. It was mathematics. When Anni looked for a book from her desk she saw there a small box. When she opened the box there was a ring inside. After school she showed the ring to Juho. Juho took it carefully from the box. He put it immediately back because he did not see anything amazing in it. Not even then when Anni told how she had found it. Anni did neither see anything amazing in it. At home when they did their homework the ring started to shine and showed all multiplication tables and answers in it. They did their homework fast and decided that they will keep the ring as a secret. The end. Hilary

For a researcher to work with young students´ stories is like receiving a message from them. To evaluate what the students have to say is an intriguing and empowering experience.

THEORETICAL FRAMEWORK

Main concepts

As important it is at school to accumulate facts and learn academic skills, it is also important to realize that at school students´ basic motivation towards competence and achievement is established (Elliot 2002). Students form beliefs about their performance and themselves as poor or good students based on feedback of themselves, their parents, other students or teachers. These beliefs affect profoundly on their eagerness to learn, on their attitude towards challenges, and on their ability of persistence when facing difficulties (Dweck 2002).

A belief, which also includes feelings, is a rather stable subjective knowledge of a certain object or concern (Pehkonen 1998). According to Pehkonen (1995) students´ beliefs of mathematics can be categorized on beliefs about mathematics, beliefs about mathematics learning, beliefs about mathematics teaching and beliefs about oneself within mathematics that includes self-efficacy beliefs as well. Students´ beliefs about the self; self-efficacy beliefs, control beliefs, task-value beliefs and goal-orientation beliefs have an essential influence on mathematical problem solving (Op ´t Eynde, de Corte & Verschaffel 2002).

Self-efficacy is based on judgements of personal capabilities. Those judgements that a student creates, develops and holds are vital forces in her/his success or failure in all endeavours. (Pajares & Schunk 2002.) For example a student with low self-efficacy believes that problems are tougher than they actually are, she/he feels anxiety, stress and is not able to find an easy solution to solve a problem. Correspondingly a student with high self-efficacy feels serenity towards a problem and is optimistic, and the problem promotes her/his resilience and raises her/his self-confidence.

In figure 1 below is visualized how self-efficacy beliefs form and develop in a learning process where a student´s personal knowledge of mathematics and own learning skills activate her/his motivation towards the learning situation when having a certain assignment. Depending on the student´s judgements of her/his personal capabilities the learning circle can affect profoundly in good or bad into her/his achievement. A motivating assignment has a key role in keeping the student´s learning circle in progress and supporting student´s achievements.

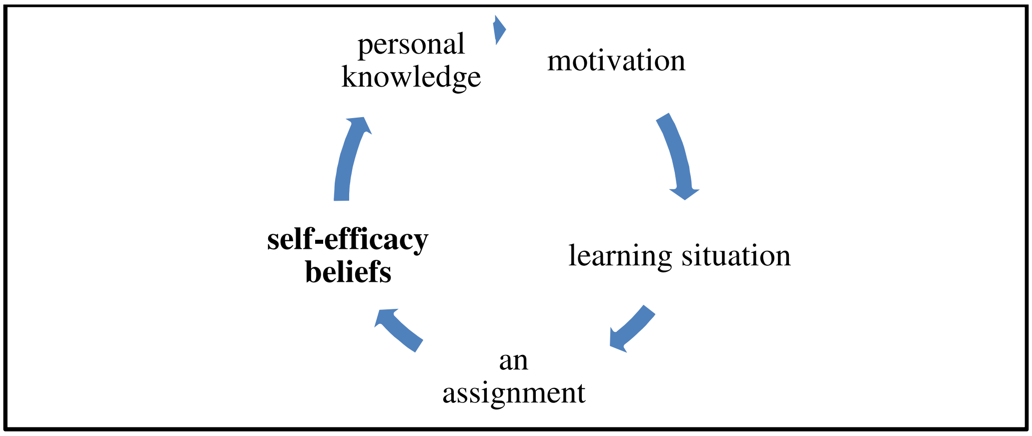

Figure 1: A Learning Circle of how student´s self-efficacy beliefs form and develop.

Dweck (2002) stresses that it is very important to focus on processes that create achievement. To support a student´s sense of self as a mathematics learner is essential in order to promote her/his performance in all mathematical assignments and challenges. Students´ beliefs are dominant, they govern actions for good or sometimes regretfully for bad.

Haven (2000) has stated that telling stories in learning situation create more vivid, powerful and memorable images in students´ minds than many other pedagogical ways of teaching a same issue. Stories can be utilized in mathematics to achieve an environment of imagination, emotion and thinking (Zazkis and Liljedahl 2009). Different methods such as stories, drawings, fairy tales and creative writing help young students to relax and become engaged.

> When students are provided an educational climate that nurtures their fundamental psychological needs, they likely attain not only high levels of achievement but also growth, development, and well-being (Deci & Ryan 2002).

This study is based on students´ narrative mathematics stories. Narrative research pass and produce information, it values a multiplicity of perspectives and its strength is the power of an audience to interpret a meaning (e.g. Coffey & Atkinson 1996, Clandinin & Connelly 2000, Lyons & La Boskey 2002, Kaasila 2007). Narrating is not only describing events or actions; it also relates events and actions, organises them into sequences or plots, and then attaches them to a character (Kaasila 2007).

The main reason for choosing stories to be analysed is that they are powerful lenses for working with young students. They help a reader to relate to students´ experiences. There is a great possibility to captivate young students´ voices, ideas and beliefs and pull a reader along as the story unfolds towards its conclusion. Researcher can learn a lot more when interpreting students ideas and valuing their imagination and exploring their viewpoints (Fawcett 2009).

Egan (2004, 2008) defines a story as a narrative unit that can fix the affective meaning of the elements that compose it. There can be seen similar elements as in problem solving; a beginning that sets up a conflict or expectation, a middle that complicates it, and an end that resolves it. Stories captivate emotions.

The story in this study is defined as a narrative unit that starts either once upon a time or has fictive characters and a fictive plot.

Focus of the study

The purpose of this study is to illustrate 3rd grade students´ self-efficacy beliefs on mathematics. The study concentrates to find out answers to the following research questions:

What kind of stories do young students write about mathematics in an open writing assignment?

What kind of self-efficacy beliefs on mathematics are there present in the stories?

METHOD

The study is a part of a larger Finland–Chile comparison research project, financed by the Academy of Finland (project #135556). The aim of the three-years longitudinal study (2010–2013) is to develop a model for improving the level of mathematical understanding, skills and self-confidence of comprehensive school´s lower grades teachers and students when involved actively with open-ended problems.

Within the follow-up project, the same students and classes are monitored three years, from grade 3 to grade 5. Each class and student has once a month mathematics lesson that involves students actively with open-ended problems. This lesson is video-taped. Same measurements for mathematical skills and self-confidence are implemented in the beginning and at the end of the project. This study deals with the background studies (beginning measurements) for students' beliefs on mathematics.

Participants

The participants were Finnish 3rd grade students (N=53) from the Greater-Helsinki area during their autumn term 2010. The students were nine years old when they wrote these stories. The students were from four different classes taught by four different teachers of the Finland–Chile comparison project. These classes and students were chosen as they volunteered to write mathematics stories.

Indicators

After a first video-taped mathematics lesson a researcher gave to each teacher a written document of a student assignment, a mathematics story. The task was to write a mathematics story. It could also be a cartoon, a letter, a card, a poem or a rhyme. No other advice was given.

Each teacher introduced the assignment to her/his students as it was given to the teacher. The students had the possibility to choose their own way to create a mathematics story. The students themselves decided what to write. The assignment was done in autumn 2010 during one mother tongue lesson taught by the same teacher who taught them also mathematics.

Data gathering

Once all the stories were written the researcher collected them from the teachers. She also thanked the students for their stories when she visited the classes a while later.

The stories were put in different categories by the researcher according to what the students had written, what the stories were like. The following groups were found: a fictive story, a cartoon or a drawing with some text in it, an evaluation report or a letter of how the student was doing in mathematics or what she/he thought about mathematics and something else.

A fictive mathematics story started with a phrase “once upon a time” or had fictive characters and a fictive plot. A cartoon or a drawing had speech bubbles or some text in it. It could have a fictive plot, too. An evaluation report/letter of mathematics was about the student´s own thoughts of mathematics as a subject or how she/he was doing in mathematics. The last category was for those stories that did not fit into the any other categories. Only in this category the stories did not have any mathematics-related content.

According to what kind of self-efficacy beliefs on mathematics were found in each story, cartoon or evaluation report/letter that had some mathematics in it, they were divided into five categories by the researcher: anxiety, stress, serenity, optimism and no self-efficacy beliefs of mathematics. The categorising is based on Pajares and Schunk -theory (2002) of how self-efficacy beliefs influence on student´s engagement and confidence.

If a student or a character was worried, not happy with her/his performance in mathematics or the results on mathematics or there was a need to learn more she/he was evaluated to be anxious. Any stronger emotions were categorized under stress. On the other hand, a student was categorized to be serene if the character was happy or serene with mathematics. When a student or a character of the story was really satisfied with her/his performance in mathematics and results on mathematics she/he was categorised as optimistic.

Examples of the different categories in detail are presented in the results section.

RESULTS

What kind of stories did young students write about mathematics?

The students´ stories as data produced rich and variegated results. The assignment to write a mathematics story inspired the 3rd grade students to write mostly fictive mathematics stories with plots. Some of the cartoons had fictive plots, too. Boys preferred stories and cartoons or drawings with text more than girls whereas girls wrote evaluation reports more often than boys.

There were only two stories that did not have any mathematics in them, a fictive cartoon and a card. A cartoon was about farting powder and how it could explode easily. And in the card to a friend there was a message to send some more cards and to meet the friend tomorrow at the concert.

Most of the stories had a problem or puzzling situation that was solved in the end, typically in school. The following example shows how children can imagine persons with magical power:

> One day a class did not like to do mathematics for example A x A + A=20 or B x B + B x B=?. One day a teacher told the students that if they do not do mathematics a witch will come and take them along. The students did not believe the teacher. However they always walked together as a class to home and to school. Still they did not do mathematics. One day they saw the horrible witch and because they were afraid of it they started to work. The teacher wondered why do they do mathematics so fast? One day the students were so tired to do no more mathematics and were really wondering if the witch is true or not. But suddenly the witch came and took all the students and told them that they need to work immediately on mathematics. And from then on the students always worked hard on mathematics. The end. Fara

Mathematical content in the stories was simple arithmetic, mainly multiplication exercises, that had been taught during the beginning of the 3rd school year. There were some more difficult multiplication tasks like 100 x 100 or bigger amounts of money like 50 000€ or 500 000€ where the students wanted to challenge themselves or to achieve more.

Money did interest the students, specially the boys. Many of the money stories were fictive. The students wrote about themselves or somebody else wanting to have something like pets, sweets and sport items but also bread and butter, beer, a fighter plane, a car and fuel and a sofa with certain amount of money.

> Robert has 50 000 €. He buys from the army a fighter plane. It costs 45 000 €. He has 5000 € left. James

In the students´ stories fictive characters were mainly other children, but they could also be animals like bears, squirrels and mice. Animals had a role to help and support. They were characters that knew mathematics well and enjoyed mathematics a lot.

One girl wrote about a squirrel that taught her to count like the other children. There were also a vampire and a witch that made children to work harder and achieve better.

In few stories a teacher or parents were present. Their role was either to encourage a student to work harder or to support when there were problems but also vice versa a student could help her parents.

Like in the stories also in the students´ cartoons there were problems or confusing situations that were solved in the end, just like in a stick figure´s mathematics test.

> A friend: - Yeah, tomorrow is a mathematics test.- A stick figure: – Is it?- Later in the evening. The stick figure: – Huh I forgot it.- At night. The stick figure: – I cannot sleep.- At school. The stick figure: – I do not want to have a bad mark in the test.- At home. The stick figure´s father: - How did the test go?- The stick figure lies: - It is tomorrow.- The father: - Ah, so.- The stick figure: - What do I do now?- The father: - What did you say?- The stick figure: - Nothing.- The father: - Okay.- In the evening. The stick figure: - Father.- The father: - Yes son.- [There is also a mother in the picture.] The stick figure: - The test was today.- The father: - Well show me.- [The result is *5*-, just barely better than fail.] The stick figure´s father and mother: - It is okay.- The stick figure: - Yeah.- The end. Andy

In the evaluation report or the letter the students wrote more about how they were doing in mathematics and what they thought about mathematics.

> I am not doing so well in mathematics. But sometimes I can do well. Mathematics is a bit boring but it is also quite nice. Mathematics is a bit difficult but not too much. I like mathematics when we play with a computer. I do not like to be in the weakest group. My poorest mark is 5- and the best 9+. My mother asks me to practise every day. Ann

> How are you doing in mathematics mother? If you are not doing so well I can help you. Can you count 8 x 7? If not, so how about 5 x 4? If you are not so weak in mathematics, this letter is also to the father. Can you count 5 x 8? ... Ida

What kind of self-efficacy beliefs on mathematics were there present in the stories?

Anxiety, stress, serenity and optimism could be identified in the stories. There were only two stories that did neither have any mathematics nor beliefs in them.

Difficult homework, a group, tests and games, not to be able to count like the other students made students anxious and caused also pressure towards mathematics. There was, for example, a message to a friend:

> Hi Viola! I am working on my homework. The mathematics exercises are really difficult. I do not know how much is 6x6? Can you come and help me? Emily.

The students expressed even stress in some stories. There was the stick figure in a cartoon that could not sleep because he forgot the multiplication test and to practise for it, and the test did not go well. He lied to his parents first that he did not have a test yet. He was afraid of what the parents were going to say or do.

A girl described how it was scary to go to the mathematics lesson because students were disappearing there. In the story about the witch, children were afraid of the creature and that it would come if the kids did not work hard in mathematics. There was one boy who died because he was so disappointed to his result (see Figure 2 below, three first pictures). The previous result was 10, the best possible. Then he got 4 and failed.

However in most of those stories where we identified some stressful thoughts, the end was better and more serene: the parents were forgiving and understanding even if the test did not go well or when children worked harder the witch did not return.

Serenity and optimism were reflected in many stories. Mathematics was the way to solve the problems. Students were satisfied when they got what they wanted if it was money that made it possible or a test that went fine. There was one girl with great optimism. She was so pleased on her results in mathematics that she even offered to help her parents in mathematics in her letter.

In the students evaluation reports they wrote about how they were doing in mathematics and what kind of results they had had in the tests. The evaluation criteria were nice, easy, not so well, boring and difficult. The students liked computer exercises, a small group or their teacher. One girl did not like to be in the weakest group but there was a serene boy who preferred the small size of the same group in his letter to his father.

> It has been nice to study mathematics. Because I have learnt new things. When I am in a smaller group it is easier to study mathematics. It is also somehow nicer to study in the smaller group. Jack

There were students who were really happy about mathematics. One girl considered herself being a mathematics genius. Great optimism!

> A story about a mathematics genius and a poor mathematics student. Mathematics genius is very good in mathematics. A poor mathematics student cannot count at all. I am a mathematics genius. My friend is good in mathematics too. There are other good students, too, and also poor students. My favourite subject is mathematics. I would like to have mathematics always. It would be so wonderful. The mathematics tests are nice. Sometimes mathematics is difficult but also sometimes easy. I do not know how the other students feel about mathematics... Fiona

There were also two drawings in the story above: a picture of a mathematics genius and a poor mathematics student saying that

> I do not know anything.

In another picture (see Figure 2, the last picture) there is a heart and a text inside:

> My favourite subject is mathematics. Mathematics is the best.

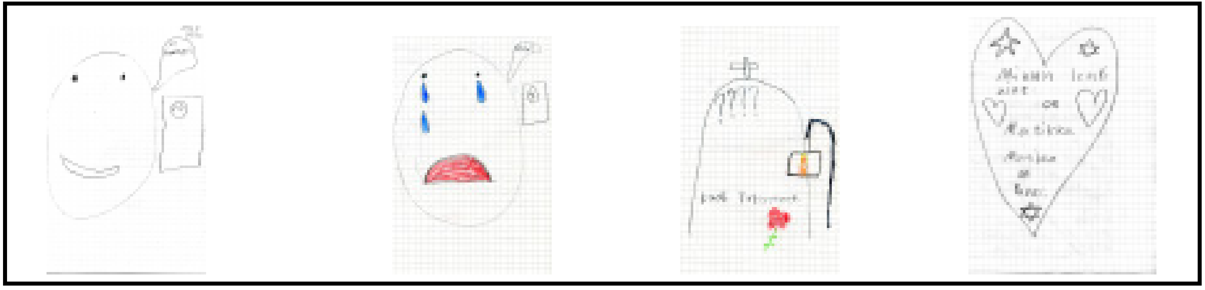

Figure 2: The students´ pictures in their stories.

CONCLUSION

The students´ basic knowledge of mathematics and their learning skills and beliefs have been engaged in a learning situation by the motivating and effective assignment, story writing. Stories really captivated emotions. It was evident that mathematics was a meaningful subject to these students. It provoked strong emotions and the beliefs that exist were quite dominant. Even though mathematics was not always easy and it did trigger strong emotions there was, however, a strong will that things will become better. When there is a will it is likely that a way out will be found.

We are aware that the emotions of the characters in the stories are not necessarily the emotions of their narrators. However, the narrator does perceive these emotions possible for someone and hence, we believe these emotions to reflect the narrators´ perception of mathematics.

This time the stories were written after a problem solving lesson but how about having story writing sessions before the problem solving lesson? Would story writing as an assignment inspire students´ achievements and commitments in mathematics? Would story writing reveal these hidden "mathematical rings", maybe it could even open locks of those "I cannot" –beliefs? At least in this study story writing assignment affected profoundly to the students´ behaviour, motivation and achievements. The students expressed themselves openly and wrote about their personal issues that were important to them as students of mathematics.

Each student´s learning circle in mathematics is important and individual. There should be more focus on assignments that encourage students to do self-evaluation and to share their own thoughts about their learning. It is easier to challenge oneself when one can freely express her/his thoughts of mathematics.

Let us hope that every student could have a chance to find such a mathematics ring from her/his desk to experience that "I got it" -feeling in mathematics.

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ESTONIAN SCIENCE AND NON-SCIENCE STUDENTS' VIEW OF MATHEMATICS AT UNIVERSITY LEVEL

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The current paper investigates the attitudes and beliefs toward studying mathematics by university level students. A total of 970 randomly chosen, first year, Estonian bachelor students participated in the study (of which 498 were science students). Data was collected using a Likert-type scale questionnaire and analysed with a respect to field of study (science and non-science). Results of this study show that science students have more positive view of towards studying mathematics.

INTRODUCTION

One of the strategic objectives of Estonian higher education for the period 2006 to 2015 is to put higher education in the service of Estonian development and innovation. Scientific research and education is aimed at the needs of Estonian society and economy. Based on world developments and the current situation in Estonia, one of the strategic objectives of Estonian higher education for the next 10 years is: satisfy the needs of Estonian society for a highly qualified workforce, taking into account the integration of the Estonian economy into the Nordic countries' economy, with preferential development of studies in the natural and exact sciences and in technology (Estonian Ministry of Education and Research, 2008).

At the same time, relatively little attention has been paid to study this situation in Estonia (Estonian Ministry of Education and Research, 2008). The effectiveness of mathematics teaching at the tertiary level is an issue; the student drop-out rate is high and motivation is relatively low. Therefore the focus of this paper is on university level mathematics teaching and learning.

Teaching and learning mathematics at university level is a big challenge in most European countries (Baumslag, 2000), even though mathematical applications have become more common during the course of studies in many subjects such as economics, technology and science (Baumslag, 2000). The use of computers and mathematical programs in the workplace have become commonplace and therefore the educational aims in mathematics have changed. Simple arithmetic has become less important and being able to form real-life mathematical models and use software has become more dominant (Vogt, Hocevar & Hagedorn, 2007; Petocz & Reid, 2006). Teaching mathematics at university level, as it stands, is based like many other subjects on the system of lectures. The huge quantities of work covered by each course in such a short space of time make it difficult to understand and elaborate. The pressure of time seems to take away the essence of mathematics creativity and does not allow any true understanding of the subject. University mathematics is often presented in a formal way that causes many students to cope by memorizing that which they perceive as a fixed body of knowledge rather than learning to think for themselves (Yusof & Tall, 1999). The number of students with excellent mathematical skills is declining; therefore the effectiveness of teaching mathematics is becoming more essential (Biehler, Scholz & Strasser, 1994; Abiddin, 2007) and students' beliefs, attitudes and motivation towards mathematics teaching and learning are playing an important role in reflections on the teaching of mathematics (McLeod, 1992).

The study of students' mathematical beliefs has received much attention in recent years. Most studies of beliefs have been carried out with a separate focus on cognitive, motivational, or affective aspects and only a few contributions explicitly address beliefs as a system (Op 't Eynde & De Corte, 2003). In order to emphasize the present focus on studying the structure of students' mathematical beliefs, a belief structure which combined previously published instruments, is used in this study (Kaldo and Hannula, 2011).

The aim of the study is to analyse correlations between factors on students' belief structures towards mathematics and the differences between students' beliefs and attitudes according to their field of study (science and non-science) at university.

Theoretical framework

A framework for mathematics-related beliefs forming a belief system has been suggested, adopted and adapted by a number of researchers in the field (Hannula et al., 2005; Op 't Eynde & De Corte, 2003). Belief systems can be characterized by how beliefs in multiple categories work together to influence thought and guide behaviour. Schoenfeld (1985) describes mathematical belief systems as: "one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks."

In their study, Rösken, Pehkonen, Hannula, Kaasila & Laine (2007) primarily focused on the systematic character of beliefs as they were interested in dimensions describing such a view of mathematics. They obtained seven dimensions structuring this construct. Both studies mentioned above (Op't Eynde & De Corte, 2003, Rösken et al, 2007), miss one important aspect of beliefs and attitude towards mathematics; namely, motivation. Some items on motivation were included (Hannula, Kaasila, Laine & Pehkonen, 2006), but they failed to form a reliable component. Motivation plays an important role in the studies of mathematics. The prominent questions are why it is necessary to teach a specific topic and why it is relevant. The questions become important in trying to find practical implementations. To understand student behaviour, one needs to know their motives. And this is an especially important task for the lecturers in Estonia, because the number of students is decreasing. Students cannot be ignored.

A study of Flemish junior high students reveals that high achieving students have more positive beliefs concerning the relevance of, and their ability in, mathematics than low achieving students (De Corte & Op 't Eynde, 2003). Thorndike-Christ

(1991) affirm De Corte and Op ’t Eynde’s results with United States middle school and high school students. In the Thorndike-Christ (1991) study, students in advanced classes held significantly more positive attitudes toward mathematics as measured by the Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976) than students in regular and remedial classes.

Students’ mathematics-related belief systems are characterized by their beliefs about mathematics education, beliefs about self, and beliefs about the class context (Op’t Eynde & De Corte, 2003). This framework is similar to that described by McLeod (1992), but one difference between the two frameworks is McLeod’s fourth category: beliefs about mathematics teaching. This category could be part of mathematics education as it relates to teaching practices, or it could be part of classroom context if the beliefs are related to one’s personal experiences in being taught mathematics.

In order to emphasize the present focus on studying the structure of students’ mathematical beliefs, I use the term *view of mathematics* in this paper. This term was originally introduced by Schoenfeld (1985) and later adapted by others (Pehkonen, 1996; Pehkonen & Törner, 1996). The students’ view of mathematics is a result of their experiences as learners of mathematics and as such, it provides an interesting window through which to study the teaching of mathematics. The *view of mathematics* indicator has been developed in 2003 as part of the research project “Elementary teachers’ mathematics” financed by the Academy of Finland. It has been applied to and tested on a sample of student teachers and was modified for the present sample.

Summarising here, I am interested in students’ views of mathematics as a result of their experiences as learners of mathematics in tertiary level. With regard to this focus, I pay attention to the cognitive component described by beliefs as well as to motivational aspects. The choice of concept draws on the following aspects: first, beliefs are often considered to be on a more cognitive side of the affect (e.g. McLeod, 1992). Using ‘‘view’’ instead of ‘‘beliefs’’, I want to emphasize that not all dimensions I address are cognitive ones. Second, I consider the term ‘‘view’’ more appropriate to capture the structural properties of the affect–cognition interplay in social learning situations. In some sense, the term ‘‘beliefs’’ is separate while ‘‘view’’ is holistic (Roesken, Hannula, & Pehkonen, 2011).

Some studies about students’ and teachers’ attitudes in comprehensive schools, or in upper-secondary schools have been carried out in Estonia (Lepmann, 2000; Lepmann & Afanasjev, 2005; Pehkonen & Lepmann, 1994; Kislenko, 2009). For example, the study by Lepmann & Afanasjev (2005) revealed that high-ability students have considerably greater faith in achieving success in mathematics learning than low-ability students. Compared to other students, high-ability students are considerably more desirous of each student being able to work according to his or her ability. They want to develop their ability and are ready to do more work in the name of success. However, low-ability students are more disposed to giving up than students with high abilities. More recent results by Kislenko's (2009) indicate that students in comprehensive and secondary schools perceive mathematics to be important, but studying it tends to be difficult and boring. However, there has not been a study of science and non-science students' beliefs of mathematics in Estonia at the university level and thus to date it is an unexplored area.

Design of the research

Pilot study

A pilot study was carried out in Estonia in spring, 2009 with 93 students. Before carrying out the pilot study, the questionnaire was translated into Estonian and then back to English. As one of the aims was to make a comparative analysis it was deemed important that the translation had to be carried out with a high degree of accuracy

Since the aim of the pilot study was to confirm the earlier scales on beliefs (Rösken et al., 2007; Diego-Mantecon, Andrews & Op 't Eynde, 2007), attitudes (Yusof & Tall, 1994) and motivation (Midgley et al., 2000), a confirmatory factor analysis was performed. The pilot study gave a positive signal about the usefulness of the instrument, as the component structure remained stable for different populations (Kaldo, 2011).

Main study

Questionnaire

After undertaking the pilot study and in order to develop a valid and reliable assessment tool, a draft questionnaire was prepared from 62 statements. Items describing the background of students and lessons were not included. The factor analysis was carried out using SPPS. Some factors containing only two items, low Cronbach alphas or items which had communalities less than 0.3, were removed. Finally 35 items were left, in 7 factors, for the main survey (Kaldo & Hannula, 2011). The structure of students' view of mathematics was assumed to consist of the following seven factors:

- F1 performance-approach goal orientation,
- F2 mastery goal orientation,
- F3 relevance,
- F4 personal value of mathematics,
- F5 student competence,
- F6 teacher role,
- F7 cheating behaviour (see Appendix, Table 5).

(Principal component analysis gave the results close to similar surveys (Rösken et al. 2007, Op ’t Eynde & De Corte, 2003; Midgley et al. 2000); the same component names were used.

Participants filled in the questionnaire on paper, responding to a 4 option Likert scale (1- strongly disagree, 2- partly disagree, 3- partly agree, and 4-strongly agree). Students were given 40 minutes to complete the questionnaire anonymously.

Research participants

The total number of bachelor students in Estonia is 31691 and first year bachelor students is 8770 (Estonian Ministry of Education and Research, 2011). To provide valid estimates of student achievement, the sample of students had to be selected in a way that ensured sufficient representation of the full target population. The target population are students who study at least on course of mathematics as part of their studies.

The main survey covered a sample of students drawn from the first year mathematics course from one private and four public universities: Estonian Business School (*EBS* hereinafter), Tallinn University (*TLU* hereinafter), Tallinn University of Technology (*TUT* hereinafter), Tartu University (*UT* hereinafter) and University of Life Sciences (*ULS* hereinafter). In total, 970 students who had taken at least one compulsory first year mathematics course participated in the study on a voluntary basis. A response rate of 69% was achieved. The age of participants ranged between 18 and 34; average age was 20 with 96.6% of all students between the ages of 18 and 23. Of the participants, 508 were male and 462 female; 88.5% were Estonian. 10.1% Russian and 1.4% were from different nationalities. There were 498 (260 male and 268 female) science students and 472 non-science students (see Table 1). The sample of science students were studying natural science, mathematics, physics etc. while the sample of non-science students were from business, economics, etc.

University	Frequency	Non-science students	Science students	Per cent	Cumulative per cent
EBS	91	91	0	9.4	9.4
ULS	228	228	0	23.5	32.9
TU	103	2	101	10.6	43.5
TUT	314	134	180	32.4	75.9
UT	234	17	217	24.1	100.0
Total	970	472	498	100.0	

Table 1: Characteristics of participants

Results of Research

The students' beliefs about mathematics were structured. The main interest was in the correlations between factors for the seven factors. These are given in Table 2.

Factors	F1	F2	F3	F4	F5	F6	F7
F1 Performance-approach goal orientation	1						
F2 Mastery goal orientation	0.288**	1					
F3 Relevance	0.131**	0.607**	1				
F4 Personal value of mathematics	0.141**	0.551**	0.723**	1			
F5 Student competence	0.200**	0.489**	0.538**	0.409**	1		
F6 Teacher role	0.085**	0.356**	0.324**	0.276**	0.325**	1	
F7 Cheating behaviour	-0.023	-0.280**	-0.267**	-0.186**	-0.365**	-0.176**	1

***Correlation is significant greater than 0 at the level 0.01 (2-tailed)*

Table 2: Correlations between the factors

Table 2 shows that nearly all correlations were statistically significantly greater than 0. The correlation matrix indicates that the correlation between factor *F4, relevance* and *F5, personal value of mathematics* was the highest. The correlation is moderate between the factors: F2, *mastery goal orientation* and *F4, relevance;* F2, *mastery goal orientation* and *F5, personal value of mathematics; F2, mastery goal orientation* and *F6, student competence; F6, student competence* and *F4, relevance; F6, student competence* and *F5, personal value of mathematics.* The other correlations were considered weak.

In the Table 3, the lowest average values are for factors F7, cheating behaviour and F1, performance approach goal orientation. A neutral position, close to agreement, is shown for factors F5, students' competence and F6, teacher role. The highest averages are given for factors F3, relevance of mathematics, F2, mastery goal orientation and F4, personal value of mathematics.

The t-test assesses whether the means of two groups are statistically different from each other. In table 4, most factors are relevant for both science and non-science students who participated in the survey. Both groups perceive the subjects equally. T-tests for equality of means (independent samples) for each of the factors show that factors, F2 mastery goal orientation; F3, relevance; F4, personal value of mathematics; F5, student competence and F7, cheating behaviour show a significant difference for science and non-science groups. No significant difference was found for factors F1performance-approach goal orientation and F6, teacher role.

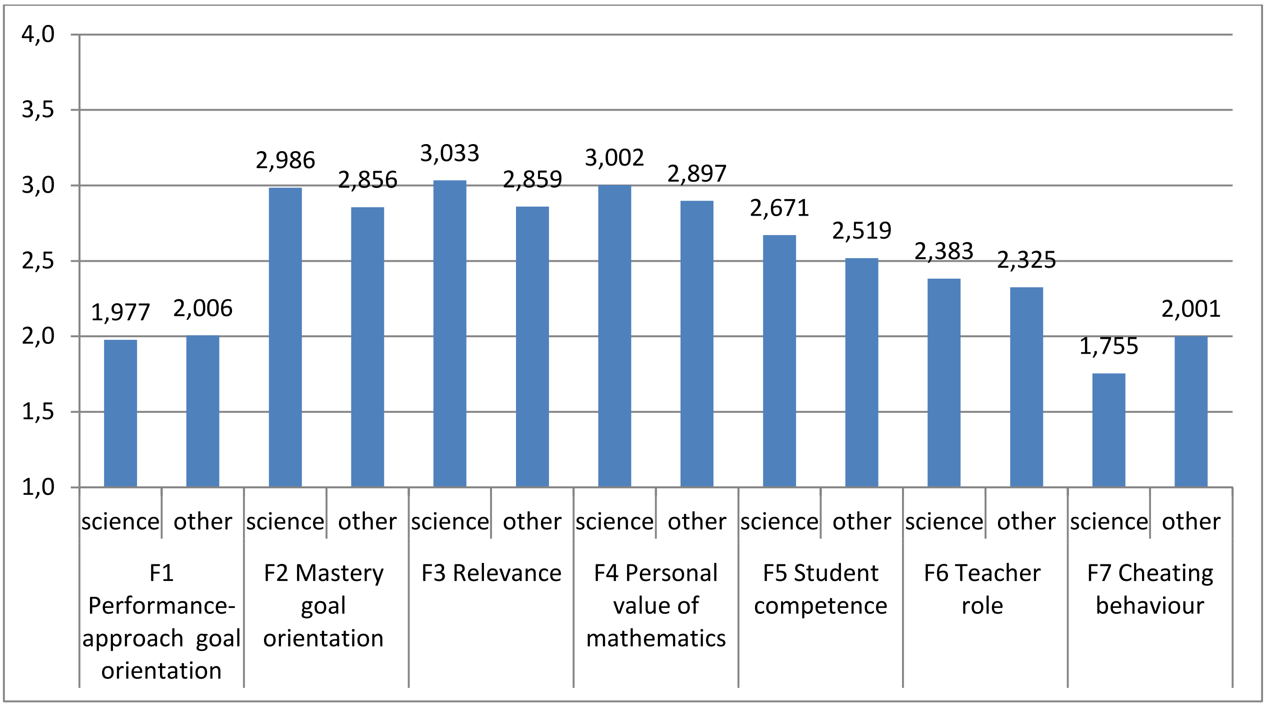

Table 3: Mean value science and non-science students responses to items in the questionnaire by factors

	t-test	Sig. (2-tailed)	Mean difference	Std. error difference
F1 Performance-approach goal orientation	-0.690	0.490	-0.2893	0.04193
F2 Mastery goal orientation	3.694	0.000	0.12957	0.03507
F3 Relevance	5.422	0.000	0.17390	0.03196
F4 Personal value of mathematics	2.613	0.000	0.10443	0.03997
F5 Student competence	3.692	0.000	0.15162	0.04106
F6 Teacher role	1.461	0.144	0.05817	0.03983
F7 Cheating behaviour	-4.628	0.000	-0.24636	0.05335

Table 4: Results of t-test for determining the equality of mean responses between science and nonscience students

Discussion and conclusions

In the current study, it was concluded that the components mastery goal orientation, relevance, personal value of mathematics and students competence seemed to form the 'core' of the belief structure, each having a high correlation with one and another. The highest correlation (0.723) between relevance and personal value of mathematics indicates that these components are essentially measuring the same thing. Students with high mastery goal orientation level, typically consider themselves to be competent and mathematics to be important. They also have a positive attitude towards mathematics, want to perform well and have a positive view of their teacher. Moreover, they typically do not cheat. Moderate correlation is also shown between students' competence and relevance (0.538).

For most factors (six of seven) in this study, science students have more a positive view of mathematics than non-science students. The difference between means is not great, but is statistically significant in five factors. Both science and non-science students have a positive attitude to the factors F2, Mastery Goal Orientation; F3, Relevance; F4, Personal Value for Mathematics and F5, Students Competence. They think that knowledge of mathematics is important; it helps us to understand the world. They study mathematics because they know how useful it is. They feel that they are good in mathematics and are motivated to study mathematics. They think that mathematics is an important subject. The Teacher Role holds a neutral position close to disagreement suggesting the teacher has not inspired them to study mathematics very well. Additionally, this study shows that science students cheat less than non-science students. Statistical non-significance was found between the factors F1, Performance-approach goal orientation and F6, Teacher role.

According to this research, there is a significant difference in students' view of mathematics, with science students having a more positive attitude than non-science students. The results of this study are in agreement with the studies of De Corte & Op't Eynde (2003) and Thorndike-Christ (1991).

In Estonia there is a declining percentage of students studying science, technology, engineering and mathematics. Kislenko's (2009) study shows that for students' mathematics is important, but studying it tends to be difficult and boring. Therefore there is a need to study, in more detail, the mathematical attitudes of science vs. non-science students. Such an analysis can help mathematics teachers better understand the needs and characteristics of students from different specialities, which in turn can help mathematics teachers better serve and advise students.

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Appendix

F1 Performance-approach goal orientation (Cronbach's alpha =0.78)	Loadings
15. It's important to me that other students in my class think I am good at my class work.	0.576
16. One of my goals is to show others that I'm good at my class work.	0.674
26. One of my goals is to show others that class work is easy for me.	0.514
27. It's important to me that I look intelligent compared to others in my class.	0.843
F2 Mastery goal orientation (Cronbach's alpha =0.74)	
17. It's important to me that I improve my skills this year in mathematics.	0.335
52. I am very motivated to study mathematics.	0.677
64. It's important to me that I thoroughly understand my class work.	0.379
65. It's important to me that I learn a lot of new mathematical concepts this year.	0.559
71. One of my goals is to master a lot of new skills this year.	0.390
78. One of my goals in class is to learn as much as I can.	0.429
F3 Relevance (Cronbach's alpha =0.82)	
22. Some knowledge of mathematics helps me to understand other subjects.	0.471
28. Knowing mathematics will help me earn a living.	0.302
29. I think mathematics is an important subject.	0.613
34. Studying mathematics is a waste of time.	-0.451
49. I can use what I learn in mathematics in other subjects.	0.623
59. I study mathematics because I know how useful it is.	0.472
69. Mathematics enables us to understand better the world we live in.	0.731
73. I can apply my knowledge of mathematics in everyday life.	0.526
F4 Personal value of mathematics (Cronbach's alpha =0.70)	

23. Knowledge of mathematics is important; it helps us to understand the world	0.797
30. Mathematics is useful for our society.	0.510
74. After graduating university I have many opportunities to apply my mathematical knowledge.	0.484
F5 Student competence (Cronbach's alpha =0.82)	
24. Mathematics was my worst subject in high school.	-0.455
25. Mathematics is a hard for me.	-0.514
46. I am good at mathematics.	0.778
47. I think that what I am learning in mathematics is interesting.	0.688
48. Compared with others in the class, I think I am good at mathematics.	0.574
55. I understand everything we have done in mathematics this year.	0.552
F6 Teacher role: *(Cronbach's alpha =0.72)*	
37. My lecturer explains why mathematics is important	0.468
53. The lecturer has not been able to explain the processes we were studying	-0.563
54. My lecturer has not inspired me to study mathematics	-0.497
60. My lecturer tries to make mathematics lessons interesting	0.577
68. In addition to mathematics, the lecturer teaches us how to study	0.419
F7 Cheating behaviour (Cronbach's alpha =0.81)	
42. I sometimes copy answers from other students during tests.	0.788
43. I sometimes cheat whilst doing my class work.	0.690
62. I sometimes copy answers from other students when I do my homework.	0.400

Table 5: Questionnaire used in the main study

WHAT MAKES AN INTERESTING MATHEMATICAL PROBLEM?

A PERCEPTION ANALYSIS OF 22 ADULT PARTICIPANTS OF THE COMPETITION MOVEMENT

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The two aims of the paper are: (1) to identify the goals that the adult participants of the competition movement wish to achieve with an aim of the problems given at mathematics competitions, and (2) to identify characteristics of an "interesting competition problem" which contribute to achieving these goals. The data were collected from 22 adult participants of the competition movement from seven countries. The findings stress the proximity of the competition movement to the mainstreams in the field of mathematics education, and support adapting its successful practices for a broad range of students. Some practical and research applications are discussed.

INTRODUCTION

When Professor Tibor Rado reflected on the mathematical problems given at the first Eötvös Competitions, he said:

> […T]he problems are selected, however, in such a way that practically nothing, save one’s own brains, can help…. The prize is not intended for the good boy; it is intended for the future creative mathematician. (Rado, 1932, p. 86)

This view of competition problems is shared by many participants of the competition movement (e.g., Kenderov et al., 2009). For an outsider, this view might create an impression that there exists a secret bank of exceptional problems which have some “magic powers” to make people admire and enjoy solving them. This paper makes an attempt to detect the sources of these “magic powers”.

The aim of the paper is twofold: (1) to identify the goals that the adult participants of the competition movement wish to achieve with an aim of the problems given at mathematics competitions, and (2) to identify characteristics of an "interesting competition problem" which contribute to achieving these goals. Some mathematics educators can say that an attempt to identify general problems’ characteristics which have the potential to make some sort of positive effect on an intended solver is doomed to failure, since “everything is contextual”. This paper does not oppose this approach. However, it argues that at least in the case of competition problems there is a “built-in kernel” which has the potential to be appreciated by the intended solvers. Resorting to a metaphor, when millions of people around the world are amazed by the Renaissance art, it is at least partially because of some fundamental structures which are mutually shared by the representatives of this art style.

LITERATURE REVIEW

In the first subsection of the review we compare between different kinds of mathematics competitions focusing on their general goals and apparent differences in the problems' styles. The next two subsections are dedicated to problems' cognitive demand and surprise, the notions which are commonly used when challenging problems are under discussion.

ON DIFFERENT KINDS OF MATHEMATICS COMPETITIONS

Eötvös Competition of 1894 is often addressed as the first official national competition for high-school students (e.g., Rado, 1932). It also appeared to be a role model for many other competitions which followed its steps. Kenderov et al. (2009) referred to this kind of competitions as *exclusive* since they were intended for a relatively small number of exceptional students interested in mathematics; "baby-mathematicians" in words of Rado (1932). Some of these competitions were intended only to invited participants ("closed" competitions), and the others were open for all. However, in both cases the expected knowledge base of the participants went ways beyond standard school syllabi. In the second half of the last century there was a rise of *inclusive* competitions, which were intended to much broader audience. Kenderov et al. (2009) said that *exclusive* competitions are perceived as a strong identifying tool of mathematical ability as well as a way of nurturing it, whereas *inclusive* competitions are meant to raise public awareness of mathematics by providing recreation and fun through mathematics problems.

The structure of a questionnaire in exclusive and inclusive competitions is also different. The exclusive questionnaires are characterized by a relatively small number of open problems and relatively big amount of time for their solutions (for instance, four hours for the solution of three problems at Eötvös Competition). In inclusive competitions, a relatively large number of closed problems is given for the solution during a relatively small amount of time (for instance, in International Mathematical Kangaroo the students are given three hours for the solution of 25-30 multiple choice problems).

We suggest addressing the notions of exclusive and inclusive competitions as a continuous scale rather than dichotomy, since many competitions reputed to be exclusive have the attributes of inclusive competition, and the opposite is also true. Interestingly, sometimes the same attributes are in use. For example, in the Tournament of the Towns (a more exclusive competition) and in International Mathematical Kangaroo (a more inclusive competition) the problems have different top scores. This enables to involve more students in a competition by combining easier problems and still to challenge the more progressive students. However, the intended participants of these competitions are still different.

ON COGNITIVELY DEMANDING PROBLEMS

In the above section, the notion of a *problem* was introduced. Schoenfeld (1992) stressed the difference between problems and exercises in the following way: "[...] exercises organized to provide practice on a particular mathematical technique that, typically, has just been demonstrated to the student" (p.11). Problems, on the other hand, are often characterized by a gap between the given initial and the goal state, when the means for achieving the goal are unclear in advance for a particular solver (NCTM, 2000). Frederiksen (1984) said that when handling the cognitively demanding problems the solver can not apply known algorithms and schemes and needs to approach the problems heuristically.

A possible operational approach to the structure of cognitively demanding problems can be made by combining the literature on mathematics education with psychological literature. Namely, Selden, Selden, Hauk, and Mason (2000) argued that every problem provokes a *problem situation image* – "[...] a mental structure possibly including strategies, examples, non-examples, theorems, judgments of difficulties, and the like, linked to the problem situation" (p.145). The variations of these constructs are present in the psychological literature on the insight problems. Namely, it is noted that when working on the insight problem a solver often gets a feeling of being in an impasse, and a significant change approach is needed for the solution (e.g., Bowden & Beeman, 1998). In other words, the initial problem situation image does not contribute or, as it will be argued shortly, even mislead a solver in her way to the solution.

The psychological literature agrees that solving an insight problem involves some kind of restructuring of the initial problem representation. Weisberg (1996) differentiated between *discontinuity* and *restructuring*. Generally speaking, *discontinuity* includes a thinking change in a search for the solution strategies, when a *restructuring* is a radical change in representation of problem situation image (e.g., a re-conceptualization of the initial stage and the goal of the problem).

ON SURPRISING PROBLEMS

We are naturally attracted to aesthetically appealing problems. The characteristics of aesthetical appeal are highly subjective and driven by the personal mathematical taste and other factors. Koichu and Berman (2005) pointed out that in this research area such subjective and vague term as beauty, elegancy, ingenuity, interest and etc. are often defined by means of synonyms that are also subjective and vague. This paper resorts to the notion of *surprise* – an integral part of aesthetical appeal, since surprise is operationally elaborated in the fields of mathematics education and psychology.

The psychological literature refers to *surprise* as a kind of a cognitive conflict, which follows from an occurrence or an absence of the expected phenomenon. Ludden, Schifferstein, and Hekkert (2009) wrote that a feeling of surprise can be followed by a change in a face expression, vocalization, stopping of the ongoing activity, focusing on the surprising event and inquiring this event. The researchers noted that when the first two actions are made unconsciously, the remaining ones are initiated and conducted at the result of the inner motivation. Hence it is not surprising that the power of surprise was acknowledged and implemented by many mathematics educators for the learning purposes (e.g., Movshovits-Hadar, 1988).

Movshovits-Hadar (1988) presented and illustrated a list of ten possible sources of surprise in school mathematics: (1) a common property in a random collection of objects (2) a small change that makes a big difference (3) unexpected existence and non-existence of the expected (4) a rare property becomes generalizable (5) analogies which prove not-analogous (6) plausible reasonable that fails (7) refutation of a conjecture obtained inductively (8) the limit process yields an altogether new findings (9) a single algorithm solves infinitely many problems (10) mathematical paradoxes.

From the abovementioned list, one may conclude that the presence of surprise is highly sensitive to the manner of organizing a potentially surprising task. Therefore, not all sources are equally relevant in the case of the problems for mathematics competitions (e.g., see the tenth item). For the further purposes, let us focus the reader's attention to the first, forth and ninth items. Possibly in these cases the sense of surprise follows from the unexpected gap between the suitability of a *particular* characteristic or method to a *wide range* of objects or situations.

METHOD

Twenty two adult participants of the competition movement took part in the study. Three of them identified themselves as problem composers, three – as coaches for the mathematics competitions, and the rest of the participants combine the previous roles with organizing the competitions. The participants were from Australia ($n=1$), Bulgaria ($n=1$), Israel ($n=9$), Lithuania ($n=1$), Russian Federation ($n=6$), Spain ($n=1$) and USA ($n=3$). All the participants are active in the national or regional mathematics competitions, some of which serve as a means for choosing national teams for international competitions. Six participants are related to more inclusive competitions, and the rest are related to more exclusive ones.

To the best of our knowledge, the adult participants of the competition movement have not been studied before. Thus, when gathering the research data we tried to convince as many participants as possible to take part in in-depth interviews. In case of the Israeli participants, we succeeded in conducting face-to-face interviews in all 9 cases. The interview questions were sent to the participants by e-mail a week before the interview. With participants from other countries, conducting similar interviews telephonically or by Skype could be less comfortable for them. Therefore, open questionnaires, which addressed the central issues of the interview, we sent. After getting the answers, we suggested the participants to take part in follow-up interviews. In 9 cases, follow-up interviews were conducted. In sum, face-to-face and phone interviews lasted between 60 to 90 minutes. All the interviews were audio-taped and transcribed.

In the framework of the interviews and the questionnaires, the participants were explicitly asked to address the question: "What are the characteristics of an interesting problem for a mathematics competition?" They were also asked to exemplify their answers with some problems. The data analysis was conducted according to grounded theory paradigm with partially predefined categories (Strauss & Corbin, 1990) presented in the literature review.

RESULTS AND DISCUSSION

(1) The intended goals of the competition problems

The participants referred to four main goals that could be achieved with an aim of a competition problem:

The intended goal	Quotations
(1) to supply opportunities for learning meaningful mathematics	Shuki: "The students should be enriched with something after solving the problem. They should learn something new from it."
(2) to strengthen a positive attitude towards a particular problem and mathematics in general	Avi: "The problem must be beautiful! It is necessary for having an aesthetic enjoyment of mathematics."
(3) to create a cognitive difficulty	Feodor: "Usually you need some brilliant idea or a trick for the solution. The problem shouldn't be solved straightforwardly".
(4) to surprise	Leo: "The students should say "wow!" at the response to the problem – the problem should surprise them"

Table 1: The intended goals

The aforementioned goals are compatible with the instructional pedagogical goals. For instance, Vale and Pimentel (2011) interpreted the NCTM's (2000) approach to a "good" task as "[...] when it serves as an introduction to fundamental mathematical ideas that constitutes an intellectual challenge to students and allows different approaches" (p.5). The pedagogical goals mutually shared by the participants of the competitions movement and by the policy-makers, point out that the competition movement is located in the mainstream of mathematics education, and an adaptation of its successful practices for a broader range of students should be more widespread.

(2) The identified characteristics of an interesting competition problem

The participants of the study referred to the four aforementioned categories as the goals by their own. However, the following list of four characteristics of an interesting problem will show that the first two goals can be seen as the major ones while the last two serve as catalyzing means for achieving the first two.

a. Meaningful mathematics beyond the problem

All participants agreed that a problem for a mathematical competition is supposed to have several approaches for the solution. In this way it becomes accessible for a greater amount of students who can eventually compare and learn from each other. In the opposite case, it becomes a knowledge quiz question with no learning opportunities (see the first intended goal).

In addition, the solution of each problem equips the student with some mathematical facts of different levels of generality and applicability to other situations. The participants related to more exclusive competitions mentioned that facts and methods discovered during the solution of the particular problem should be focused on a wide range of mathematical objects that have something in common, that is, a some kind of surprising invariant (see the first, forth and ninth item on Movshovits-Hadar's list in the literature review section). The participants said that in this way the students get a feeling of meaningful mathematics (see the first intended goal).

The participants related to more exclusive competitions also noted that an interesting problem connects several mathematical topics that are not usually connected. Shuki explained that when a problem leads the student to the solution through different mathematical topics and areas, it exemplifies that mathematics is an interconnected field despite the way it is usually taught.

The participants related to more inclusive competitions did not address these issues explicitly. However, most of the problems in these competitions are particular cases of much more general phenomena; it seems that due to the stricture of the questionnaire in inclusive competitions, it is the best that could have been done.

b. The misleading problem situation image

All the participants agreed that a competition problem is supposed to have some kind of trick that a student has to figure out during the solution (e.g., see Feodor's citation in Table 1). Moreover, a student should be set in the position in which she would appreciate the existence of such a "trick"; then, finding or even being exposed to this "trick" could be enjoyable (see the second intended goal). In this scenario it is also possible that a student will integrate this "trick" in her knowledge base since its value has been appreciated (see the first intended goal).

In order to set a student in such a delicate situation, a proper combination of a problem formulation and its solution should been found. Namely, problem formulation should create an initial problem image that will put cognitive obstacles to reach the solution (see the third intended goal). Next, we present two problems followed by the evaluation of their embedded cognitive obstacles.

The convex polygon problem

This problem was shared by Avi. Avi said that he found it in a set of problems from Olympiad of Budapest of 1899 for the 8th graders:

> Prove than in the convex polygon there can be no more than three acute angles.

Avi addressed this problem in the following way:

> At first glance, the problem is not very accessible: we know that the sum of the angles of the convex polygon is $180^\circ(n-2)$ [when n stands for the number of vertexes] and it was said that there are no more than 3 acute angles… So how does it help? It doesn't!

In Avi's scenario the usage of the words "polygon" and "acute angles" in the formulation of the problem is supposed to provoke an algebraic problem situation image. Therefore, a solver might introduce the parameter of n and get an expectation that some algebraic manipulations or algorithms (like a principle of mathematical induction) can lead to the solution. In this scenario, the idea to turn to the geometrical concept of an exterior angle is unexpected.

The proof can be easily derived by a contradiction and by the fact that the sum of the exterior angles of the convex polygon is a constant equals to 360°. Therefore, if a convex polygon with more than three acute interior angles exists, then each of their exterior angles is obtuse. This makes their sum greater than 360° ■. It is even more surprising that this geometric approach leads to the solution suitable for every polygon (see the first and the ninth items on Movshovits-Hadar's list in the literature review section).

The envelope game problem

This problem was shared by Rom who said that he had found it in one of the books of Peter Winkler.

> Dan chose two different whole numbers, wrote them on notes and put each note in a different envelope. Nina chooses one of the envelops, opens it and looks at the written number. She should guess if the chosen number is the greatest or the smallest one between the two. Does Nina has a way of guessing correctly with a probability greater than 50%?

Rom evaluated this problem in the following way:

> This is one of the most beautiful problems that I've ever seen. I know it for a several years and it still astonishes me. It's amazing that there is a way to make the probability greater than 50%! When you see the solution you understand its mathematics, but still, it is so anti-intuitive. It's unbelievable that almost without anything she can be right in more than half cases.

Indeed, this is a wonderful example of a surprising problem. At first glance, it seems that knowing one of the numbers does not contribute any new information to the initial problem situation. In other words, the problem situation image is perceived as equivalent to the situation when Nina is supposed to guess in which envelope there is a greater number from the beginning. Obviously a probability of a correct guessing in the latter situation is 0.5; hence it is so surprising that there is a way to increase it in the former one. A solver needs to invest a lot of effort to get rid of this blocking image and to reconstruct the new, more accurate one.

Due to length limitations of this paper, a solution to the Envelope Game Problem or any other examples of problems cannot be presented in this paper; the plan is to do it during the presentation. However, it is worth mentioning that the participants of the study related to more inclusive completions presented problems which were more or less similar to the Convex Pentagon Problem, i.e., when a relatively local change approach is needed in order to get to the solution (*discontinuity* in terms of Weisberg, 1996). The participants related to the more exclusive competitions focused on problems similar to the Envelope Game Problem; i.e. problems, which demanded a relatively radical change in the initial problem situation image (*reconstruction* in terms of Weisberg, 1996).

c. A-typicality and novelty of problem's formulation

The participants acknowledged the importance of *a-typical* formulations. For instance, Michael said that: "Most of the inequalities in the competitions are cyclic. Hence, when I see something a-symmetrical it immediately catches my attention." The preference for the a-typicality does not always go hand in hand with the previous characteristic. For instance, Koichu, Berman and More (2006) used so-called *seemingly familiar problems*. The formulations of these problems were typical for routine-instructional problems and provoked problem situation images which included routine solution methods. The solutions to the problems, however, were non-standard. Thus, when in the case of a-typical problems the students could be surprised immediately, in the case of the seemingly familiar problems the students can get surprised by the unexpected a-typicality of the solution (see the forth intended goal).

d. The dilemma on the problem's "wrapper"

The participants used the word "wrapper" when addressing the wording that embeds the mathematics given of the problem. Their perceptions on an appropriate problem's wrapper were not uniform. The participants related to more inclusive competitions emphasized the role of a "nice" story, which may include amusing names, can represent some real life situation, can be related to students' interests or to the current competition. For instance, the number of a current year frequently appears in the competition problems. The participants related to more exclusive competitions did not argue with this view, as long as the problem formulation remained brief. In words of Leo:

> The ideal case is when the formulation takes 2-3 lines. In some cases there is no other choice and 5-6 line formulations appear. ... We want to check students' ability to think mathematically and not their reading comprehension skills.

The difference between the participants can be explained by the complex combination of personal taste, the assumptions about the mathematical taste of the potential solvers and, again, by the structure of the competition questionnaire. Indeed, the participants supposed that the students who come to the more inclusive competitions will enjoy a balanced mixture between mathematical and non-mathematical elements of the problem. The participants connected to more exclusive competitions suggested that the students enjoy the beauty of pure mathematics and its structures. These arguments are in line with Silver and Metzger (1989), who argued that an ability to appreciate mathematical beauty attributed to exceptional persons; in this case, these exceptional students will probably be more attracted to the exclusive competitions. Moreover, brief problem formulation has a potential for surprise because of the gap between a modest number of givens, and the complexity of the solution. In addition, a structure of the questionnaire in more inclusive competitions (many problems in little time) does not give a real chance to appreciate the internal structures of the problem. Therefore, in these cases the appreciation (see the second intended goal) can be achieved by other means like humour.

SUMMARY AND APPLICATIONS

The identified goals underlying the competition problems serve as an additional empirical argument that the competition movement is located in the mainstream of mathematics education. However, when the characteristics of interesting problems like significant mathematical facts, multiple solutions, connectedness and applicability are widely acknowledged in general mathematics education (e.g., NCTM, 2000) they are widely applied in the area of mathematics competitions. Thus, the study fully supports adapting successful practices from the competition movement to other educational contexts.

The study shows that in order to achieve the intended goals, like supplying opportunities for learning meaningful mathematics or strengthening a positive attitude towards mathematics, the competition movement selects the problems according to very high criteria of quality. This should remind us as teachers that every problem that enters our mathematics classroom participates in the formation of students' mathematical taste and perception on the notion of prototypical mathematical problem. Therefore, it is important to introduce cognitively demanding and surprising problems in the classroom. An obvious source for such problems is an enormous bank of competition problems. Moreover, the variety of mathematics competitions allows assuming that an appropriate problem for any educational context can be found in that bank. Another source for such problems can be a teacher herself, under the condition she will develop her problem composing skills and absorb the perceptions and norms prevailing in the competition movement. We hope that this study can be helpful for those teachers.

The final remark is concerned with the participants of the competition movement. Unlike the students who participate in mathematics competitions, it seems that the adults who stand behind the competition scenes got lost to mathematics education research among teachers and practicing mathematicians. This study shows that they have a perspective of their own which deserves a further research. We currently work in this direction.

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(NAÏVE) BELIEFS AND AFFECT ASSOCIATED WITH CREATIVE MATHEMATICAL TASKS: THREE CASES

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Engaging students with appropriate mathematical tasks contributes to the promotion of their mathematical creativity. This study investigates task features and cognitive demands of tasks that are believed by graduate education students to promote mathematical creativity. It also investigates affective dimensions associated with these tasks. Three cases are presented.

INTRODUCTION

Mathematicians as well as mathematics educators agree that creativity plays an essential role in doing mathematics. For example, Sriraman (2009) claimed that, "mathematical creativity ensures the growth of the field of mathematics as a whole" (p. 13). Various studies related to different characteristics of mathematical creativity such as divergent and flexible thinking (Haylock, 1997), "unusual and insightful solutions to a given problem" (Sriraman, 2009, p. 15), and employing non-algorithmic decision-making (Ervynck, 1991).

Promoting mathematical creativity is one of the aims of mathematics education. Towards this end, several studies have pointed out the importance of having students engage in appropriate tasks that may encourage some of the different aspects of mathematical creativity mentioned above. For example, Leiken (2009) claimed that multiple solution tasks offer students the opportunity to solve problems in many different ways, in turn encouraging three hallmarks of mathematical creativity in school: fluency, flexibility, and novelty (Silver, 1997). Choosing which mathematical tasks to implement with students is central to the work of teachers (Ball, Thames, & Phelps, 2008). These choices are not only based on teachers' knowledge but on their beliefs as well. In this study, tasks that have the potential to promote mathematical creativity are called creative mathematical tasks. The aim of this study is to explore the beliefs of graduate education students regarding creative mathematical tasks.

THEORETICAL BACKGROUND

There are three main issues which are relevant to this study: mathematical tasks, mathematical creativity, and affect. This section begins by reviewing how mathematical tasks in education may be analysed. It continues by describing aspects of mathematical creativity associated with school mathematics. Finally, it differentiates between some affective constructs relevant to this study.

In their study of mathematical tasks used in reform classrooms, Stein, Grover, and Henningsen (1996) analysed mathematics tasks in terms of their features and cognitive demands. Task features included the number of solution strategies, number and kind of representations, and communication requirements (e.g., demand for students to communicate and justify their procedures) of the task. One may also consider features that are basically surface characteristics, such as whether or not the task includes visual aids, the use of manipulatives, or is set in a real-life context (Arbaugh & Brown, 2005). Cognitive demands include memorization, the use of procedures and algorithms, and the employment of strategies. One may also describe tasks in terms of the security or insecurity they provide for students (Krainer, 1993). Secure tasks provide a secure path to the learning of concepts while insecure tasks allow for investigation and discovery on the part of the student.

Many studies refer to three key components of mathematical creativity: fluency, flexibility, and originality. Silver (1997) measured fluency by “the number of ideas generated in response to a prompt” (p. 76). A person who is fluent exhibits continuity of ideas and associations. He claimed that the use of ill-structured and open-ended problems in instruction may encourage students to generate multiple solutions developing fluency. Flexibility, according to Silver (1997) refers to “apparent shifts in approaches taken when generating responses to a prompt” (p. 76). Leikin (2009) evaluated flexibility by establishing if different solutions employ strategies based on different representations, properties, or branches of mathematics. At times, it helps to think of flexibility as overcoming fixation or breaking away from stereotypes. Originality is often related to creating new ideas. With regard to mathematics classrooms, this aspect of creativity may manifest itself when a student examines many solutions to a problem, methods or answers, and then generates another that is different (Silver, 1997). In this case, a novel solution infers novel to the student or to the classroom participants. Leikin (2009) measured the originality of a solution based on its level of insight and conventionality according to the learning history of the participants. For example, a solution based on a concept learned in a different context would be considered original but maybe not as original as a solution which was unconventional and totally based on insight.

The above characterizations of mathematical creativity did not take into account possible related affective issues. McLeod (1994) differentiated between three affective constructs: beliefs, attitudes, and emotions. To this, DeBellis and Goldin (2006) added a fourth subdomain, values/morals/ethics. In this study we investigate participants’ beliefs related to creative mathematical tasks. “Beliefs involve the attribution of some sort of external truth ... to systems of propositions” (DeBellis & Goldin, p. 135). Another aspect investigated is the emotions participants believe may be elicited by those who engage in the tasks. Emotions are less stable than beliefs and may range from mild to intense feelings (DeBellis & Goldin, 2006). Finally, the values that participants consider as they choose creative mathematical tasks are noted. Values refer to “’personal truths’ or commitments cherished by individuals” (p. 135).

This study presents three cases of graduate students who were requested to find and present a creative mathematical task and explain their choice. For each individual, the following questions are addressed: (1) What task features are believed by the individual to promote mathematical creativity? (2) What cognitive processes and problem solving strategies are believed to be associated with mathematical creativity? (3) What affective domains are believed to be associated with creative mathematical tasks? In this paper, initial results are reported.

METHOD

Participants and tool

The three individuals were all graduate students working towards a Masters degree in *Mathematics, Science, and Technology Education*. None of the participants had previously taken a formal course related to creativity. In this sense, their beliefs regarding creative mathematical tasks may be considered naïve. The research tool was an assignment given to each participant. The assignment was: 1) Choose a task or activity from a mathematics text book or workbook that in your opinion promotes mathematical creativity. 2) Photocopy the task and write down its source. 3) Write one paragraph to explain why, in your opinion, this task has the potential to promote mathematical creativity. All students handed in the assignment within a week.

Data analysis

Two strands of data resulted from the assignment. The first strand included the actual mathematical tasks chosen by participants; the second included participants' reasons for choosing these tasks. Each strand is analysed on its own. Regarding the actual tasks presented, analysis includes the type of source from where the task was taken and obvious task features. The type of source may be a classroom textbook, an enrichment book, an internet site, a teacher resource book, or other type of source. Obvious task features include the number of possible solutions to the task, if the task explicitly requests the student to seek different solutions, and communication requirements.

Analysis of each participant's reasons for choosing these tasks was conducted along three paths. First, task features mentioned by the participant are itemized. Then the cognitive processes and strategies which the participant mentions in connection with the task and/or in connection with mathematical creativity are analysed. Finally, affective issues, including emotions and values are analysed.

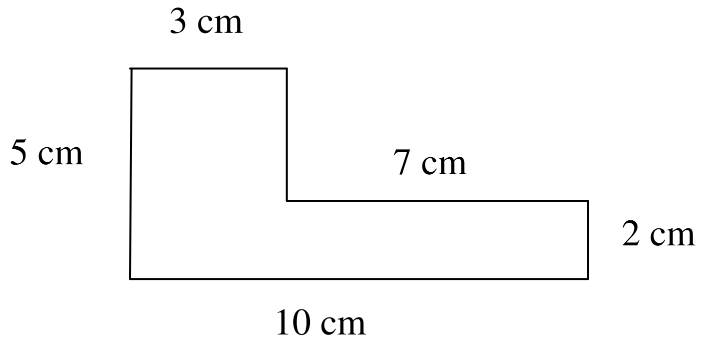
RESULTS

Three cases are presented and analysed. For each participant, the actual task is presented (translated from Hebrew) along with its analysis. Then the participants' reasons for choosing the task are given along with its analysis, as described in the previous section.

Randy

Randy is an elementary school teacher with 17 years of experience. She chose the following task, taken from the fourth grade mathematics textbook entitled *Geometry for the Fourth Grade* (The Center for Technology in Education, 2006):

Find the area of the polygon. It may be helpful to divide the polygon into rectangles. (Draw the dividing lines on the diagram.)



Computation: ______________________

The area of the polygon is: __________________

Discuss: Are there different ways in which you can divide this polygon? If so, what are they? From the different ways of dividing the polygon do you get different areas of the polygon?

First, it should be noted that the problem posed in the task is, what may be called, a standard measurement problem. That is, it was taken from the classroom mathematics textbook and not from a mathematics enrichment book, and was meant for all students to solve. The problem itself has one correct answer. However, there are several ways of solving this problem. One method is hinted at in the directions – that of dividing the area into rectangles. And yet, this one method may also yield several possibilities. The explicit direction to consider several possibilities is given at the end of the task. Thus, although the individual student is not directed to solve the problem in several different ways, there is explicit discussion regarding multiple solution methods, most probably aimed at whole class discussion. Of course, the problem itself may be solved in other ways not suggested in the task directions. For example, one may build up the polygon into a 5 by 10 rectangle and then take away the extra 3 by 7 rectangle. The polygon may also be divided into non-rectangular shapes.

Randy gave the following reasons for choosing this task as one which would promote creativity:

> The teacher can ask questions that invite discussion such as: Can the polygon be divided in different ways? If yes, then how?
>
> The task encourages the student to solve the problem in different ways.

> The task involves the topic of rectangular areas, a topic from the school curriculum, in a challenging way.
>
> Every student is active in solving the task, according to his own level of understanding and his own knowledge.

I will sum up and say that this task promotes mathematical creativity among students because it exposes them to different solution strategies and connects different mathematical concepts and ideas. These processes contribute to the promotion of mathematical knowledge as well as the development of creativity, especially when accompanied by classroom discussion.

How may Randy's reasons be categorized? Randy notes that the task has several possible solution methods and that it explicitly calls for class discussion. These are task features. She also claims that the task encourages the student to solve the problem in different ways. To actively solve the problem in different ways may be considered a cognitive demand. In other words, that the task may be solved in different ways is a task feature, independent of whether or not the student will indeed solve it in different ways. However, the demand or even encouragement to solve the task using different methods may be considered a cognitive demand of the task. She also claims in her summary that this task connects different mathematical concepts and ideas, another cognitive demand. Finally, Randy also points out the challenging way the topic of rectangular areas is reviewed. Being that challenge is relative to the person involved, and that one person may feel challenged while another may not, this point is considered to be an affective issue. Randy also makes a point of mentioning that every child may take an active part in solving the problem. It seems that Randy is concerned that every child feels involved. This may illustrate an issue of values for Randy, who considers the value of promoting mathematical creativity in all students. It may also be that Randy is considering how the task may emotionally impact on students and believes that a creative mathematical task should make the student feel included, rather than excluded. To summarize, Randy noted features of the task as well as cognitive demands which are, at least to some extent, also inherent in the task. To these, she also added affective issues which she believes may add to the promotion of mathematical creativity.

Sylvie

Sylvie had three years teaching experience in elementary. She chose a task from an extra-curricular activity book entitled *The I Hate Mathematics! Book* (Burns, 1975). The task, 'Cutting Sidewalks' (translated into Hebrew and pictured below in Figure 1), encourages the child to go outside with a piece of chalk and draw, first one line, then two lines and eventually three and more lines, dividing the rectangular or square area of pavement into different amounts of sections.

At different points in the activity, the child is asked to consider the following questions: (1) Let's say we add another line, how many sections will we get? (2) If we draw the same amount of lines but in different ways, will we get the same number of sections? (3) If you draw three lines, how many ways are there to divide the rectangle or square? (4) If you draw three lines, what is the smallest possible number of sections that you would get? What is the largest possible number of sections that you would get? (5) What if you were to organize a neighborhood tournament where a square is divided by 10 lines and the winner is the one with the most sections?

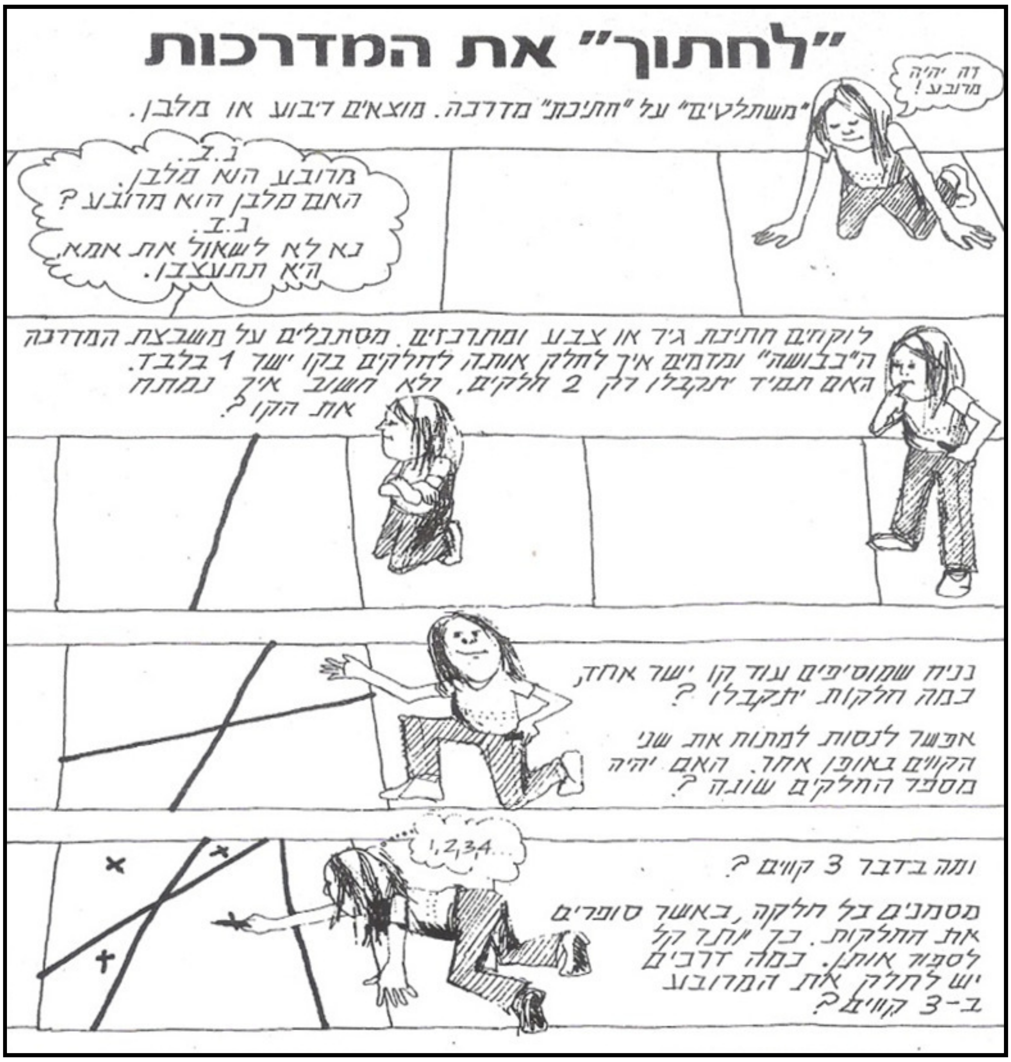

"לחתוך" את המדרכות
זה יהיה מרובע!
"משתלטים" על "חתיכת" מדרכה. מוצאים ריבוע או מלבן.
נ.ב. מרובע הוא מלבן. האם מלבן הוא מרובע? נ.ב. נא לא לשאול את אמא, היא תתעצבן.
לוקחים חתיכת גיר או צבע ומתרכזים. מסתכלים על משבצת המדרכה ה"כבושה" ומדמים איך לחלק אותה לחלקים בקו ישר 1 בלבד. האם תמיד יתקבלו רק 2 חלקים, ולא חשוב איך נמתח את הקו?
נניח שמוסיפים עוד קו ישר אחד, כמה חלקות יתקבלו?
אפשר לנסות למתוח את שני הקווים באופן אחר. האם יהיה מספר החלקים שונה?
1,2,3,4...
ומה בדבר 3 קווים?
מסמנים כל חלקה, כאשר סופרים את החלקות. כך יותר קל לספור אותן. כמה דרכים יש לחלק את המרובע ב-3 קווים?

Figure 1: Sylvies' task

What are some of the features of this task? First, we note that this is a multi-step task or activity. That is, it includes several mini-tasks, which are meant to be enacted in a certain order. Some of these tasks contain problems with one solution while others consider problems that have several solutions. In addition, in steps 2 and 4 there are explicit requests to think of several possible solutions to the problem.

Sylvie wrote the following reasons for choosing this task:

In my opinion this task promotes mathematical creativity in several ways. One way is that it takes place outside the classroom. I believe that by just being outside the classroom walls frees you from the shackles of convention, which in turn encourages all kinds of creativity.

An additional way is the, what seems like, non-mathematical activity of dividing the squares into sections. When it comes to finding the greatest number of sections, the task becomes a sort of competition, which can encourage creativity among learners. And when the task calls for dividing the square into the greatest amount of sections with a limited number of lines, it encourages creative thinking of space, which is part of the world of mathematics. In addition, this challenge invites the child to partake in trial and error actions as well as thought experiments.

The first task feature noted by Sylvie is the outside environment in which the task is supposed to take place. This is, essentially, a surface feature in that it is not at all relevant to the mathematics involved. The task could very well take place inside the classroom using pencil and paper. Yet, for Sylvie, this is an element of the task which she believes to be significant in promoting mathematical creativity. When explaining her view on this matter we realize that for Sylvie, creativity is related to breaking away from convention. In the same vain, Sylvie seems to believe that the activity does not appear to be a mathematical activity, and that this is another reason why the task may promote creativity.

Sylvie does not note that the task contains multi-solution problems. Nor does she note that the task explicitly encourages the children to investigate different possibilities. Sylvie does mention two cognitive aspects she believes are promoted by the task. The first is visual thinking which she calls, "creative thinking of space". The second is the use of a trial and error strategy when solving problems. Sylvie also remarks on two affective aspects which she believes stem from the task and may encourage mathematical creativity. The first is, in her opinion, the competitive nature of the task. The second is the challenging nature of the task.

Erwin

Erwin was studying towards his teaching degree at the same time he was doing his Master's degree and had no teaching experience. Like the second participant, he did not choose a task from a classroom mathematics textbook. Instead, he chose a task taken from a book written about problem solving by the mathematician Polya:

To number the pages of a bulky volume, the printer used 2989 digits. How many pages has the volume? (Polya, 2004, p. 234).

First, it is important to note that this problem has one correct solution. Second, perhaps because the book was not written with the intention of being used in class, there is no direct request to solve this problem in different ways nor does the problem invite open inquiry and investigation. Then why did Erwin choose this task as one which has the potential to promote mathematical creativity?

Erwin gave the following rationale for choosing this problem:

The task encourages mathematical creativity because it presents a question different from what the students are used to and they (the students) do not have a familiar method with which they can solve it. Therefore, students will have to search for a new way to cope with a mathematical problem, which in turn will open the way for creative thinking. In addition, the mathematical tools that are necessary for answering this task are quite basic and do not require difficult technical calculations. This will allow for a relatively easy time coping with the problem which will allow the students to concentrate on finding a creative solution.

Erwin does not choose this task because of its features. Instead, Erwin chooses this task because of the cognitive demands he believes the tasks will promote. Specifically, because he believes the problem calls for a non-standard or unfamiliar solution method, students will be encouraged to search for new ways to solve the problem and thus promote their creative thinking skills. In addition, Erwin points out that the solution does not require difficult-to-use mathematical tools. It seems that Erwin believes that technical difficulties may hamper the promotion of mathematical creativity. Finally, it seems that Erwin does not relate to any affective issues.

SUMMARY AND DISCUSSION

What were some of the task features believed by the participants to be associated with promoting mathematical creativity? The first two tasks included problems that had several solution methods and explicit directions to consider the different methods in solving the problem. The first participant mentioned these features. Although the second participant pointed to the surface features of the task, it is still noteworthy that the task itself would be considered a multiple-solution task (Leiken, 2009) or even an open-ended task (Kwon, Park, & Park, 2006) and thus a task which has the potential of promoting fluency, flexibility, and originality.

All three participants noted different cognitive demands when explaining why they chose their task. The first participant explicitly related to the processes of searching for and discussing different solution methods. The second participant related to a trial and error strategy, which may also be considered part of the search for different solutions. The third participant related to the demand for thinking of new directions. In essence, all three participants were concerned with processes which promote divergent thinking (Haylock, 1997). The third participant was particularly concerned with promoting new ways of thinking. This is reminiscent of Sriraman (2009) in that mathematical creativity invokes insightful solutions.

Regarding affect, different issues were raised by the first two participants. The first participant took into account the value of promoting creativity in all students. The second participant mentioned the competitive nature of the task. Both participants raised the issue of challenge. While the third participant did not specifically relate to emotions or values, he did consider the newness of the task which may be a contributing factor to the relative challenge felt by a student.

In general, this study found that although the participants may have been lacking in formal vocabulary or terminology, many of the factors believed by the participants to promote mathematical creativity were indeed factors pointed out by the research community. Another important finding of this study was that participants considered the issue of affect when choosing a creative mathematical task and did not only look for the cognitive demands of the task. Teachers' values, such as providing challenging tasks for all and not just a few select students, may also influence how they choose creative mathematical tasks for their classes.

While a task may have the potential to promote mathematical creativity, living up to its potential may depend on other factors such as teachers' mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), teachers' beliefs related to mathematical creativity, and sociomathematical norms established in the classroom. The next step would be to explore how participants implement the tasks they chose with students and what, if any, additional beliefs and affective issues arise when implementing tasks.

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THE *THEORY OF* CONCEPTUAL CHANGE AS A *THEORY FOR* CHANGING CONCEPTIONS

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It has become widely accepted that what and how mathematics teachers teach is linked to what it is they believe. What teachers believe, however, is not always in alignment with contemporary notions of mathematics and the teaching and learning of mathematics. As such, it is important for teacher educators to help facilitate changes in teachers' beliefs in ways that will enable them to become more effective teachers of mathematics. In this article I present the results of a research project designed to examine the feasibility of using the theory ***of*** *conceptual change as a theory* ***for*** *changing mathematics teachers' conceptions about key aspects of mathematics and the teaching and learning of mathematics. The results indicate both that the theory of conceptual change is a viable theory for designing interventions for the purpose of changing beliefs, and that the implementation of these aforementioned interventions resulted in the rejection of participants' a priori beliefs.*

INTRODUCTION

It has become widely accepted that what and how mathematics teachers teach is linked to what it is they believe about mathematics and the teaching and learning of mathematics (Beswick, 2009). What teachers believe, however, is not always in alignment with contemporary notions of mathematics and the teaching and learning of mathematics (Green, 1971). As such, it is important for teacher educators to help facilitate changes in teachers' beliefs in ways that will enable them to become more effective teachers of mathematics. The research literature, as well as my own experience with such efforts, has informed me that these are both worthy and challenging pursuits.

Teachers' beliefs can be difficult to change. Too often have I encountered situations where the in-service mathematics teachers I work with have agreed (or acknowledged) that, for example, there is more to mathematics than mastery of the collection of outcomes in the current curriculum, that mathematics is about problem solving and inquiry and reasoning, only to then have these ideas demoted to lower levels of importance for the sake of "preparing students for the exam" or "to save time". Along with this demotion comes a privileging of more traditional teaching methodologies that are seen to be more efficient. What has happened in such situations is that, although the teachers have been willing to assimilate additional views of mathematics and mathematics teaching and learning into their belief structures, the old views have not been eradicated. Schommer-Aikins (2004) points out that beliefs are “like possessions. They are like old clothes; once acquired and worn for awhile, they become comfortable. It does not make any difference if the clothes are out of style or ragged. Letting go is painful and new clothes require adjustment” (p. 22). So, in the end, there is a return to the old, out of style, and ragged beliefs.

But, what if instead of going through a process of assimilation the teachers had gone through a process of replacement – first rejecting their old beliefs and then adopting the new beliefs. In such an instance there could not be a return to the old beliefs – they would have already been discounted as viable. Such a change in beliefs can be seen as a form of accommodation, although the lack of compromise and blending of the old and the new make it a very specific form of accommodation. A more accurate description of such a process is one of conceptual change – a process by which a current conception is first rejected and then a new conception is adopted.

In this article I present a research project designed to examine the feasibility of using the theory of conceptual change as a theory for changing mathematics teachers' beliefs about key aspects of mathematics and the teaching and learning of mathematics. In what follows I first present the theory of conceptual change and argue its applicability in changing conceptions among in-service mathematics teachers. I then present and argue for a theoretical turn in which the theory of conceptual change is not just used as a *theory of* how conceptions may have been changed, but also how it can be used as a *theory for* changing conceptions. This is followed by a description of the methodology used in the aforementioned research project and finally the presentation of some of the results of this project.

THEORY OF CONCEPTUAL CHANGE

The theory of conceptual change emerges out of Kuhn's (1970) interpretation of changes in scientific understanding through history. Kuhn proposes that progress in scientific understanding is not evolutionary, but rather a "series of peaceful interludes punctuated by intellectually violent revolutions", and in those revolutions "one conceptual world view is replaced by another" (p. 10). That is, progress in scientific understanding is marked more by theory replacement than theory evolution. Kuhn's ideas form the basis of the theory of conceptual change (Posner, Strike, Hewson, & Gertzog, 1982) which has been used to hypothesize about the teaching and learning of science. The theory of conceptual change has also been used within the context of mathematics education (Vosniadou, 2006).

Conceptual change starts with an assumptions that in some cases students form misconceptions about phenomena based on lived experience, that these misconceptions stand in stark contrast to the accepted scientific theories that explain these phenomena, and that these misconceptions are robust. For example, many children believe that heavier objects fall faster. This is clearly not true. However, a rational explanation as to why this belief is erroneous is unlikely to correct a child's misconceptions. In the theory of conceptual change, however, there is a mechanism by which such theory replacement can be achieved – the mechanism of 'cognitive conflict'.

Cognitive conflict works on the principle that before a new theory can be adopted the current theory needs to be rejected. Cognitive conflict is meant to create the impetus to reject the current theory. So, in the aforementioned example a simple experiment to show that objects of different mass actually fall at the same speed will likely be enough to prompt a child to reject their current understanding.

The theory of conceptual change is not a theory of assimilation. It does not account for those instances where new ideas are annexed onto old ones. Nor is it a theory of accommodation, *per se*, in that it does not account for examples of learning through the integration of ideas. The theory of conceptual change is highly situated, applicable only in those instances where misconceptions are formed through lived experiences and in the absence of formal instruction. In such instances, the theory of conceptual change explains the phenomenon of theory rejection followed by theory replacement. The theory of conceptual change, although focusing primarily on cognitive aspects of conceptual change, has been shown to be equally applicable to metaconceptual, motivational, affective, and socio-cultural factors as well (Vosniadou, 2006). And, it has been argued that it is applicable to teachers' beliefs about mathematics and the teaching and learning of mathematics (Liljedahl, Rolka, and Rösken (2007). For purposes of brevity, I will not articulate these arguments here.

THEORY OF → THEORY FOR

In the aforementioned example of a child developing naive views of the effects of gravity on objects of different masses and then changing that view on the heels of a demonstration the theory of conceptual change gives a viable explanation of the learning that that particular child has experienced. When a teacher is aware that there may be a number of children in her class that have a similar misconception decides to run such an experiment for the purpose of changing her students' conceptions she is using this theory for the purposes of promoting learning. That is, she is using the *theory of* conceptual change as a *theory for* changing concepts. More generally, she is using a *theory of* learning as a *theory for* teaching.

In this aforementioned example such a move on the teacher's part is both intuitive and natural. She needs not have any special knowledge of the theory of conceptual change to know that challenging the students naive views would be a pedagogically sound move. However, in proposing this shift as a change from a *theory of* learning into a *theory for* teaching I suggest that it is equally applicable in situations where sophisticated knowledge of a learning theory is needed. Simon (1995), for example, uses deep knowledge of the theory of constructivism to guide his planned and implemented teaching actions. This is not without challenge, however:

> Although constructivism provides a useful framework for thinking about mathematics learning in classrooms and therefore can contribute in important ways to the effort to reform classroom mathematics teaching, it does not tell us how to teach mathematics; that is, it does not stipulate a particular model. (p. 114)

To overcome this lack of a model Simon must "hypothesize what the [learner] might learn and find ways of fostering this learning" (Steffe, 1991, p. 177 as cited in Simon, 1995, p. 122). It is within this hypothesizing that the sophisticated knowledge of constructivism comes to bear as Simon designs mathematics pedagogy on the constructivist view of learning (paraphrased from Simon, 1995, p. 114). I propose that Simon's actions in this regard can be described as turning the *theory of* students' constructing knowledge into a *theory for* promoting knowledge construction.

There is a need in mathematics education to have a way to discuss the distinctions between theories with respect to learning and with respect to teaching. Whereas there is an abundance of theories to discuss learning – from constructivism (Piaget, 1970) to commognition (Sfard, 2008) – these same theories don't explain teaching.

> While theory provides us with lenses for analysing learning (Lerman, 2001), the big theories do not seem to offer clear insights to teaching and ways in which teaching addresses the promotion of mathematics learning. (Jaworski, 2006, p. 188).

In fact, it can be argued that there can never exist such a thing as a theory of teaching.

> Theories help us to analyse, or explain, but they do not provide recipes for action; rarely do they provide direct guidance for practice. We can analyse or explain mathematics learning from theoretical perspectives, but it is naive to assume or postulate theoretically derivative models or methods through which learning is supposed to happen. Research shows that the sociocultural settings in which learning and teaching take place are too complex for such behavioural association (Jaworski, 2006, p. 188).

Yet at the same time, teaching is perpetually informed by theories of learning.

> It seems reasonable that the practice of teaching mathematics can and should draw on our depth of knowledge of mathematical learning, and learning theory, but to theorise teaching is a problem with which most educators are struggling. (Jaworski, 2006, p. 188).

I propose that the source of this tension between theories of learning and theories of teaching is the assumption that theories should play the same role in teaching as they do in learning. This does not need to be the case. Teaching and learning are inherently different activities. And to theorize about them requires, not (necessarily) the use of different theories, but the use of theories differently. The *theory of/theory for* distinction is a manifestation of these ideas.

METHODOLOGY

As mentioned, in this article I explore the feasibility of using a particular learning theory – the theory of conceptual change – as a theory for changing teachers' conceptions about mathematics and the teaching and learning of mathematics. The research for this is situated within a course for in-service secondary mathematics teachers wherein the participants are subjected to six interventions designed to change their beliefs about six core aspects of mathematics education: (1) the nature of mathematics, (2) the nature of mathematics teaching, (3) the nature of assessment, (4) the nature of student knowledge, (5) the nature of student learning, and (6) the nature of student motivation. In what follows I first describe the general setting of this research and then detail the particular aspects of the methodology used to answer the research questions.

Setting and Participants

Participants for this study are in-service secondary mathematics teachers who were enrolled in a master's program at Simon Fraser University in Vancouver, Canada. The program was specifically designed to help teachers develop insights into the nature of mathematics and its place in the school curriculum, to become familiar with research on the teaching and learning of mathematics, and to examine their practice through these insights. The particular course that this study took place in is called *Teaching and Learning* and was the second course in the program. This particular course was designed to examine closely the teaching of mathematics from the perspective of, and with the goal of, students learning mathematics. There were 14 teachers enrolled in the course, all of whom agreed to be part of the study.

Data Sources and Analysis

This study began with an examination of the feasibility in constructing interventions designed to create cognitive conflict and to promote the rejection of beliefs that I anticipated some participants may have[1]. Careful records of this planning process were kept. A journal was started in which initial ideas, tentative plans, apprehensions, and unresolved questions were recorded. Once the course began, this journal became the place where field notes were recorded. Records were kept of the observed effects of each planned intervention with specific focus on things that were predicted and things that were surprising. Also recorded in the field notes were pieces of discussions with and among participants.

With respect to the changes in beliefs of the teachers enrolled in the course the data is constituted of four sources. The first has already been mentioned – the field notes produced from observations within the classroom setting. The second and major source of data came from the journals that each of the participants kept. As part of the requirements for the course the teachers were to keep a reflective journal within which they were to respond to specific prompts. The third source of data was informal interviews that were conducted with each participant at varying times during the course. The final source of data was from one of the course assignments in which the teachers had to write an essay. The details of this essay will be discussed in a subsequent section.

These data were sorted according to the specific planned interventions wherein all the data pertaining to a specific planned activity were grouped together and analysed. This data were recursively coded for emergent themes and analysis focused on changes in beliefs as related to each of the specific intervention. This began with a search for anticipated outcomes. Evidence of cognitive conflict and belief rejection was sought out as was evidence pertaining to more assimilatory behaviour. But the process was open and as this cursory analysis progressed themes beyond the anticipated effects began to emerge. This was continued until no more themes were forthcoming.

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[1] As the course designer and researcher I am heavily implicated in this research. In particular, with regards to what beliefs were chosen to be targeted and the relative worth of these beliefs vis-à-vis the ones I wished to promote. Although my decisions can be supported in the literature, they are still *my decisions*.

RESULTS AND DISCUSSION

As can be expected, the results from the aforementioned analysis are many. As such, only two of the interventions have been selected for presentation in this article. In what follows, results from the interventions pertaining to (1) the nature of mathematics and (2) the nature of mathematics teaching are presented in their own subsection. Each of these begins with a detailed description of the intervention and how it is designed to promote cognitive conflict and the rejection of beliefs. This is then followed by exemplification from relevant data regarding the effect of this intervention on the participants' beliefs.

Nature of Mathematics

Experience, prior research (Liljedahl, 2010), and literature (Beswick, 2009) show that teachers' beliefs about mathematics are often anchored firmly in the context of the school mathematics curriculum learning outcomes. Such a view is often punctuated with a belief that mathematics is about facts and procedures, where facts are to be memorized and procedures are to be mastered. This is a rather narrow view of mathematics and one I felt was important to change. Adhering to the theory for conceptual change being implemented here, any such change needs to be preceded by a rejection of any such *a priori* beliefs.

Lockart's Lament (2008) was chosen for this purpose. The first few pages of this essay tell the tale of first a musician and then a painter who wake from their own respective nightmares in which their craft has been reduced to a compulsory curriculum of skills to be practiced and mastered. Lockhart's purpose for doing this is clear. Using reductionism he effectively showcases the absurdity that results when complex activities such as music and painting, and by analogy – mathematics, are reduced to the mastery of the tools of the trade. These first few pages were selected to be read by the participants during class in week 2 of the course. This was then followed by a 15 minute quick-write where the teachers responded to the prompt – "... meanwhile on the other side of town a mathematician wakes form a similar nightmare. What nightmare did he wake from?" This was succeeded by a whole class discussion. The full text of Lockhart's Lament was then assigned as homework as was the journal prompt – "what is the relationship between the curriculum learning outcomes and mathematics?"

... results

The quick write produced not only interesting, but also creative, results. Common among the 14 nightmares were intonations about students being shown, and being required to practice, algorithms without any explanation of what its purpose was or why it worked. This is succinctly demonstrated in an excerpt from Jenny's passage wherein a high school mathematics teacher is telling her student:

> Don't worry about it dear, you'll learn what it is for next year.

This was a recurring phrase in her nightmare appearing in four separate places. It was also a recurring theme in Alicia's and Marcus' nightmares. During the follow-up whole class discussion it was mentioned, and agreed to by many that not only has mathematics been reduced to a collection of curricular skills but it has been reduced to a collection of pre-requisite skills that need to be mastered now.

Analysis of the journal entries revealed that this reduction of mathematics to curricular learning outcomes was layered situated, and in many cases surprising. In his journal Chad wrote:

> I never really thought about it before, but for me math is all just about what I'm teaching tomorrow, or the next day, or last week. Even when I'm talking to the kids it's all about "you have to learn this ... it's important ... you'll need it next year!" After last class I started looking at my top students and thinking about why they like math and I realized that they don't even know what math is. They think math is this stuff I'm teaching them and they like the fact that they can be completely right at it and know that they are completely right. Then I started to think about why it is that I like math and I realized that I had forgotten that I used to love to figure out logic puzzles and solve difficult problems. I don't do that anymore, but I should.

Chad's entry shows how his own view of mathematics is bound up with the view of his students. They are inseparable, situated within the context of the mathematics that they are experiencing on a daily basis. The difference, however, is that Chad has a memory of mathematics as something else and for this reason he is surprised that he has allowed his views to stray from what it is he used to enjoy about mathematics.

Lori also talks about memories of a distant experience, albeit quite different.

> The thing about the music really struck a chord for me (ha-ha). I used to play the piano when I was a kid. I hated all the practicing and I wanted to quit. My mom would tell me that I just needed to get through it and then I would start enjoying it. But I didn't have the patience and so I quit. The ironic thing is that I was the one who wanted to start taking piano lessons. I wanted to play the piano. I wanted to be able to play the music. But I gave up. Isn't this what we are doing to our students? How many of my students have given up? We have reduced math to a bunch of drills. No wonder kids hate it. Math needs to be more than this. Kids need to have an opportunity to play maths as well, to forget about the drills and just enjoy it.

Although Lori does not make clear what she means by *play math* or what mathematics *needs to be* she is very clear about what mathematics shouldn't be.

All of the teachers had journal entries similar to these – entries that lamented the systemic reduction of mathematics to curriculum and with it, the reduction of their own views about mathematics. They also, uniformly rejected this view even if they did not have an alternate view to replace it with. By all accounts, the first few pages of Lockhart's Lament (2008) created a conflict for many of these participants. The reduction of a craft to a collection of discrete and closed skills in such a stark fashion created enough of a disturbance for them that they rejected their self professed understanding that mathematics is learning outcomes.

Nature of Mathematics Teaching

One of the implicit goals of the masters program within which this course is set is to provide the enrolees with the knowledge, the will, and the ability to teach mathematics using more contemporary and progressive teaching methodologies. The achievement of this goal is made difficult by the fact that many teachers that come to the program are often very traditional teachers, are deemed to be very qualified with high teaching abilities, are seen as leaders within their schools and districts, and are teachers who like the way they teach and feel that they do it well. For many, these more traditional practices will have originated in their own successful experiences as learners of mathematics (Ball, 1988). Taken together, most of the teachers believe that they are good at teaching the way that they do and they believe that what they do is effective for student learning. It is this second belief that was targeted for intervention.

Jo Boaler's book Experiencing School Mathematics (2002) was chosen for this purpose. In this book, Boaler presents the results of her doctoral research in which she compares two very different teaching methods employed at two otherwise very similar schools. At Amber Hill teachers use a more traditional method of teaching while at Phoenix Park they construct their teaching around problem. What is powerful about this book is that Boaler's descriptions of the traditional teaching practices and classroom norms of Amber Hill are so rich that they reflect back at a reader their own teaching practices while at the same time exposing the consequences of this style of teaching in terms of students' attention, retention, performance, and enjoyment. At the same time Boaler counters this stark depiction of traditional teaching with an equally descriptive account of an antithetical teaching style, associated classroom norms, and subsequent student learning.

In week 1 of the course the participants were introduced to the book and assigned a 2500 word essay on the following:

> It can be said that when we read a book we read ourselves into the text. In what ways do you read yourself into Boaler's book? Speak about your own teaching practice (past, present, and future) in relation to the book.

This essay was due in week 6 of the course at which time we engaged in structured debate on the merits and demerits of teaching and learning in the dichotomous settings of Amber Hill and Phoenix Park. This was followed up by a journal prompt asking them to reflect on the most powerful aspect of the book.

... results

The most common theme that emerged from the data regarding this intervention was the unanimous declarations that the participants saw themselves at least partially reflected in the teaching of Amber Hill. Lori's, Kris', and Alicia's journal reflections are succinct examples of this.

> It was as though I was looking at my own teaching.
>
> I couldn't help but think that Boaler was describing my classroom.
>
> ... it was a really good description of the classroom I did my practicum in.

Nicholas, in an interview, was not as committal, choosing to temper his acknowledged traditional teaching styles with some of the more progressive aspects of his practice.

> Nicholas: I certainly would fit in well with the teachers at Amber Hill, especially with the focus on testing. But I'm not exactly the same. I tend to make more use of group work especially during project work.

Also common among participants was how incongruous their reading of the book was. In an interview immediately after the essay was due, Chad commented on his experience in reading the book.

> Interviewer: So, what did you think of the book?
>
> Chad: It was good ... it was eye-opening. As I was reading it I kept trying to identify myself with Jim at Phoenix Park but I kept coming back to Amber Hill. It was really troubling when I finally realized that I was an Amber Hill teacher.

Ten of the participants commented, either in their essay, their journals, or in the interviews on a similar experience of trying to will themselves into the more progressive classroom but not being able to avoid the reality that their teaching is more traditional than they may have been willing to admit ... even to themselves.

Each of the participants wrote at length in their essays and their journals about the negative effects of traditional teaching portrayed in Boaler's book. In particular, they commented on the issues of retention:

> If the students are not going remember the stuff we teach them then have they really learned? And if not, then what was the point in the first place? (Ingrid)

attention:

> If the students are not engaging with the lesson then there is no way that they can learn. (Eric).

and issues pertaining to student affect:

> Math needs to be fun. Sitting in rows and listening to the teacher is not fun. (Alicia).

In so doing they were rejecting the traditional teaching paradigm, and by extension, many of their beliefs about what mathematics teaching should look like.

Like the intervention designed around Lockhart's Lament (2008), the impactful aspect of Boaler's book (2002) seems to be the starkness of the picture it paints. This can be seen in both Chad's and Frieda's comments (above) and was evident in most of the data collected. It seems as this starkness makes explicit that which has previously been implicit, and brings to light exactly what the participants beliefs about teaching are. The evidence against the effectiveness of such a teaching style makes it almost impossible to sustain their traditional views of teaching and rejection seemed inevitable.

CONCLUSIONS

The results of the analysis of data pertaining to the two aforementioned interventions, as well as the four not presented here, indicate that: (1) the *theory of* conceptual change is a viable *theory for* designing interventions for the purpose of changing conceptions, and (2) implementation of these interventions resulted in cognitive conflict and eventually rejection of the participants’ *a priori* beliefs. The cognitive conflict that precipitates this belief rejection seems to be greatly affected by the starkness of the images present in some of these interventions – especially when those images are both troubling and undeniably reflective of the participant's practice. Further, the data is replete with evidence that the participants not only rejected beliefs pertaining to their current practices, but that often they did so without an immediate replacement at hand.

In the introductory sections of this article I argued that learning and teaching were inherently different constructs, and as such, needed to be explained, not necessarily with different theories, but with theories in different ways. In this article, the *theory of* conceptual change was used as a *theory for* designing teaching. The empirical data and analysis of results show that within the aforementioned context this process was effective in producing the desired changes in teachers' beliefs. As such, this study adds to the small but important body of literature that attempts to bridge the gulf between theory and practice in practical (as opposed to theoretical) ways. More such work is needed in order to fully capitalize on the rich and abundant theories of learning that we have accessible to us.

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FINNISH ELEMENTARY TEACHERS' ESPOUSED BELIEFS ON MATHEMATICAL PROBLEM SOLVING

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According to the Finnish curriculum (NBE 2004), improving pupils' problem solving skills is an important objective in teaching mathematics. Teachers' role is crucial in carrying out the objectives of the curriculum. Especially their conceptions influence decisions they make when teaching mathematics. The purpose of this paper is to find out what kind of conceptions elementary teachers (teachers for grade 3) have concerning problem solving and its teaching in mathematics, as well as what is their understanding about problem solving and how they implement it. The data was gathered with a questionnaire consisting of open questions during November 2010. The questionnaire was sent to a random sample (N = 100) of all Finnish grade 3 teachers. Two rounds of posting produced 63 answers. According to teachers´ responses, problem solving in mathematics means in the first place word problems, and problem solving is mainly done collaboratively.

In Finland we have a nine-year comprehensive school where all children learn in heterogeneous classes, also in mathematics. The class size varies between 20–30 pupils, and therefore, teachers have difficulties in balancing between low-achievers and successful pupils, especially in upper grades (grades 7–9). See more on mathematics and teacher education in Finland in Pehkonen, Ahtee & Lavonen (2007).

THEORETICAL FRAMEWORK

Problem solving has been emphasized in the mathematics curriculum of presented in the national core curriculum for basic education (NBE 2004). In the core curriculum, it is defined that mathematics teaching in the comprehensive school delivers to pupils mastering of mathematical concepts and the most common solution methods in basic mathematics. Another objective of teaching is that it should lead pupils to find, elaborate and solve problems. Mathematics teaching influences pupils' spiritual growth and teaches purposeful performance. In the descriptions of key contents and good achievement for different grades, it is repeated the emphases of mathematical problem solving and thinking skills, from even the very first years of school (ibid).

Problem solving in mathematics teaching

Problem solving has generally been accepted as a mean for advancing thinking skills (e.g. Schoenfeld 1985). But the basic concepts, 'problem' and 'problem solving' seem still to be rather ambiguous in the mathematics education. Sometimes a 'problem' is understood to be a simple arithmetic task that can be solved in a routine way, whereas at other times it means a more complex situation. The fuzziness of problem solving concepts is discussed e.g. in Pehkonen (2001).

We shall adopt here the following interpretation that is widely used in the literature for the concept of problem (e.g. Kantowski 1980): A task is said to be a *problem* if its solution requires that an individual combines previously known data in a new way (new at least for the solver).

The nature of problem solving has been described in the literature with the help of problem solving models (e.g. Polya 1945, le Blanc & al. 1980, Mason & al. 1985, Schoenfeld 1985, Aebli 1985). Polya's 60-year-old four-step model is still the most common one: Understanding the problem, Devising a plan, Carrying out the plan, and Looking back (Polya 1945). The shortage of Polya's model has been its oversimplified structure, it looks like a receipt. Mathematical problem solving (e.g. doing new mathematics) is not possible via following such a scheme, instead the solver needs to use their creativity.

Therefore, Polya's model has been modified by other researchers, usually by refining one or more steps. One of these modifications is the model that Schoenfeld (1985) has used. Another one is offered by Mason, Burton and Stacey (1985). Their model has only three phases: Entry, Attack, Review, since they have combined the two middle steps in Polya' model. But their key idea is that between the phases Entry and Attack there is a mulling circle. The mulling in this circle will end until when the solver finds a correct way out, i.e. when one solves the problem (ibid, 131). Similar ideas had earlier been published by Kiesswetter (1983).

Mason and colleagues' interpretation of problem solving is compatible to constructivist understanding of learning (e.g. Davis, Maher & Noddings 1990). Aebli emphasizes the importance of creativity in forming a new concept. That is the difference with Polya who says, that a task is a problem if its solution requires that an individual combines previously known data in a new way (new for the solver). Le Blanc, Proudfit and Putt (1980) identified for elementary school two types of problems: standard textbook problems and process problems.

One may state that thinking of problem solving from a 'models' point of view is not relevant to the study on elementary teachers' problem solving. We disagree this, since according to our observations from elementary classrooms same elements of problem solving can be seen in small pupils' work, althrough they are not so sofisticated. Similar aspects can be read in published research papers (e.g. Kolovou, van den Heuvel-Panhuizen & Bakker 2009).

On teachers' conceptions

In earlier studies, it has been noticed that teachers' conceptions are of paramount importance when trying to understand teaching situation (e.g. Grouws, Good & Dougherty 1990).

Teaching in school class happens in a large group, but learning is a personal event, and therefore, it affects each individual differently. Ultimately, a teacher's task is to create learning environments that allows high-quality learning for all. Martin Hughes

(1986) have presented a perspective on children´s attempts to understand mathematics. It is important that children can create their own strategies for understanding mathematics using problem solving. In order to understand teachers' decisions, it is important to know what a teacher thinks about teaching and ways to help pupils learn, i.e. teachers' conceptions.

Conception is problematic as a concept, since it is not clearly defined (cf. Furinghetti & Pehkonen 2002). Here we understand an individual's beliefs in a rather wide sense as their subjective, experience-based, often implicit knowledge and emotions on some matter or state of art. Furthermore, *conceptions* are explained as conscious beliefs. In the case of conceptions, we understand that the cognitive component of beliefs is stressed, whereas in basic (primitive) beliefs the affective component is emphasized (Pehkonen 1998).

Sivunen & Pehkonen (2009) implemented a study about elementary teachers' conceptions on problem solving and its teaching in mathematics, in the case of one school district (N = 43). According to these teachers, problem solving in mathematics means various problems, strategies, mathematics in everyday situations, pupils' own thinking and applying previously learned skills. Here we want to generalize these results to the whole Finland using a statistically representative sample.

The focus of the paper

This paper examines Finnish elementary teachers' (grade 3) conceptions about mathematical problem solving. Thus the research questions can be formulated as follows: (1) How do teachers understand the concept 'problem'? (2) What do teachers understand as problem solving? (3) What kind of examples do teachers provide for problems? (4) How often do teachers use problems in their classes?

METHOD

The empirical section is a part of a larger research project called "*On the development of pupils' and teachers' mathematical understanding and performance when dealing with open-ended problems*". It is a joint three-year project with Chile (2010–13), financed by the Academy of Finland (project number 135556). Within the project we will try to develop a model for improving the level of understanding, the skills and self-confidence of teachers and pupils, with the active participation of teachers, using certain open-ended problems.

The study dealt with in this paper is a nationwide survey about problem solving. This background study aims to single out what kind of conceptions grade 3 teachers in Finnish elementary schools have concerning problem solving. Then, in the case of random sample, the results can be generalized to all Finnish elementary teachers, since they do not specialized in one grade level, as in some countries, but usually teach all grades in elementary school (grades 1–6).

Indicator

The questionnaire was prepared for this study with the help of some existing papers: Polya (1945), le Blanc & al. (1980), Kantowski (1980) and Aebli (1985). There are three levels of understanding the concept ´problem´; firstly as a word problem, secondly that an individual combines previously known data in a new way (new for the solver), and thirdly formatting a new concept.

The questionnaire was also made up to distinguish how differently teachers used the steps in Polya's model. In addition, we were interested in problem solving in mathematics context and in frame of reference. There were five open questions in the questionnaire. Some alternative answers were offered to the main questions, and an additional option for respondents to write their own answers (cf. Tables 1 and 2). In the beginning of the questionnaire. Our ten experimenting teachers (3 grade) tested the questionnaire, and it was corrected.

The questionnaire in Finnish was sent to a random sample (N = 100) of all Finnish comprehensive schools asking their headmasters to give the questionnaire to one grade 3 teacher. If there were many teachers in school, then to the teacher who is first on their list of grade 3 teachers.

Participants

The data was gathered with a questionnaire in autumn 2010. It was selected at random 100 schools from Finland's comprehensive schools. The questionnaire was posted to the headmasters of these schools in the middle of November. A total of 63 responses was received from schools, thus the response rate was good.

Among the 63 respondents, there were 51 female teachers and 12 male teachers, the share corresponding roughly the gender division in teacher education. Since the share of the male teachers was so small, we have kept all teachers as one group. Most of the participants were experienced teachers, the mean value of their school experience was 16.8 years, and it varied from 2 months to 35 years.

Methods of data analysis

Ten first papers were used to create a categorization of answers. After classifying all answers into these categories, they were written into a SPSS file, in order to calculate statistical indicators (modes etc.) and analyze relevant explanatory factors.

PRELIMINARY RESULTS

In the following, we will present the 3rd grade teachers' conceptions of problem solving, and some examples of problems they stated to be using in their lessons.

The question: What is a problem?

In the first question, the teachers were asked "How would you describe the concept of problem", and they were given four alternatives from which they should number the most important option and the second option (Table 1).

Table 1: The teachers' answers to the question "What is a problem?".

Alternatives given in the question	*The 1st option*	*The 2nd option*
A problem is a verbal task, where the pupils apply their previously knowledge	27	15
A problem is a verbal task to be solved using reasoning	17	19
A problem is a task the solution of which is not in my mind	15	12
A problem is something I want to find out but where I have gaps in my knowledge	8	12
What else might a problem mean	1	5

Some teachers gave more than two responses as their 1st option, and therefore, the sum of first column is bigger than 63. In the following, the frequencies of choices are given in brackets (the 1st choice; the 2nd choice). The mode is in the first line: The teachers seem to understand problem as a verbal task (27;15) in which pupils apply their earlier knowledge. Many teachers answered that a problem is a verbal task that will be solved by reasoning (17;19). Thus, altogether 44 respondents (70%) attached a verbal task as their first alternative a verbal task to the concept 'problem'.

The remaining 19 respondents (30 %) linked with the concept 'problem' following ideas: For about one fourth of the teachers, problem is a task for which the solution is not known (15;12). Problem was also defined as something you would like to know, but you have gaps in your knowledge base (8;12). Some representative additional statements are, as follows: "Problem should be understood more broadly than only as solving of verbal tasks." In additional answers, one teacher states that "Problem in mathematics is all things above, depending on the class /on pupils´ mathematical knowledge / skills and abilities."

Summarizing, the most common understanding among the grade 3 teachers was that problem is a verbal task.

The question: What is problem solving?

In the second question, the teachers were asked "What is problem-solving in mathematics?" Here we developed from to the most important options and the second options of the alternatives the following table (Table 2).

Table 2: The teachers' answers to the question "What is problem solving?"

Alternatives given in the question	*The 1st option*	*The 2nd option*
Helping pupils to find out their solutions for the task	33	15
Using previous knowledge to solve a task	20	16
Defining the conditions and choosing solving methods	9	16
Exercising learning procedures and methods	2	8
Solving a verbal task together with pupils	0	6
Evaluating the solution found	0	1
What else could problem solving be	0	2

Some teachers gave more than two answers, and therefore, the sums of both columns are bigger than 63. The frequencies of choices are given in brackets (the 1st choice; the 2nd choice) in the following.

The mode is in the first line: Most of the teachers would like to see themselves as the pupils' guide in helping them to find solutions to problems (33;15). Many teachers said that problem solving is applying previously learned skills (20;16). Some of the teachers thought that problem solving is a determination of the terms of the task, and a selection on the solution (9;16). Some of the grade 3 teachers said that problem solving is to exercise procedures and methods (2;8). Still other teachers stated that it is solving a verbal task together with the pupils (0;6).

In the teachers' free answers ("What else…") one can see many interesting details that indicate the teachers' multi-faceted understanding on problem solving. For example, many teachers used at least one of the following concepts: *attitude, sensibility, motivation, creative, flow, discuss* that were not given in the questionnaire.

> In practical situations I connect also an attitude. I don´t give answers but the children have to find solutions themselves, in that way the sensibility and the motivation will be preserved in the schoolwork.
>
> It is a creative individual/ group activity.

How to implement problem solving?

The fifth question of the questionnaire “Give an example of the way of teaching you like to use in a mathematics lesson when dealing with problem tasks” inquired on the way how the teachers will implement problem solving in their class.

When the answers were classified according to two school-practice variables or kinds of working in class (individual work – teamwork; teacher-centered – pupil-centered), the following four-field matrix (Figure 1) was found:

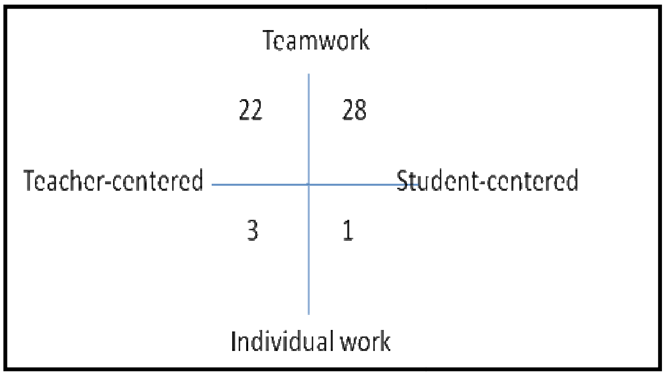

Figure 1: Practices in school classes.

The sum is here not 63, because some teachers did not answer this question. Another researcher classified the responses independently, and the reliability for the classification was found to be 93 %.

Here I give some examples about this four-field matrix. To the teacher-centered teamwork, there are classified answers like this.

> Made a task together on the table is often rewarding.

In the student-centered teamwork, there are the answers like this.

> Group of 3-4 pupils. They are trying to work together to solve the problem which has been given. The teacher guides the final solution, if the group can not solve it.

The teacher-centered individual work is like this.

> I read the problem to the pupils and give the necessary data/numbers. I give each of them a label to solve the problem. At the end we are discussing about the answer.

The pupil-centered individual work is like this.

> A week´s puzzle. We are checking the answer at the end of the week and sometimes rewarding.

In the four-field matrix above, we can clearly see the emphasis on teamwork. Thus on one hand, the majority of the teachers (50) considered problem solving to be teamwork. On the other hand, teacher-centered and a pupil-centered working seemed to be equally popular.

The question: What kind of examples teachers give for problems?

We also asked teachers for examples that they like to use in their problem solving lessons. These answers can be classified into five categories; use of tools, mathematics in everyday situations, verbal tasks, use of textbooks, and examples in meta level. In the following the frequency is given in brackets.

A common belief among teachers was the idea of the usefulness of tools (21). As we saw before, 70 % of the grade 3 teachers considered the problem to be a verbal task, but only about half of them offered here a word problem (15). These problems seem to be the same as in the textbooks. Mathematics in everyday situations (12) was one of the reasons for choosing the problem. The idea here was to prepare pupils to use mathematics in everyday life. There were some teachers (8) whose pedagogical reflection was in meta-level. They wanted to help their pupils in thinking. Some teachers were using textbooks and teacher guides (7) when they chose their problems, because most of the problems they highlighted in the questionnaire seemed to be from the textbooks. According Kolovou & al. (2009) elementary textbooks contain only few problems. As teachers, however, use a lot of textbook tasks, then one should consider how the textbooks could be improved.

The question: How often do teachers use problems in their lessons?

In the questionnaire, it was asked how often did the teachers use problems in their lessons. Altogether 79 % of the grade 3 teachers tell that they use problem solving at least once a week, and 38 % of them answered that they use it in all their lessons. Furthermore, it is questionable that all the teachers said that they use problem solving at least occasionally (19 %). Those who said that they use it occasionally, meant that it requires a suitable context, and therefore there is no regularity.

DISCUSSION

The goal for mathematics teaching is that pupils learn to understand also the structures of mathematics, not just mechanical calculations (NBE 2004). Higher-level thinking in learning can be reached in working with such a problem where an individual has to integrate existing data in a new way in order to find a solution.

The survey results indicate that all grade 3 teachers said that they use problem-solving. However, they mean different things by problem solving and problem-solving tasks. A word problem and a problem-solving task are not automatically synonymous. On one hand word problems can be solved fairly routinely, and on the other hand traditional product tasks can be solved according to the problem solving principles. More than the type of task is the question of how to guide pupils.

Additionally with the use of a questionnaire, we can reach only teachers' espoused conceptions. They may act in their teaching practice totally differently. This has been noticed already some twenty years ago, e.g. Ernest (1989) wrote on two kinds of teacher's conceptions, namely 'espoused' and 'enacted'. But also teachers' espoused conceptions reflect on their actual thinking, and therefore, are worthwhile to study.

It seems that in Polya´s model the first step (Polya 1945), understanding the problem by defining the conditions and choosing solving methods of a task, was not emphasized in the teachers' answers. This step is important for two reasons in problem solving. In the interpolation problems, the identification of missing elements activates the necessary concepts, and in open problems orientation step, a student might limit the size of the problem. Another reason is related to motivation. When the teacher is preparing a lesson he/she should think about what motivates pupils, and how their interest can be maintained. (cf. Aebli 1985). In a learning situation, teachers regulate with questions the level of knowledge to be learnt. Genuine listening to pupils (cf. Pehkonen & Ahtee 2005) and focusing on pupils' thinking helps the teacher to do such questions to pupils that assist them into the heart of the problem.

In Polya´s second and third step (Polya 1945), devising a plan and carrying out the plan, was focus in the teachers' answers. All the grade 3 teachers answered that they use the problem solving at least occasionally. Also the majority of the teachers considered problem solving as teamwork. They stated that they use everyday situations as the problems, and word problems were very popular (70%). When we looked at the tasks they said to have used, we saw that most of them have named routine verbal tasks to be problems. We cannot see in this study, how the teachers implement their lessons, but careful estimates based on our experiences as teacher educators suggest that about half of the teachers mention the use of routine tasks in Kantowski´s (1980) meaning.

The study of Sivunen & Pehkonen (2009) dealt only the elementary teachers in one town, i.e. it was a case study, and therefore, its results were not generalizable. Our study was based on a random sample of grade 3 teachers in Finland, and the response rate was rather good (63 %). Thus its results can be generalized to all Finnish elementary teachers, since Finnish elementary teachers do not specialized in one grade level but usually teach all grades after each other in elementary school (grades 1–6).

So we can say that because half of the grade 3 teachers use routine tasks, their pupils' learning stays on the routine level. It is difficult to change wrong learning habits, and therefore, also the elementary teachers should understand what problem and problem solving means.

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VARIATION IS GOOD - MATH BOOK IS BAD
CONNECTING TYPOLOGIES TO DICHOTOMOUS WORDING

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The statement in the title "variation is good – math book is bad" is part of a dichotomy created based on Swedish novice mathematics teachers' examples of good and less than good mathematics lessons. In this paper, I will illustrate how otherwise seemingly static statements in that dichotomy, when connected with three typologies, offer a multifaceted meaning to the dichotomy. Both the dichotomy and the typologies were created based on the analysis of two years of ethnographic inspired field work in a study investigating the professional identity development of novice primary teachers. The conclusion in this paper is that there is a need for socially orientated, ethnographic inspired research to discover multifaceted meanings in otherwise seemingly static statements.

INTRODUCTION

The empirical material and the results presented in this paper are from a study investigating the development of novice primary school mathematics teachers' professional identity. The whole study is not completed and the results presented in this paper are two smaller components which have been developed during the analysis. In this paper, these two components will be connected showing how seemingly static statements and agreements regarding those statements become multifaceted when connected with typologies.

The two components to be connected are a dichotomy regarding mathematics teaching and three typologies of mathematics teachers. Both the dichotomy and the typologies are parts of the results of this study but, due to the limited space in this paper, they will only be briefly explained and after that they will be treated as starting points for the connection. The connection, and how the ethnographic approach has made the connection possible is the main focus of the paper. However, before presenting the dichotomy and the typologies, the study and some of its theoretical premises will be presented.

THE STUDY

The study is a case study with an ethnographic approach where seven primary school teachers have been followed from their graduation and the following two years. According to Gee (2000-2001), our interpretation of an individual as a special kind of person[1] comes from how we recognise the individual's combination of speaking, writing, acting, interacting, facial expressions, body language, dressing, etc. Discovering this combination requires both time and interaction and this is the reason for the ethnographic approach in this study.

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[1] To be recognised as a special kind of person in a given context is how Gee (2000-2001) defines identity.

Ethnography is not a collection of methods but a special way of looking, listening and thinking about social phenomena where the main interest is to understand the meaning activities have to individuals and how individuals understand themselves and others (Arvatson & Ehn, 2009; Aspers, 2007; Hammersley & Atkinsson, 2007). Reaching such an understanding requires, according to Aspers (2007), interaction. Interaction is not enough in itself but is an assumption and requires the researcher to be present, observe and interview informants in the field of study. That has been done in this study.

Also, time is an aspect of ethnographic studies and can be understood both as the total time for the study as well as the time used in fieldwork (Jeffrey & Troman, 2004). In this study, the time has been used in a selective, intermittent way, which means that the time taken for fieldwork, from starting to stopping, has been long (two years) but with a flexible frequency regarding the field visits.

Analysis is not a separate part of ethnography but starts in the pilot study and continues through the fieldwork and the writing process (Hammersley & Atkinson, 2007). The results presented in this paper have been developed gradually based on interplay between fieldwork and analysis of observations, interviews and self-recordings made by the subjects during that fieldwork. These different empirical materials have different characteristics but the common aim is to shed light on the professional identity development of the novice mathematics school teachers. The empirical materials are treated as, named by Aspers (2007), complete-empiricism, implying all the material constitutes a whole that the analysis is based on. The analysis has been conducted using methods inspired by the constructivists' grounded theory (Charmaz, 2006) to create categories. The dichotomy and the typologies are two such created categories.

PATTERNS OF PARTICIPATION

Since this paper does not concern the whole study regarding novice primary mathematics teachers' professional identity development, its total theoretical framework will not be presented here, instead, essential parts related to the dichotomy and the typologies are presented.

At the beginning of the study, Wenger's (1998) theories regarding communities of practices were explored in order to understand *identity* and *identity development*. To understand the *professional* part of professional identity, beliefs and mathematical knowledge for teaching were also explored. However, some combination problems occurred as research regarding beliefs and mathematical knowledge is not derived from a social perspective in the same way as Wenger's theory regarding identity and identity development.

Instead, according to Skott (2010), beliefs are generally understood as “deeply personal, conscious or unconscious, value-laden, mental constructions that carry a subjective truth value and are the result of experiences gained over prolonged periods of time” (p.194). He argues that the social orientation that has developed in other mathematics education research should also be made within belief research.

Also, Lerman (2002) argues that the absence of context and the assumption that beliefs are stable across contexts is a problem. Teachers’ actions, including their responses in interviews and questionnaires, need to be analysed in context. To achieve this, he argues that a participatory approach is required with focus on participation in social practices and identity development.

Skott (2010) suggests a shift from focusing on objectified beliefs to focusing on individuals’ patterns of participation in social practices. He describes patterns of participation as the pre-reified processes of teachers’ participation in social practices, some of them orientated towards the teaching and learning of mathematics.

Regarding research focusing on mathematics knowledge for teaching, an important aim has been to identify the professional knowledge needed to teach mathematics (Hill, Sleep, Lewis & Ball, 2007). It is, however, according to Ball, Thames and Phelps (2008) and Wilson and Cooney (2002) difficult to know what mathematical knowledge is specific to teachers and how much of that knowledge is culturally specific and depends on the teacher's teaching style. Like Skott (2010) and Lerman (2002), over argues that belief research needs to focus on participation in social practices Llinares and Krainer (2006) recommend a shift from considering mathematical knowledge for teaching as independent of context to treating teachers' mathematical knowledge for teaching as part of teaching practice.

The notion of mathematical knowledge for teaching is a reification in the same way as the notion of beliefs. Similarly, as researchers attribute beliefs to individuals based on statements or actions, researchers attribute mathematical knowledge for teaching to individuals based on statements or actions. In both cases, the individuals’ statements and actions have been objectified and, after that, are treated as objects (beliefs or mathematical knowledge for teaching) which the individuals are described as having.

The same patterns of participation, suggested by Skott (2010) as the pre-reified processes of teachers’ participation in social practices, are valid also as pre-reified mathematical knowledge for teaching. The actions and statements objectified earlier as beliefs respectively mathematical knowledge for teaching are no longer interesting. Instead, patterns of participation regarding all components within mathematics teaching as well as patterns of participation regarding the sense of being and developing as a teacher become the focus. Using patterns of participation instead of beliefs and mathematical knowledge for teaching makes a connection to Wenger’s notion of identity possible. That connection will however not be made here since identity is not the focus in this paper.

PATTERNS OF PARTICIPATION AS A DICHOTOMY

The first component of the connection to be discussed in this paper is a dichotomy. A dichotomy means a division of something into two mutually exclusive categories. Some variables have a natural dichotomy (e.g. gender) while other variables are constructed in analysis with the purpose to clarify. The dichotomy presented here is of the second kind.

In interviews shortly before graduation, the informants in the study were asked to give examples of, according to them, good and less than good mathematics lessons. In their examples, all of them, independent of each other and without being asked, motivated their examples regarding good mathematics lessons using examples of less than good mathematics lessons and vice versa. They said examples were “funnier”, “more right for the children” and “such knowledge sticks better”. In relation to those expressions, counter-questions could be asked regarding “funnier than what”, “more right than what” and “sticks better than what”? Sometimes the informants also gave examples of good or less than good mathematics teaching in terms of “teaching that is not” followed by an opposing example.

These accounts of mathematics lessons and teaching in terms of a negotiation between good and less than good can be represented as a dichotomy. Of course, the individual informants’ accounts were not identical but they, in their individual pattern of participation, gave a distinct and concurrent picture of how mathematics teaching should be best managed. The common starting point seemed to be that good mathematics lessons and teaching are *innovative* and less than good mathematics lessons and teaching are *traditional.*

The informants’ accounts and negotiations between good and less than good lessons and teaching as innovative or traditional can be further divided based on them focusing on the teacher, the teaching, or the outcome of the teaching, resulting in the dichotomy below. The author has created the dichotomy based on the grounded theory analysis but its content is from the empirical material. As such, the dichotomy is an objectification of the individuals’ patterns of participation regarding discussing mathematics lessons and teaching before graduation.

Innovative mathematics teaching	**Traditional mathematics teaching**
The teacher	***The teacher***
- reformative	- conservative
- creative	- controlled by the math book
- autonomous	- heteronomous
- daring	- weak
- up-to-date	- stagnant
The teaching	***The teaching***
- reveals different ways of thinking	- reveals only one way of thinking
- group work	- solo work
- concrete	- abstract
- student-focused	- student-distanced

- reality-based	- reality-distanced
- integrated with other subjects	- separate from other subjects
- varied	- repetitive, based on the math book
- focused on processes and problems	- focused on products
- mathematics is hidden	- mathematics is visible
- connected components	- separate components
The outcome of the teaching	***The outcome of the teaching***
- cooperating students	- students working alone
- active students	- passive students
- many solutions to a problem	- one solution to a problem
- motivated students	- uninterested students
- comfortable students	- uncomfortable students
- students having fun	- students being bored
- students understanding	- students not understanding
- mathematics becomes interesting	- mathematics becomes uninteresting

Two years after the interviews behind the creation of this dichotomy, at the end of the whole study, group interviews were held with the informants. In these interviews, they were shown the dichotomy. When reviewing and discussing its content, the division of mathematics lessons and teaching into these antipoles seemed to be unproblematic. As such the informants made a kind of confirmation of the dichotomy. Looking at it they started to talk about limitations between the two sides of the dichotomy which prevented them, and other teachers, from working in line with the left side.

Gunilla: I believe many teachers would like this [pointing to the left side of the dichotomy].
Helena: Yes.
Gunilla: But they feel unsure about covering everything.
Nina: Mm.
Gunilla: Will we reach our goals working like this? Do I dare step away from the math book, leaving this traditional and safe way of working? Or will I risk my students failing? […]
Nina: And above all, this way of working [pointing to the left side of the dichotomy] takes a lot more time for the teacher. I feel that.

PATTERNS OF PARTICIPATION AS TYPOLOGIES

The second component of the connection to be discussed in this paper is typologies. Just like the dichotomy is an objectification of the individual's patterns of participation regarding discussing mathematics lessons and teaching before and two years after graduation, the typologies are objectifications of the individual's patterns of participation regarding mathematics teaching during the two years after graduation. It is important to mention, however, that the empirical material on which the typologies are based is the complete-empiricism from the whole two years of the study, not just interviews as with the dichotomy.

Typologies differ from actual types (e.g. the informants) in that they are analytic constructions used to characterise key patterns from several actual types. None of the iformants is a typology personified. Instead, typologies are based on common features of interest regarding the informants' patterns of participation in relation to the studied phenomena (Atkinson, 1990; Hammersley & Atkinson, 2007).

Looking at the patterns of participation of the actual types (the informants in the study) using methods of grounded theory (Charmaz, 2006), key patterns became visible and identified three typologies: the coach-orientated mathematics teacher, the content-orientated mathematics teacher and the emotion-orientated mathematics teacher. The ethnographic approach resulted in data making that analysis possible. These three typologies will only be briefly described here since the purpose of this paper is not to explore them as such but connect them to the dichotomy.

The main focus of the coach-orientated mathematics teacher is to teach the student how to organise and take responsibility for their own learning. These teachers, in both talk and action, foster team spirit in the class and encourage students to help each other and they want their students to develop strategies for self-help.

The main focus of the content-orientated mathematics teacher is how the content is to be treated in the teaching and learning. These teachers, in both talk and action, focus on how different content can be represented and understood.

The main focus of the emotion-orientated teacher is the students' emotions and self esteem. These teachers, in both talk and action, focus on the students' emotional experiences of the teaching and learning and on creating positive attitudes towards mathematics.

CONNECTING THE TYPOLOGIES WITH THE DICHOTOMY

In this part of the paper, the dichotomy and the typologies will be connected. Looking at the antipoles in the dichotomy in relation to how the typologies talk and act, it becomes evident that the meaning of the dichotomy differs. Despite using the same words and agreeing about the dichotomy two years after graduation, the meaning of the dichotomy differs depending on typology.

This differences will be illustrated by using one of the antipoles in the dichotomy, varied teaching being innovative (good) and repetitive teaching based on the math book being traditional (bad).

Variation is good

All informants stress variation as important and good in relation to mathematics teaching. However, when connected to the three typologies, the meaning of and the benefit of variation differ.

According to the coach-orientated mathematics teacher, variation implies organising the learning environment in different ways with the purpose of making it possible for every student to find their best way of learning.

According to the content-orientated mathematics teacher, variation implies representing the mathematical content in different ways with the purpose of giving the students an opportunity to gain a richer understanding.

According to the emotional orientated mathematics teacher, variation implies doing lots of different things during the mathematics lessons with the purpose of the students perceiving the teaching, and therefore also mathematics, as fun.

Math book is bad

All subjects stress that repetitive teaching based on the math book is bad and is an antipole to the variation discussed above[2]. But why is the math book bad?

According to the coach-orientated mathematics teacher, the math book is bad as it is not the best way for all children to learn.

According to the content-orientated mathematics teacher, the math book is bad as it represents mathematics in one way only, which works against students gaining a richer understanding of mathematics.

According to the emotional orientated mathematics teacher, the math book is bad because it is boring and, working in the math book results in students thinking of mathematics as boring.

CONCLUSIONS

Both the dichotomy and the typologies are objectifications of patterns of participation. The dichotomy is an objectification of the informants' patterns of participation with regard to discussions about mathematics lessons and teaching before graduation and was confirmed in group interviews two years later. The typologies are objectifications of the informants' patterns of participation regarding discussing and teaching mathematics during the two years after graduation. The basis for the typologies is the complete-empiricism (observations, interviews, self-recordings) collected with an ethnographic approach

If connecting the dichotomy with the typologies, the dichotomy becomes more multifaceted than it seemed at first sight. When looking at and talking about the dichotomy in the quotation earlier, Gunilla, Helena and Nina were in an agreement. However, when connected to the typologies, their agreement may not be as simple as it looks.

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[2] The math book as something negative is also mentioned as a traditional antipole to the innovative creative teacher where the traditional teacher is described as being controlled by the math book.

The typologies are analytic constructions and no one of the informants is identical with a typology. However, even though the informants differ in many ways, their patterns of participation regarding teaching mathematics during the two years after graduation are dominated by a single typology. Gunilla, Nina and Helena's patterns of participation are not dominated by the same typology in the two years after graduation.

In the group interview, Gunilla, Nina and Helena agree about the dichotomy. They refer to the left side of the dichotomy using the term "this" in mutual agreement despite, in many ways, "this" actually meaning different things to them. Nina saying "this" way of working would take a lot longer may be true but what the extra time required would be used for by the three informants, regarding, for instance, variation, is not the same.

For the coach-orientated mathematics teacher and the emotion-orientated teacher, the extra time would, in general, imply the same use, organising the learning environment in different ways. Their reasons for doing this would, however, differ. For the coach-orientated teacher, the aim would be to offer activities where each student could find their best way of learning. For the emotion-orientated teacher, the aim would be to enable the students to have fun. For the content-orientated mathematics teacher, the time factor regarding variation would, instead, imply planning and teaching the mathematical content using different representations. The reason for this would be to give the students a richer understanding of mathematics. As such, the coach-orientated mathematics teacher and the content-orientated teacher have the same reason for variation, students should learn mathematics, but they would try to achieve this in different ways with different meanings for the word variation.

A connection like this between typologies and an antipole from the dichotomy can be made with all the other antipoles, thus uncovering a more multifaceted dichotomy. That is a work that will be done in the future. The example above shows the importance of going beyond the words of the informants by looking at their patterns of participation in social settings. The patterns of participation of the informants regarding variation described here would not have been visible without using the ethnographic approach and treating the empirical material in terms of complete-empiricism. The purpose behind objectifying the patterns of participations as dichotomies and typologies is to make them practical to talk and write about; however, the main interest, really, is the patterns of participation behind those objectifications.

Finally some words about the use of patterns of participation. In recent years, one focus in belief research has been the problems within that research. For instance, Lerman (2002), Lester (2002), Speer (2008) and Skott (2010) highlight the problem of circular reasoning where practice is explained by beliefs while beliefs are determined from observations of the same practice. In addition, the problems regarding the absence of context and the presumption that beliefs are stable across contexts, as mentioned earlier, are considered problems. Lester (2002) and Speer (2008) stress the development of research methods as a solution to these problems.

I, however, do not see the use of the ethnographic approach, which treats the empirical material in terms of complete-empiricism with a purpose to investigating patterns of participation with regard to mathematics teaching, as the development of a research method. I see the use of patterns of participation as another approach to research other than the earlier approaches used in belief research and research regarding mathematical knowledge for teaching. The purpose is not to find an underlying structure of beliefs or mathematical knowledge for teaching within the individual but to explore patterns of participation in social practices. This approach makes new demands on fieldwork requiring socially orientated, ethnographic inspired research.

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PUPILS' CONCEPTIONS ON MATHEMATICS LESSONS REVEALED VIA THEIR DRAWINGS

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Here we consider the drawing material from the starting test (autumn 2010) of the Chile – Finland comparative study. Altogether pupils from 10 classes (N = 153) have drawn a picture on mathematics lesson. Thus we tried to reveal third-graders' conceptions on mathematics and its teaching from these drawings. In our study, we restrict us in the communication to be seen in drawings. A typical communication in class seems to be built mainly on three themes: mathematics is nice/ easy, mathematics is dull/ difficult, and pupils can do mathematics.

There is a nine-year comprehensive school in Finland, where all children study in heterogeneous groups, including in mathematics. Teaching in schools is regulated by the national curriculum (NBE 2004). Teaching mathematics in elementary grades is usually concentrated around the use of textbooks (cf. Perkkilä 2002). Details on mathematics teaching in Finland can be found e.g. in the book Pehkonen & al. (2007). Here we aim to clarify third-graders' (about nine-year olds) conceptions of mathematics and mathematics teaching through their drawings.

PUPILS' DRAWINGS AS A RESEARCH OBJECT

Kress, Jewitt, Ogborn, & Tsatsarelis (2001) have studied the multiplicity of modes of communication (speech, image, gesture, action with models, writing, etc.) that are active in the classroom when the teacher presents the subject matter and seeks to shape pupils' understanding of it through the interactive processes of classroom teaching and learning. According to them communication always draws on a multiplicity of modes of representation, of which language is only one and not necessarily the dominant one. Learning is much more than a matter of speaking or writing; it is a dynamic process of transformative sign making. Drawing is an alternative form of expression for children. Barlow, Jolley and Hallam (2010) have noted that free hand drawings helped children recall and express more details about events they depicted. Drawings tend to facilitate the recalling of events that are unique, interesting or emotional, but not routine events or isolated bits of information that are not part of a narrative. Finally, we want to emphasize that pupils' drawings open an holistic way to evaluate and monitor the classroom communication.

About twenty years ago, it was observed that pupils' classroom drawings opened a non-verbal canal to children's conceptions. Many researchers (e.g. Aronsson & Andersson 1996; Weber & Mitchell 1996) have used pupils' classroom drawings, and realized that they form rich data to rearch children's conceptions on teaching. For example, Weber & Mitchell (1996) state that a part of everything that we experience

– what we have seen, know, think or imagine, remember or reject – will be convoyed in drawings that thus reveal sub-conscious connections or inaccuracies. According to Harrison, Clarke & Ungerer (2007) pupils' views can be captured with drawings more meaningfully than with other research data.

Ruffell, Mason & Allen (1998) question surveys as a method to investigate children, since small children do not necessary understand the words and statements used in questionnaire in the way a researcher means. This observation was confirmed about ten years later by Bragg (2007). It would be optimal for a child in question to have an opportunity to formulate his/her conceptions into words. According to Hannula (2007) it is not easy to get from young children verbally rich answers to questions, since young children tend to give monosyllabic answers to questions that they do not consider relevant to them (Tikkanen, 2008). Young children may have some difficulties with reading surveys and expressing themselves clearly in writing. Within interview contexts, children of all ages may be hesitant to discuss with an often relatively unknown researcher.

Picker & Berry (2000) have collected from several countries a total of almost 500 drawings on a mathematician in his/her work. The pupils in question were from grade 7 or 8, i.e. 12– 13-year olds. These drawings send a message about pupils' negative images on mathematics teaching. In about one fifth of the drawings, a mathematician was described as a school teacher, who forces pupils' to learn by intimidation, threaten or violance. Drawings of the teacher showed that he does not always master the teaching group nor the topics to be learned. The teacher in drawings seemed to be cleverer and better than his pupils, but he lacked common sense, style and calculation skills. The teacher in drawings might have supernatural forces, and thus he could use magic and magic potion.

Reseachers in mathematics education (e.g. Tikkanen 2008, Remesal 2009, Dahlgren & Sumpter 2010) suggest that a practical method to evaluate teaching is to ask pupils to draw a picture on a mathematics lesson: If teaching has been teacher-centered, pupils often draw a blackboard and a teacher in front of the class. Very seldom the drawings contain hints on communication between pupils. When pupils have participated activating teaching, they produce pictures where discussions between pupils and activating methods are emphasized. Drawings also tell about beliefs, attitudes and emotions connected to mathematics. Furthermore Murphy, Delli & Edwards (2004) have observed, when letting pupils to draw classroom events, that pupils begin, as early as in the second grade of elementary school, to form beliefs about good teaching.

In the analysis of drawings, the most important point is the meaning that we give them, as well as their interpretation. According to Blumer (1986) and symbolic interactionism, the meanings that pupils have given to happenings and matters will influence their actions, how they interprete different situations, and what they put into their drawings. Giving meanings is a continuous process. Different pupils derive different meanings from the very same happenings and matters. Meanings can be related to physical things, such as the blackboard or the pupil's desk in the classroom, to social interactions such as working alone or in groups, or to abstract things, as mathematical concepts or emotions evoked by teaching. Working methods used in teaching organize activities between the teacher and his/her pupils as well as between pupils. As a result of experiences gained from teaching a pupil may evaluate himself/herself to be poor and his/her classmates good in mathematics.

Additionally, drawing has other strengths (Tikkanen 2008): Drawings as research instruments decrease linguistic barriers in comparisons at the international level. Drawings provide a window into children's views of mathematics, mainly because they reflect beliefs and emotions of the drawers. As a technique for exploring views of mathematics, drawing taps holistic views and prevents children from trying to match their knowledge with that of the researcher. It is also a useful alternative form of expression for children with a communication impairment. Many young children like to draw pictures, completing them quickly, easily and in an enjoyable way. According to Bonoti and Metallidou (2010), children improve in drawing performance with age, but their feeling of liking and the estimation of correctness of the drawings significantly decreases, especially after the second grade.

On pondering the reliability and validity of mathematics drawings, Stiles, Adkisson, Sebben & Tamashiro (2008) concluded that drawings offer more opportunities to show a stronger and more personal opinion than responses to the questionnaire statement ”I like mathematics”. In pictures pupils may draw hearts when loving mathematics, or a gun in order to destroy mathematics showing that they don't like mathematics. In cases like this, the data based content analysis of drawings can concentrate on both a qualitative exploration of what is drawn, as well as quantitative inspecting how often particular themes/categories appear. Furthermore, the focus of content analysis on inter-rated reliability is the potential for researchers to check credibility of drawings. Also Dahlgren & Sumpter (2010) consider pupils' drawings as a reliable means to catch pupils' conceptions of mathematics teaching.

Conceptions on mathematics

Conception is problematic as a concept, since it is not clearly defined (cf. Furinghetti & Pehkonen 2002). Here we understand an individual's beliefs in a rather wide sense as his/her subjective, experience-based, often implicit knowledge and emotions on some matter or state of art. Furthermore, *conceptions* are explained as conscious beliefs. In the case of conceptions, we understand that the cognitive component of beliefs is stressed, whereas in basic (primitive) beliefs the affective component is emphasized.

Since conceptions are one of the main concepts of the MAVI meetings, we will not go here into details, but only point out additional information to some publications in the list of references: Dahlgren & Sumpter (2010), Hannula (2007), Murphy & al. (2004), Perkkilä (2002), Remesal (2009), and also to older MAVI proceedings.

The focus of the study

We aim to clarify pupils' conceptions of mathematics and mathematics teaching looking at the communication in drawings, and thus will get answers to the research problem: "What is communication like in drawings of mathematics classrooms?" In this paper we will restrict ourselves in those parts of the drawings where communication can be found. Thus we enter the research problem through following three research questions:

(1) What kind of communication can be found between the teacher and pupils?

(2) What kind of communication is within the class?

(3) What kind of quality has the communication in class?

METHODS

The paper at hand is connected with the three-year Finland – Chile comparison study that is financed the Academy of Finland for years 2010–13 (project #135556). Its aim is to clarify the development of pupils' mathematical understanding and problem solving skills, from grade 3 to grade 5 when using open problem tasks at least once a month. The data dealt with here is pupils' drawings from autumn 2010 that belongs to initial measurements in the project.

Participants and data gathering

Project participants are third-graders (N = 153, about 8– 9-year-olds) from classes taught by 10 different teachers in five elementary schools in Great-Helsinki. Pupils did the drawing task during their mathematics lessons in the beginning of the school year (autumn 2010). The task for the pupils was, as follows: *"Draw your teaching group, the teacher and the pupils, in a mathematics lesson. Use speaking and thinking bubbles to describe discussion and thinking."* Drawing were collected and analyzed from a total of 131 pupils; thus the non- response rate is 14,4 %. Those pupils delivering their drawings were comprised 69 boys, 60 girls and 2 who did not indicate their gender. The contents of the speaking and thinking bubbles enabled the researchers to investigate the communication in class.

Data analysis

The starting point for classification of pupils' drawings is the analysis method used in Pirjo Tikkanen's dissertation (2008) that she has since developed. According Tikkanen's method a drawing as an observational data can be divided into content categories. A content class means the phenomen on which data are gathered. Here we selected our content classes to be the following: *a teacher's communication with his/her pupils (1), communication within in class (2), quality of communication in class (3)*. For the analysis, the content classes were specified into following sub-categories.

1. A teacher's communication with his/her pupils: gives orders; maintains order; teaches; gives feed-back; follows quietly pupils' working.

2. Communication within class: a pupil makes/ asks/ thinks a remark in connection to teaching; a pupil asks for help; pupils discuss with each other; a pupil makes/ thinks an improper remark.

3. Quality of communication in class: a pupil says/ thinks ”mathematics is nice/ easy”; a pupil says/ thinks ”mathematics is dull/ difficult”; a pupil can do the mathematics; a pupil helps another pupil; a pupil cannot do the mathematics.

Two researchers classified all pupils’ drawings, and the cases with differences were looked through again. All drawings were classified carefully, and the estimation for concensus was received by calculating the differences between two classifiers. The analysis of the drawings was qualitative, and therefore, it can be classified as inductive content analysis (cf. Patton 2002). The drawings were looked through focusing on one content class at time. In each content class, we looked at each drawing, to see whether there are sub-classes belonging that content class. In every content class, the last sub-class was ”not recognizable”. The concensus of the two classifiers was in all sub-classes very good, i.e. more than 90% (variation about 91% – 95%).

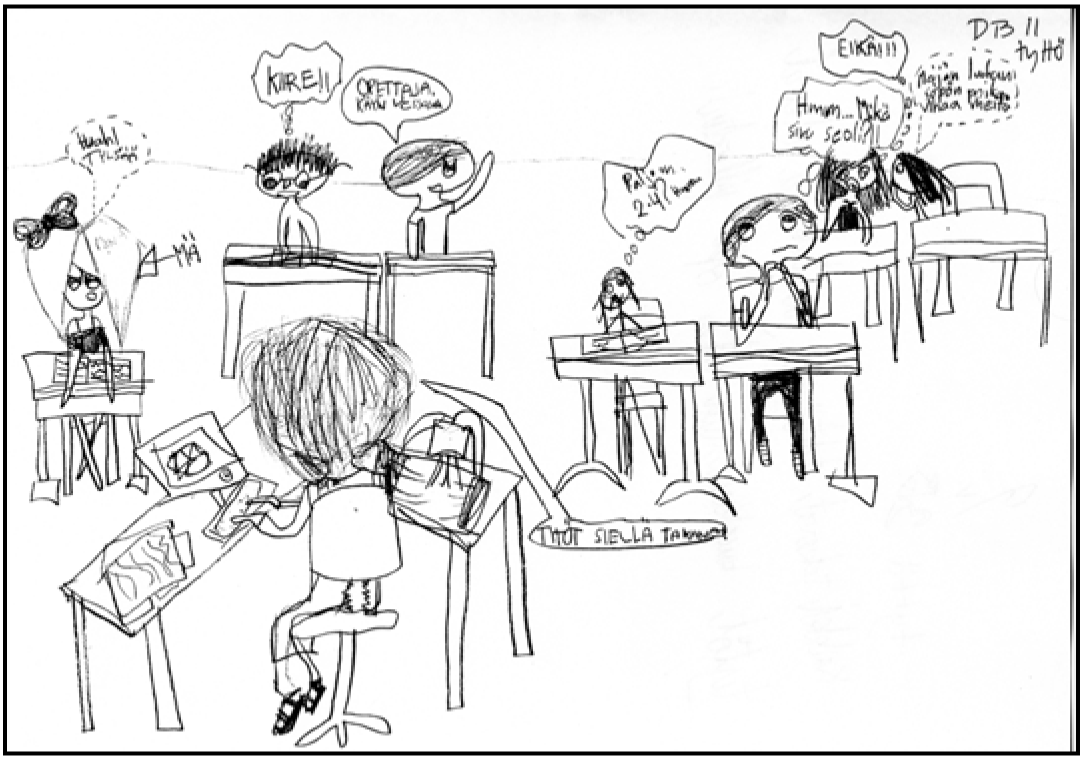

Figure 1: An example of a pupil’s drawing, its analysis is below.

The example in Figure 1 of pupils’ drawings is very informative. In many drawings, one can see only stick figures, in some cases hands beginning from the head, and in some drawings there are only pupil desks representing pupils. However, some third-graders are very talented in drawing, and then in pupils’ drawing one can see several details. In speaking bubbles pupils present ideas about mathematics lessons and their atmosphere. But pupils' way of presenting a saying (loud or wispering) and thinking is not always consequent.

The analysis of the drawing in Figure 1 is, as follows. In many content classes there is not only one feature, but many: 1. A teacher's communication with his/her pupils (a teacher maintains order in class). 2. Communication within class (pupils make/ ask/ think a remark on teaching; pupils' remarks/ thoughts are improper). 3. Quality of communication in class (a pupil says/ thinks ”mathematics is dull/ difficult”).

RESULTS

In this study, we try to answer with the help of the drawing analysis to the three research questions: (1) What kind of communication can be found between the teacher and pupils? (2) What kind of communication is within the class? (3)What kind of quality has the communication in class? It is helpful to notice that in the categories of the classification, the frequency is bigger than the number of the pupils (131), since in many drawings one can find several features.

Firstly, we discuss the content class ”A teacher's communication with his/her pupils” (cf. Table 1). Since in the drawings of many pupils there were several indicators, the total frequency is 145. This totality is divided rather uniformly between several factors. The mode (36; 25 %) in this category is ”teaches” that contain both a teacher's own questions and expository teaching. But the frequencies are almost as big in the categories ”follows quietly pupils' working” (33; 23 %) and ”not recognizable” (28; 19 %). Thus most of the pupils have an impression that a teacher makes questions and delivers knowledge in mathematics lessons.

Secondly, we take the content class ”Communication within class” (cf. Table 1). Since in the drawings there were several indicators, the totality is 191. The biggest frequency is in the sub-class ”a pupil makes/ asks/ thinks a remark in connection to teaching” (65; 34 %). The next biggest frequency (48; 25 %) is in the sub-class ”a pupil makes/ thinks a unproper remark”. The frequencies of the rest of the three sub-classes are under half of the maximum frequency. Therefore we could say that the communication within the class is a compound of pupils' remarks where the biggest share form the remarks connected to teaching/ learning of mathematics, but there is also a great many of improper remarks (cf. Figure 1).

Thirdly, the content class ”Quality of communication in class“ (cf. Table 1) will be taken under consideration. Since in the drawings, there are several indicators, the total frequency is 149. In this content class there is no clear modal class, but three sub-classes compete with each other: “a pupil can do mathematics“ (33; 22 %), “a pupil says/ thinks ”mathematics is nice/ easy“ (32; 21 %), “a pupil says/ thinks ”mathematics is dull/ difficult“ (31; 21 %). But the share of non-recognizable drawings is remarkable (42; 28 %). Therefore, we could state that the quality of communication in class is not clearly recognizable.

Table 1. The relative frequencies in the three content classes: A teacher's communication with his/her pupils (N = 145), Communication within class (N = 191), Quality of communication in class (N = 149).

A teacher's communication with his/her pupils (%)		Communication within class (%)		Quality of communication in class (%)	
teaches	25	connected to teaching	34	can do mathematics	22
follows quietly	23	is improper	25	mathematics nice/easy	21
maintains order	13	asks for help	15	mathematics dull/difficult	21
gives orders	10	pupils discuss	12	helps other	4
gives feed-back	10			a pupil cannot do	4
non-recognizable	19	non-recognizable	10	non-recognizable	28

Table 1: Relative frequencies of communication in class.

In Table 1 there are relative frequencies of communication in class. If we select from each content class those sub-classes that are most popular (the modal class) in pupils' drawings, we receive from the communication of third-graders in mathematics lessons the following prototypic picture: A teacher's communication consists mainly on teaching (25 %). Pupils' communication is connected with teaching (34 %), and about in a half of pupils' drawings there is a clear attitude to mathematics: they consider mathematics nice or easy (21 %), or dull or difficult (21 %). Additionally, a fifth of the pupils (22 %) have a feeling that they can do mathematics.

CONCLUSION

It is evident that in pupils' drawings there are many kinds of influences. These drawing were made in the beginning of the third grade (autumn 2010), thus it might be that most of pupils have been thinking on their teacher and teaching in grade 1–2. Additionally, many third-graders seem to have difficulties in drawing, and therefore, they might concentrate to draw only such a situation that is easy for them.

We consider the most important observation from these drawings the result that about in every second drawing there can be seen pupils' sayings or thoughts on their attitude toward mathematics: mathematics is nice, easy, dull or difficult. It comes out spontaneously from pupils' drawings, without extra being asked for. Thus, we could conclude that pupils talk about mathematics, its liking and difficulty of tasks. This polarization on the scale nice–dull and easy–difficult is very clear already in the beginning of the third grade, although according to Metsämuuronen's (2010) study pupils think that mathematics is nice and one can do tasks, even at the end of the second grade and in the beginning of the third grade. Since research results (e.g. Aunola, Leskinen, Lerkkanen & Nurmi 2004) indicate that individual differences in calculation skills are growing when moving to upper grades, it is important that teachers recognize as early as possible those pupils who have difficulties in mathematics. Such a drawing might help a teacher to understand his/her pupils' thinking.

In a further study, it would be interesting to find out, whether pupils themselves experienced mathematics as nice, easy, dull or difficult, or whether they put the words in somebody else's mouth. In the analysis we have combined nice and easy, as well as dull and difficult, in order to get a first approximation of the results. But we are aware that there are some pupils who might consider mathematics nice, but difficult, and similarly for some other pupils mathematics can be at the same time dull and easy. Although in our data pupils used only one of the alternatives nice, easy, dull or difficult, however, it is possible that pupils may think mathematics lessons are dull when the tasks are too easy for them. This might be another interesting topic for a further study.

No negative attitude toward the teacher could be found in these drawings. This is different from the results of the study Picker & Berry (2000). Instead of that pupils often ask teachers for help. Of course, a teacher commands somewhat when maintaining order. The interaction between a teacher and pupils seem to be in drawings positive, and this is important, since pupils are in cooperation with their teacher about 4–5 lessons during the school day.

Altogether two thirds (67%) of pupils produced drawings where pupils' thinking/ speaking and action can be seen. Also according Lindgren (1990) and Tikkanen (2008) mathematics lessons seem to contain many activities. It is to understand that pupils draw a teacher in the first place in front of the class, although classroom observations show that the teacher walks around among the pupils.

Such a drawing task can help a teacher to understand his/her pupils' thinking, and also to hint how the teaching should be developed. Furthermore, such a ”window” to pupils' thinking is very easy to open also within a lesson, and it will not demand from a teacher so much additional work.

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THE ROLE OF BELIEFS AND KNOWLEDGE IN PRACTICE

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In this paper we analyse a teacher's practice from a cognitive point of view, discussing the role of the teacher's beliefs and knowledge in carrying out the task of teaching. It aims to deepen our understanding of some of the critical factors that influence practice, how they are interrelated and, ultimately, how they impact on the learning opportunities made available (or not) to students for developing their knowledge. From the analysis of a (primary) classroom sample we obtain a set of relationships between the teacher's beliefs, knowledge and actions. These relations provide an insight into the teacher's practice and the way she envisages her and her students' role in the (mathematics) teaching-learning process.

INTRODUCTION

The roles of teacher and students, and their interactions, are directly related to the teacher's beliefs, knowledge and goals. These dimensions and the manner in which they are put into effect, depend, amongst other factors, on the teacher's experience and personal investment in the professional, and are reflected in the classroom interactions (e.g. Baturo, Warren and Cooper (2004), Shulman (1986)). As a result, and also because the process of teaching is too complex to be analysed as a whole, it is necessary for researchers to select only those dimensions considered crucial to this process.

In order to deepen our understanding of what seems to be happening in the classroom and of the factors that influence practice (the hows and whys), it is especially important that we focus our attention on the actions of the teacher and the cognitions that these make evident. This focus provides us with a fuller understanding of the teacher's perspective regarding what takes place in the classroom, and how their decisions and consequent actions influence and are influenced by their cognitions, as the more we know about these and the relationships between them, the more we know about the teaching process (Ribeiro, Monteiro & Carrillo, 2009; Schoenfeld, 1998b). In order to put this aim into effect, we developed a model (which we denominate cognitive), based on that of Schoenfeld (1998a, 2000) with adaptations by Monteiro, Aguaded and Carrillo (2008). Throughout its development the model underwent constant adjustments as new data was compared with previous analyses, and this led to a broader vision of the relations between the constituents. These constituents include cognitions (beliefs, knowledge – specifically Mathematical Knowledge for Teaching or MKT – and goals), teacher's actions, the types of communication promoted, and the resources and pupils modes of working employed. In addition, a record was made of whether the teacher intended each situation.

The purpose of this cognitive model was not simply to obtain some kind of personal profile of the teacher, but essentially to gain an insight into how teachers' knowledge, beliefs, actions and attitude to mathematics directly or indirectly influence practice (e.g. Ernest (1989), Pajares (1992), Thompson (1992)). Objectives, whether short or long term, are prioritised and given shape according to the teacher's beliefs and the knowledge they bring to class (or believe they do), as the teacher can promote certain situations over others. Hence these factors condition the choices to be made with regard to the process of teaching and, consequently, the opportunities for learning (Hiebert & Grouws, 2007) made available to students and the nature of tasks which are designed and implemented (e.g. Charalambous (2008) Stein, Smith, Henningsen and Silver (2000)).

The principle objective of our research project was to explore teachers' beliefs and MKT as revealed by their classroom practice, and to consider the potential of this approach for teacher training. We focused our attention on practice and the factors influencing it, giving special importance to the teacher's cognitions. There were two inter-related motives for this. On the one hand, recent changes implemented in Portugal in teacher education (in accordance with the Bologna Process and the creation of the Programme for Ongoing Training in Mathematics) provided us with an opportunity to contribute actively towards improving the teacher training available to both practising and trainee teachers. On the other hand, consistent with ideas put forward by Tichá and Hošpesová (2006), we felt that improvements in teacher training would be of far greater value if teachers were confronted with situations that they could consider their own, to which effect it would be essential to analyse, discuss and reflect upon actual classroom situations and the role of beliefs and knowledge in the practice illustrated by these.

One of the objectives of the research project of which this paper forms a small part, is that of gathering information which will enable us to identify the beliefs and knowledge (limited here to the perspective of MKT) which are manifested in practice, and to discover how they inter-relate and the role that these relations take on in practice, and hence in students' learning opportunities. Here we discuss some emergent relationships between the dimensions considered and their impact on practice. We also consider implications for achieving a fuller understanding of these dimensions and their relevance to teacher training. This discussion has as its starting point a review episode taken from a lesson delivered by a primary teacher we shall call Maria in year 4. The aim of the episode was to go over, with the whole class, a worksheet on the concept of a tenth which pupils working individually.

BELIEFS, KNOWLEDGE (MKT) AND ACTIONS

A teacher's professional knowledge can be considered to consist of a set of multiple dimensions (Shulman, 1986) which, directly or indirectly, influence their teaching.

The teacher's beliefs and MKT lie at the heart of this professional knowledge and together inhibit or promote certain hypothetical opportunities for students to learn (Ribeiro, 2011) and consequently the objectives that these embody. (We consider objectives and the types of communication promoted as core elements of professional knowledge along with beliefs and MKT.) These core dimensions are envisaged here as externalized through the actions put into effect by the teacher at all stages of the lesson. This section summarises our position with respect to each dimension.

In terms of the mathematics teacher's mathematical knowledge, we opted, from amongst the various approaches that have emerged in recent years, for that of Mathematical Knowledge for Teaching (MKT) (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005) and its various sub-domains. The selection of this conceptualisation over others derived from the nature of our aim, which was to identify, from observed practice, what knowledge the teacher was deploying at each specific moment, and consequently the system for making this identification played a key role. Also advantageous was the fact that MKT embraces a focus of knowledge in action, and is hence highly compatible with the way in which we conceptualise the teacher's beliefs (see below).

This knowledge influences, and is inevitably influenced by, the objectives pursued by the teacher. Objectives, then, cannot be considered in isolation, but rather as part of a system whose various components (in the short, medium and long term) interlock, one hopes harmoniously, so that any set of actions performed for a particular purpose is coherent and effective. The teacher's objectives and MKT are available to research through their externalisation in the teacher's actions in the classroom. This is true, too, of the teacher's beliefs about the teaching-learning process (Crespo, 2003). Our approach follows the work of Calderhead (1996) who considers that the teacher's mathematical beliefs (and, at least for us, necessarily his or her MKT and objectives) have a direct impact on the way they go about organising their lessons, such as how they interact with the students and the way they respond to new educational policies (whether administrative or pedagogical). We regard beliefs as systemic, forming a part of a larger system which gives form to the professional knowledge of the teacher. Teachers' core beliefs are shaped by experience and in order for change to happen it is essential that teachers reflect on these experiences (Pajares, 1992). We are particularly interested in the beliefs that are brought into play and revealed in practice. To this effect, we follow the lead taken by Climent (2005), who deploys a set of indicators of the teachers' beliefs in respect of methodology (classroom practice, lesson activities, sources of information, individual differentiation, use of manipulatives, objectives of the teaching process and programming), school mathematics (orientation, content, how it is regarded and its purpose), learning (how it takes place and in what way, what processes are used, what is the role/importance of student discussion, teacher/student/materials interaction, types of student grouping), the role of the pupils (participation in the planning stage, responsibility for learning – transfer key T-L, what is done, how it is done and why it is done) and the role of the teacher (what he or she does/how he or she does it/methodology or pedagogical attitude/how he or she behaves in relation to validating information). We consider them as manifestations of beliefs in that they are grounded in observable classroom performance, that is, specific actions dealing with the students' knowledge building.

By focusing on manifestations of belief, our intention is not to catalogue inconsistencies, but to understand the causes of any apparent lack of congruity. Teaching is a complex phenomenon, and we concur with Leatham (2006), that beliefs systems are essentially sensible, and apparent incongruence may be due to a lack of understanding of the complexity of the phenomenon on the part of the researcher. For this reason our approach to analysing practice drew on Climent's (2005) instrument; as Schoenfeld (1998b, 2000) remarks, the teacher's set of beliefs largely defines their view of the teaching process and frames their classroom performance. This kind of influence will be reflected in the day to day running of the class, as beliefs underpin what the teacher considers plausible, possible or even desirable for their students.

Wilkins (2008), when discussing his model states that beliefs have a positive effect on teachers' actions. In this research we take a broader view, considering that actions and beliefs (and MKT) influence each other, that is, the teacher's actions are influenced by their beliefs (and MKT), and beliefs (and MKT) are ultimately influenced by (and influence) the type of action put into effect.

CONTEXT AND METHOD

This paper forms part of a broad-based study of professional development which focuses in particular on aspects of professional knowledge (beliefs, goals, MKT, and communication promoted), and explores how these interact, their impact on practice, and how they change over time. The larger study concerns two primary teachers who took part in a collaborative group which met for part of the year, and who had selected various topics for consideration specifically because they felt weak in these areas.

It takes an instrumental case study approach combined with a qualitative methodology. Data consists of audio and video recordings of lessons, with the focus on the teacher and informal conversations conducted before and after each class (the collaborative discussion sessions were also audio recorded but were not yet analysed). Data was gathered in three different phases during the school year, always at points in which teachers were to introduce a new topic. The second phase consisted of the execution of lessons which had been discussed and planned in the collaborative group.

The audio recordings were transcribed and complemented with video viewings, which enabled a fuller record of the teacher-student interactions to be made (Brophy, 2004). The transcriptions were then segmented into episodes, whereby each episode was identified with a unique and specific (mathematical) goal (Ribeiro et al., 2009). All the individual episodes were analysed for indications of implicit beliefs, the presence of partial or complete items within the sub-domains of MKT, and the identification of the teacher's actions.

Identification of the indicators of beliefs corresponding to each specific goal was effected with the analytical instrument developed by Climent (2005). In order to ensure internal consistency, this was complemented by an analysis across the teacher's whole practice, matching indicators to their corresponding actions. This analysis identified the indicators common to all situations pertaining to the same cluster of goals (in the case under scrutiny here, the main goal was to review an item of mathematics through worksheet-based individual work and whole class feedback). To reinforce the triangulation further, it was decided to limit the final list of indicators of each type of episode to those that were common to all the episodes of that type. With respect to the teacher's MKT deployed in each situation (that is, an episode pursuing a specific mathematical goal), identification was achieved through an exhaustive analysis of all the episodes concerning the particular area of mathematics under study. Once the sub-domains of MKT had been assigned to specific situations and specific content, we took a closer look at one of the episodes (reviewing the concept of one tenth) to consider the network of relations.

Through this kind of analysis and discussion (in collaboration with the teachers) we aim to lay the foundations for a database of "real classroom situations" providing examples of the way beliefs, knowledge and actions interact, and their impact on practice and ultimately students learning. This we hope will contribute to an improvement in teacher training and will seize the opportunity presented by the current Portuguese panorama.

EMERGENT RELATIONSHIPS BETWEEN BELIEFS, ACTIONS AND MKT

One of the most frequently occurring goals of Maria's lessons is the topic review. The actual form these reviews take depends on the way students work together and the kind of resources employed. Each type of episode is thus associated with a specific set of actions and beliefs, although they have a common core. To illustrate our analysis we focus on the lessons which took place during the first phase of the study (in which the goal was essentially to gather baseline data). The sample presented here concerns the review of the concept of one tenth by means of guided individual work (via a worksheet) with whole class "feedback", and is intended to illustrate the relationships between the actions she employs and certain aspects of her beliefs and MKT.

Line		Transcription
356		So, staying in your places, you are going to represent a tenth, please,
357		drawing it however you want.
358	St	Miss, can we (inaudible)?
359	T	Yes, if you want..
360	St	In pencil or pen?

T It doesn't matter
So next, what you are all going to do is…
(T goes to the board and indicates the different representations of a tenth)
you are going to represent this…
The unit was divided into how many parts?
Sts Into ten.
T And each of these is…?
Sts A tenth.
T OK, so now...
St Miss, can I write in pen?
T That doesn't matter.
So now I'll represent the unit divided into ten which will give us a tenth.
(Students get down to solving a problem set by the teacher while she circulates around the classroom and corrects students' work individually)
T Now, if you've done that, let's carry on...
Don't forget to write down that the unit was divided into … how many parts?
Sts Ten.
T Ten, which is equivalent to one tenth.

Figure 1: Extract from transcription of an episode, the goal of which is to review the concept of one tenth

Each of the clusters is associated with a set of core actions that together express a particular set of beliefs and reveal certain aspects of her MKT (Ribeiro & Carrillo, 2011b). To those core actions can be added others that embody the specific type of review carried out and are associated with the teaching strategies considered appropriate to the topic (resources, pupil configurations, correction strategies). This cluster incorporates a set of six separate actions. The parentheses (...) indicate periods of talk with pupils.

[T explains worksheet] - [T distributes worksheet] - [T (...) making reference to worksheet, reviews subject matter] - [T (...) making reference to worksheet, clarifies subject matter.] - [T (...), making individualised corrections] - [P (...), making whole class corrections]

The actions of reviewing and clarifying take on a core role within the revision clusters, and are identifiable from the use of the worksheet and the type of correction employed (individual and whole class). Associated with this set of actions are indicators of beliefs that shape, and are shaped by, practice. (Remember that these indicators of beliefs are indicative of each cluster and are hence not the only ones occurring in each specific situation.) The core actions (reviewing and clarifying) are associated with beliefs about learning and the role of the teacher. They indicate the assumption on the part of Maria that the teacher is responsible for validating the information circulating in the classroom, as a result of which the majority of interaction is mediated by her.

These indicators of beliefs and the actions they generate are intrinsically related to the sub-domains of MKT revealed by the teacher. Some of these may be detected at certain phases of the episode (via specific lines in the transcription), while others emerge in the course of the episode as a whole (Ribeiro et al., 2009).

The table below summarizes the relationships between Maria's actions, beliefs (expressed as indicators of beliefs) and sub-domains of MKT. These relationships reflect a particular way of approaching and conceiving of the teaching process.

Actions	Indicators of beliefs	Sub-domains MKT	
T explains worksheet	(**Methodology – curriculum**) – Sequential, structured, closed. (Learning – achieved by) – Incremental memorisation.	**KCT** – considers use of worksheet requiring Stds to write fraction represented by shaded areas of various shapes appropriate to content review	**CCK** – knowledge that each of ten equal parts into which a unit is divided unit is one tenth, and that one tenth is equivalent to ten hundredths **KCT** the activity whereby Stds match numeric and pictorial representations should be done individually; T considers worksheet appropriate for revision of concept of a tenth.
P distributes worksheet	(**Methodology – manipulatives**) – not used. (**Teacher's role –pedagogical philosophy**) – transmission via demonstration, using organised strategies that seek to be engaging. T acts as a 'content planning technician'		
T (…) reviews subject matter using worksheet	(**Teacher's role – validating information**) – teacher provides validation of the information.	**KCT** – it is important that Stds register how they overcame difficulties re. concept of a tenth, and that concomitant learning is highlighted	
T (…) clarifies subject matter using worksheet	(**Learning – interaction: T/Std/material**) – Stronger flow toward teacher › student that the inverse		
T (…) provides individual correction	(**Stds role – what they do**) – Reproduce and imitate		
T (…) provides whole-class correction	(**Learning – significance of std discussion**) – Demonstrates assimilation of contents.		

Table 1: Indicators of beliefs associated with Maria's actions in an episode from the cluster reviewing content via a worksheet

This system of illustrating the relationship between cognitions and actions brings to the fore the fact that actions and/or beliefs (indicators of beliefs) are activated mutually without one necessarily taking precedence over others. Hence, applying the conceptualization of MKT presented by Ball et al. (2008) to the case in point – reviewing the concept of a tenth – the teacher draws on different sub-domains of MKT, albeit with a predominance of Pedagogical Content Knowledge. These two components of cognition (MKT and the objective pursued) are realised through the teacher's actions, which are also guided by her beliefs. There is then a dialectic between these dimensions of professional knowledge, and they can be considered as inseparable (though distinct) items within a single set.

SOME FINAL NOTES

The kind of focus and analysis of classroom practice presented here, focusing on the teacher and what she reveals at every turn, we believe enables us to arrive at a broader understanding of what is going on in this sample of practice, and the factors involved. This data is intended to contribute to reflection upon specific aspects of practice that can be improved. Nevertheless, it is important to stress that, as Delaney, Ball, Hill, Schilling and Zopf (2008) caution, we do not make the assumption that the MKT revealed by Maria and any identifiable areas for improvement can be extrapolated to other contexts. Such data are applicable only to each individual situation – indeed, this was one of the reasons for identifying the MKT sub-domains across all episodes concerning the particular area of mathematics under study. The data on MKT allows us to focus attention on mathematically critical hypothetical situations for teachers (practising or trainee) (Ribeiro & Carrillo, 2011c), the role of teacher's beliefs in professional development and the impact of the relations between these dimensions with respect to practice. The nature of the relations between these dimensions could determine the type of opportunities for learning provided in the classroom (Hiebert & Grouws, 2007).

These dimensions and relations, are not in themselves, generalisable, even for this particular teacher; they are specific to the situation under analysis. Nevertheless, we can reflect on the role of the teacher's knowledge and beliefs in their practice and the prospect of greater awareness (and questioning by the teacher) of what they do, how they do it and why. Ongoing reflection of this kind will lead, it is to be hoped, to a questioning of one's practice and training background, as it is of paramount importance to engage teachers in a discussion about the role and impact of each of these dimensions on their own practice. It is also our desire that such a discussion contributes to the focus of training courses being directed to areas where there is a genuine need, and to the promotion of the kind of practice which is "mathematically demanding" as well as "pedagogically exciting" (Ribeiro & Carrillo, 2011a), as opposed to focusing on the teacher and on revealing KCT, and relegating student learning and mathematical considerations – supposedly central to the process of teaching and learning – to a secondary role.

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LATENCY AND MATHEMATICS: PARTICIPATION AND STATE OF MIND

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This paper discusses how a 'latency' state of mind might serve to attract a young person towards studying mathematics at university. The psychoanalytic notion of latency, understood here as characterising a typical affect-state of a primary school aged child, is explained following Kleinian object-relations theory, particularly as interpreted by Margot Waddell (1998). Through analysis of narrative style interviews, a case is made that latency attitudes may position a young person favourably for mathematical participation at times of choices and decision-making.

INTRODUCTION

It's all very well to say 'pay attention!' to a person, but the reason that each and every one of us turns to attend has an emotive driver (Hume 1739, Damasio 2003), that forms and is informed by the relationship between the person and the intended object of attention. In the case of mathematics, the emotion of panic, stimulated in many instances by 'pay attention!', is well-known as a driver away from mathematical participation (Buxton 1982) and the person's relationship with mathematics is experienced as painful or numbed. What are emotion drivers towards mathematical participation?

This paper is on attraction to mathematics to the extent that young people, aged around 17-20, opt to continue its study at university. Data that have stimulated this enquiry come from a project concerned with participation previously reported (Rodd 2011). In that paper, the concern was with how mathematics serves as a defence (following Nimier 1993). It was argued that having a strong mathematical identity was neither necessary or sufficient for choosing to study mathematics at university. This paper is an application of another aspect of psychoanalytic theory in the domain of understanding participation in mathematics (construed as a mental as well as social practice). Here the focus is on the psychodynamic concept of 'state of mind', in particular the 'latency' state of mind characterised by the post-infant, pre-pubescent child, and how understanding this notion can help give insight into people's attraction (or otherwise) to mathematics. Before the claim of this paper can be expressed, an introduction to states of mind as a psychoanalytical notion needs to be given; hence the outline of the paper is as follows: brief contextualisation; introduction to 'states of mind' following Waddell after M. Klein; statement of the claim together with stories of attraction to mathematics that illustrate the claim using data from our project; discussion of the psychological benefits (or otherwise) of mathematics relative to states of mind and implications about attraction to studying mathematics at moments of ‘choice’, which, for academically high achieving students throughout the UK, happen at age 15/16 and again when the young person applies to university.

Project context

Understanding Participation rates in post-16 Mathematics and Physics (‘UPMAP’) is a three year project and the research focus of this report is on the analysis of 50 interviews with undergraduates who narrated their personal retrospectives on how they made their decisions about their course of study. The central question the UPMAP project aims to address is ‘why do young people participate in STEM (*i.e.* Science, Technology, Engineering or Mathematics courses of study) or not?’.

METHODOLOGY

The methodology for the project overall and for the strand that was concerned with undergraduates’ retrospectives has been reported elsewhere (Reiss *et al.* 2011, Rodd *op. cit.*). For the analysis reported here, an initial reading of a subset of interviews with a defended subject lens (Hollway and Jefferson 2000) drew attention to a pattern of defensivity that was associated with the latency state of mind (Silverman 2010). Given that result, interviews with mathematics undergraduates and a selection of others were re-read with a view to locating evidence for particular states of mind.

In this report, like others in this genre, the meaning of the claim is expressed through report of analysis of cases. The specific case of focus here is that of Sophie who is reading mathematics as a solo subject at university A (a large, well-regarded UK university). Why did Sophie choose to read mathematics? Because the subject suited her in ways to be communicated below.

LITERATURE – BACKGROUND ON ‘STATES OF MIND’

Educational discourses typically consider ‘development’ as significantly correlated with chronology. For example, in cognitive discourses, the notion of ‘readiness’, developed from Piaget’s theories, is familiar and in socio-cultural discourses, often inspired by Vygotsky, the notions of induction into practices and of ways of speaking about these practices suggest a direction of flow at least – from naïve to initiate. By contrast, psychoanalytic discourse does not provide the security of a temporal narrative despite labelling some ‘states of mind’ with terms – like ‘infancy’, ‘latency’, or ‘adolescence’ – that connote developmental stages. ‘States of mind’ is a rather ephemeral term that eludes easy definition (Waddell 1998, p5). The term aims to capture an as-experienced mental attitude, as Waddell explains:

> There is a constant interplay, for example, between the states of mind which generally characterise each developmental phase. … Mental attitudes which appropriately belong to different stages of development, infancy, latency, adolescence, adulthood, will each, at any one moment, come under the sway of emotional forces which are characteristic of one position or the other, irrespective of the subject’s actual years. An adult’s state of mind may be found in the baby; an infant’s in the adolescent; a young child’s in the old man; a middle-aged man’s in the latency boy. These various mental states will take effect

> in relation to whichever emotional attitude to the self and to the self-in-world has precedence at the time. (*ibid*., p8)

For instance, terror (*e.g.*, of feeling abandoned) is associated with infancy, being “playful yet diligent” (*ibid.,* p74) is associated with latency, and preoccupation with sexual identity is associated with adolescence, although ‘state of mind’ should not be connected immediately with the feelings (such as terror and playfulness) themselves. One of the most notable things about ‘state of mind’, and what makes it so difficult to characterise in terms of a clinical description, is that exists somewhere outside a person’s momentary conscious awareness.

While the concept of ‘state of mind’ is related to that of ‘defence mechanism’, for instance, there are defence mechanisms typical of a given state of mind, it affords another way of addressing affect and participation. The focus here is on the state of mind that is characterised by a period coincident with primary schooling. This is between “the turbulent passions of the Oedipus complex beginning to subside and the time when those passions are stirred again with the onset of puberty” (*ibid*., p73).

Latency

> But now I am six, I am as clever as clever, so I think I’ll be six, now and forever. Milne, 1927/1989: 102

While latency is associated with period of primary education (approximately ages five to eleven), the latency state of mind can be experienced at all ages. Having a thirst for knowledge, being able to organise and focus on detail, developing awareness of self as distinct from others are positive descriptors associated with this state of mind, while obsessiveness and being socially anxious are less positive; the competitive nature also typical of this period might be considered positive or negative depending on its extent and the culture in which the child is being raised.

It is a state of mind when bodily passions are relatively dormant – latent, as the label suggests – and the psyche does not have to wrestle with urgent sensual impulses as in infancy or adolescence. Within this period an internal identity develops that is needed for social interaction in the world beyond the family. Waddell asserts that there can be a ‘very distinctive sense of pleasure and achievement … [for the latency girl or boy] who relishes the increasing ability to manage his world and whose activities arouse interest and encouragement from the adults involved’ (*op. cit.* p74). The school world that is managed thus by the young person includes the sort of tasks that involve ‘knowledge that’ and ‘skills for’. A mathematical education can satisfy this type of desire for ordered and controllable knowledge.

The psycho-social importance of this stage of development was recognised by Sigmund Freud who emphasised its “significance for the later normalcy of the individual subject and his insertion into the culture” (Cengage 2005, p1) due to the process of developing the social emotions that align with the society while the sensual aspects of the psyche are sublimated (*i.e.*, latent).

Defence mechanisms, which contribute to development of an individual's mind and character, are present throughout all personality stages to protect against anxiety and are essential aspects of psychological functioning. Those defence mechanisms typical of latency are organising (labelling, collecting, structuring), shyness (watching or following), being precocious (knowing differences between brontosaurus and brachiosaurus, times tables or spellings), being competitive (sports, music, exams).

Adolescence

It is worth briefly reviewing the psychoanalytic notion of adolescence, characterised as it is by the contrast with latency with regards to overt sexuality and the work to be done of establishing a stable but flexible person in the world. As Waddell remarks:

> The anxiety involved in a young person's attempt to discover who he is, or who she is, and to define more clearly their sense-of-themselves-in-the-world often arouses extremes of defensive splitting and projection. (*op. cit.,* p157)

The defences 'splitting' and 'projection', respectively, separate 'bad' from 'good' (splitting) and place the 'bad' away from the self (projecting). An example of splitting relevant to this discussion is that a young person might split mathematics into rightness and wrongness (rather than addressing confusions or acknowledging errors) and s/he might project the experience of mathematical wrongness into another, for instance his/her being 'wrong' was the 'fault' of teacher, parent or peer. And while there is no firm distinction between latency and adolescent experiences of such splittings and projection, latency suggests a deferral of an emotionally-charged self that is being put on the line, (so rightness and wrongness are internalised as being of the culture, being expressed as what teacher says), whereas adolescence suggests identity work with a sexual positioning (and rightness and wrongness is experienced as personal worthwhileness or otherwise and an indicator of sexual prowess or its lack). Whilst projection peaks during adolescence, splitting 'good' from 'bad' occurs from birth and is experienced throughout life. Coren (1997) observes markers of adolescence, compared with latency, include affective changes such as mood swings, interruptions to concentration and a less biddable attitude to authority (*ibid.*:7).

As the typical beginning undergraduate is between the ages of 17 and 20 years old, there are times (e.g., during elections) when they will be positioned as an adult but at others their dependency (e.g., financially) on their family retains the mark of childhood. This ambivalence provides opportunities for them to pursue a variety of courses including that which a latency frame is ideal to support.

SUITS YOU

Mathematics offers opportunities to display positive aspects of latency like cleverness and orderliness, as well as competitiveness and willingness to be lead (as by a teacher). Young people who are able to tap into this dependent but self-defining state are able to exploit their relationship with mathematics to positively enhance themselves. Analysis of our narratives from undergraduates who have chosen to study mathematics at university found latency mentalities communicated in various ways but clustered around two themes that are indicative of a latency state of mind:

A. Desired precision, perceived objectivity (lack of subjectivity) of the subject (mathematics);

B. Significance of relationships with parental figures with 'goodness' expressed in achievement.

Illustrations from data

The UPMAP database has interviews with 12 undergraduates from four UK universities who are reading mathematics as a major subject. Each of these interviewees expressed views about their relationship with mathematics that, on analysis of these interviews, suggested that understanding their choices through an understanding of what was satisfied in latency seemed worth pursuing. Here is a representative selection of types of utterances that alerted the analysers of the need to delve into this psychoanalytic categorisation of mental/emotional state.

Chloe, reading mathematics with French, eschews the subjectivity she perceives in the study of English and still refers to her mother as role-model:

> I like the logic that goes with maths, where you can see there's a proof, you're working its way through and you get to English and its all subjective and 'what does somebody mean', 'I don't know'.
>
> My mum's a French teacher but I don't think that that really affects why I am doing French erm … mum definitely likes maths.

Lee, reading mathematics with statistics and finance, relates how he found mathematics with the help of his father which he perceives as quite different from English in particular poetry:

> Maths, I've always been good at it. I remember back in year 1 or something like that, I was quite stupid, not stupid, but somehow all the other kids seemed to shine. Then my Dad made me learn my times tables and then after I did, suddenly maths, anything about maths seemed to make sense, anything, and then that's how I got better and better. ... So, yeah and I'm into, my brain seems to work with logic. Logic and common sense, that's why I think I'm more suited to maths because, English I'm not so great at, stuff like poetry I'm not that good because it doesn't make sense sometimes. But that's why I've always been good at maths.

Vira, reading mathematics with computer science, expresses how she perceives the power and precision of mathematics and that her parents are mathematics teachers:

> Yes definitely erm I love the way how it's all got formulas and I can use formulas to prove a million different things even though it will take a hundred formulas to prove one thing I love using all these, I love proving things. I love erm I love having an answer in the end basically and the achievement at the end of getting an answer... with English it's your opinion and although I do love like having my own opinion its doesn't give me a definite answer in the end.
>
> I think you're just grown up with, when your parents are teachers in maths you just grown up wanting to be your parents, all children do.

A CASE STUDY

Although in many of the 50 or so interviews with undergraduates from a range of majors, there was evidence of a latency state of mind, mathematics undergraduates all exhibited it in one way or another and it seemed to be positive with respect to their experiencing their career to date. A closer analysis of the interview with Sophie, who is reading mathematics, will show how a latency mentality afforded her alignment with mathematics, even though she had the qualifications to choose a humanities subject or a vocational subject such as law. Sophie went to a co-educational maintained school in the South of England and studied mathematics, further mathematics, English, French and history – a higher than usual academic load - between the ages of 16 and 18 (A level). She has two older sisters (aged 22 and 23 at the time of her interview when she was 18). Her mother was a deputy head teacher in a primary school until she had children and now works in education support; her father is a retired accountant/businessman.

A selection of extracts from Sophie's interview transcript (~10%), made on the basis of: (a) utterances that exemplify attraction to mathematics or mathematical practices; (b) utterances that exemplify important people, follows. She also talked about her immediate family, hobbies, enrichments and test results. (Cuts are indicated by '…'.)

> 1. S – I think it[maths]'s always been quite important because, I dunno, when I was younger, my sisters, I keep going into it! My sisters would like obviously be doing work and stuff and I'm quite competitive with my sisters and so I think I always knew what their strong points were and tried to be good at that too. But I just preferred it to the other ones and I had a really nice maths teacher in year 7. Who like, if I like finished early, she'd give me extra work and stuff which obviously makes you feel quite clever. She was really nice and kind of encouraged me to try hard. … I know I was obviously quite clever so I wanted to be cleverer … I'm really competitive!
>
> 2. S – And my maths teacher in secondary school was amazing. He was the Head Teacher. I think he had only one class … he took our further maths class. We had two teachers, but like, I dunno he was like really inspirational to me, I really, really liked him. … I don't think I would have got an A in further maths without him. He was so good.
>
> 3. S – In year 8 I think we had a few teachers for maths and then in year 9 we had a teacher which not many people considered to be very good. He didn't really have much class control, so it kind of ended up just, if you wanted to, you could do the work, but you wouldn't need to. If you were talking he'd just kind of say, get on with your work, but you didn't have to you could just kind of … so obviously there was a table with me and three other girls and two boys and we were all quite good at maths so we'd sit there and do the maths together but without that whole kind of teaching and everyone focussing thing, I think we all kind of didn't do as well as we could have.
>
> 4. *Int –… a little bit more about you "always knew you were going to carry maths on"?*
> S – I just like it, I mean, I've always been quite good at it [S: embarrassed laugh]. That makes it like something more, if you can do it and you get marks. But it's also nice when you get a challenge, because I like the feeling of when you get that challenge and you've accomplished it and it feels better. But, people normally say because I did English and

History and French they're more like 'writey', whereas maths is more logical. So, I think I kind of like both sides of that but stuck with maths because I like … Actually there's two sides of it. I like it because you don't have to fumble on or anything like, it's down to the point and logical and you get the right answer, it's either right or wrong. But then, that's also bad because if you don't know how to do it, it's either right or wrong and there's nothing you can do if you don't know how to do it, you're stuck!

5. Int – with maths, what do you get?
S – Logical arguments. I do really like being stuck with an argument or like, I was doing revision for my exam yesterday and it's like, you can see which areas you're having trouble with and you'll sit there for ages and ages trying to work it out. Then when you do, you just feel like 'YES, done it!'. And, once you know it, you know it. Obviously they'll take it further in the future, but once you've sussed out how to do it, that's that, you can move onto the next one. That's how it is for me at the moment anyway. I'm sure it will get a lot more confusing later on!

6. S – My Dad was a director of an Insurance company. He's retired now, he was also really good at maths. He still does loads of like Sudoku and stuff! When we were younger he'd help us with our homework and I quite wanted to go into accounting or something as well so obviously I'd need maths. I think I wanted to be a businesswoman like him. Which for me, when I was younger and I didn't really understand what he was doing I just knew it was something to do with maths.

DISCUSSION

Using the psychoanalytic lens outlined above in the literature section, analysis both of Sophie's utterances that exemplify attraction to mathematics or mathematical practices and her utterances that exemplify important people indicate her latency attitude – at least towards her choice of subject to study at university.

Attention to evidence of

A. Desired precision, perceived objectivity (lack of subjectivity) of the subject (mathematics);
B. Significance of relationships with parental figures with 'goodness' expressed in achievement.

Is pointed out in the extracts above; In extract 1. Sophie expresses a playful competitiveness and desire to emulate her sisters (A). Her cleverness is reinforced by the feeding from teacher at the beginning of secondary school (B) which gives her self-esteem and aligns with her sisters. In extracts 2. and 6. Sophie reveals her deep attachment to her father and a father figure (B). In extracts 4. and 5. Sophie cites the 'logical argument' and the 'right or wrong' aspects of mathematics as her understanding of the nature of mathematics (A).

In extract 3., when the world of the classroom is not ideal, Sophie tells of her smaller circle of like-minded children doing "the maths together". This ability to draw into the small group for identity reinforcement as well as familiar routines is characteristic of a latency child. It is also significant that she blames the outside for them (not just her) not doing “as well as we could have”.

Sophie’s attraction to mathematics was not predicated on the sorts of ‘pedagogic events’ that have been recently linked to identity formation in classrooms (Black et al. 2009). Nor did she refer to the internet, digital technology generally or expressions of youth culture; peers were mentioned in relatively few instances and then not with regards to decision-making. Instead, positive references to mathematics were delineated by reminiscences involving parents, siblings and other adult figures who figure pre-eminently in a young child’s growing up (Waddell, *op. cit.*: 81-123).

It is interesting to compare Sophie’s utterances with those of Becky as Becky had a top grade in A level mathematics (university entrance) but opted to study English:

> Becky – Maths was the easiest of A-Levels for me by far the easiest, but I think that’s cos it’s not subjective. There’s a right answer and a way of doing things and once you know how to do it, you can do it, whereas English and history you might think you know but you don’t, you might not know. I liked maths because I was good at it, and I found it challenging but I didn’t find it interesting. But English, I just found more interesting and I’d rather do something that I find interesting and challenging than something I find easy and less interesting. … one of the modules I’m doing is like a background to the whole Western culture and way of thinking, it looks at things like Plato, Marx, Nietzsche and I think I’ll get more personal understanding and it will make me a better, more knowledgeable person than knowing a lot about maths and complicated sums and things.

In this brief exposition from Becky an illustration of an alternative state of mind can be glimpsed: her ideals are not sisters or father figures but abstractions of “Western culture”. We are not saying that Becky is further on in her development than Sophie; rather Becky, at the time of her interview, was not taking in that which mathematics offers in the same way that Sophie was.

Conclusion

This paper has addressed the question ‘What are emotion drivers towards mathematical participation?’ by considering states of mind in which emotions are experienced in characteristic ways. We claim that attraction to pursuing mathematics beyond school into adult life (that the university transition for young adults signifies) is served well by ‘playful yet diligent’, competitive yet biddable, focussed and orderly traits which have a latency flavour. Of course, other states of mind are experienced too, for example, Sophie’s reflection (extract 5) that the mathematics she will be studying will become more complicated in future indicates a more adult state. Nevertheless, that these mathematics undergraduates can or do adopt a latency mentality seems to serve them well with respect to their opting to participate in mathematics.

It is not an intention that <opting to participate in mathematics at university is aligned with a latency attitude> should be read as deficit! Far from it. We are interested in the subtle influences on academically high attaining young people as they choose their course. And there will be mathematics undergraduates that do not align with latency traits; also the situation will vary in different countries – both for structural and cultural reasons. However, an ordered, delighted association with mathematics can serve as a positive defence to the psyche (Nimier, 1993), and by following on with the mathematics that a parental figure personified and which was performed beautifully, opting for mathematics may just provide a way to be “clever as clever ... forever and ever”.

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BELIEFS VS. PATTERNS OF PARTICIPATION – TOWARDS COHERENCE IN UNDERSTANDING THE ROLE THE TEACHER

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The three main lines of research on teachers are the ones concerned with beliefs, knowledge and identity. The two first are generally based on acquisitionism, the last is more participationist. This leads to some incoherence. I introduce an ongoing study that aims to develop a coherent approach, using a patterns-of-participation framework, which was initially developed in relation to beliefs. It is a multiple case study that views instruction as the simultaneous engagement in multiple, possibly conflicting social practices. The data presented are about Anna, a novice, lower secondary teacher. The interpretations of the practices that unfold in her classroom suggest that there is some potential in the patterns-of participation perspective.

INTRODUCTION

The main contribution of teacher-related belief research has been to point to the limitations of the cognitive and purely mathematical emphases that used to dominate mathematics education. Since the 1980s it has insisted on the role of meta-issues such as teachers' conceptions of mathematics and its teaching and learning for classroom practice (Leder *et al.*, 2002; Lloyd, 2005; Maasz & Schlöglmann, 2009; Schoenfeld, 1992; Skott, 2001; Speer, 2005; Thompson, 1984). In this sense it supplements research on teachers' mathematical knowledge, which is primarily concerned with the specifics of teachers' content preparation (Ball *et al.*, 2008; Davis & Simmt, 2006; Ma, 1999), and the more recent interest in teachers' professional identities (Brown & McNamara, 2005; Hogden & Askew, 2007; Sfard & Prusak, 2005).

There are, then, three main lines of research that focus on mathematics teachers, i.e. the ones related to beliefs, knowledge and identity. Besides these lines of inquiry to some extent share a methodical interest in qualitative studies of mathematics classrooms, as it is increasingly agreed that understandings of (i) teachers' beliefs of mathematics and its teaching and learning, (ii) the relevant aspects of their mathematical proficiency, and (iii) their tales of themselves as professionals must, at least in part, be based on interpretations of their contributions to classroom practice. Besides, studies of teachers' knowledge and beliefs share an inspiration from acquisitionist views of learning and knowing. Knowledge and beliefs, then, are generally understood as relatively stable objectifications of engagement in (social) practices (cf. Sfard, 2008). These object-like entities are viewed as the properties of the individual in question and expected to take on a life of their own and have significant impact on practice (Skott, 2010).

In spite of these similarities there is a somewhat surprising disconnect between research on teachers' beliefs, knowledge, and identity that is counterproductive to the development of coherent understandings of the teachers' role for classroom practice and for student learning. First, research on identity tends to adopt a participationist stance that is at least partly in contrast to research conducted in the other two fields. It maintains a processual emphasis, for instance in line with Holland et al. (1998), Wenger (1998), or Sfard & Prusak (2005). Identity, then, is generally viewed as fluid and always in the making, as tales of being and becoming as they relate to simultaneous engagement in multiple, social practices. In this sense it is used as a less contextually and temporally stable construct than knowledge and beliefs. Second, there is little connection between current understandings of mathematical knowledge for teaching and the role of teachers' beliefs in instruction in spite of the acquisitionist and somewhat individualistic underpinnings of both lines of research.

Research on mathematics teachers, then, is conducted in three relatively distinct domains. This paper presents an ongoing study that intends to develop more coherent understandings of the teacher's role for learning and life in mathematics classrooms. It does so by using a patterns-of-participation framework that was initially developed in an attempt to overcome the notorious conceptual and methodological problems in belief research (cf. Skott, 2009), while maintaining an interest in the meta-issues that constitute the field of beliefs. The patterns-of-participation framework challenges mainstream belief research by questioning the very notion of beliefs and its acquistionist underpinnings (Skott, 2010). It has subsequently been used for empirical purposes as an alternative to regarding classroom practices as a result of an enactment of objectified mental constructs, i.e. beliefs, on the part of the teacher (Skott *et al.*, 2011). In line with other more social approaches to research in mathematics education, this alternative insists on the emergent and processual character of classroom practices. It relies, then, to a greater extent than mainstream belief research on participationism as a metaphor for human functioning.

The aim of the present paper is to indicate the possible potentials of the patterns-of-participation framework for developing a more coherent approach to research on and with mathematics teachers. The idea, then, is to extend the use of the framework from meta-issues ('beliefs') to what is traditionally referred as knowledge and identity. I shall begin by outlining the patterns-of-participation framework as developed in relation and opposition to mainstream belief research. This forms the backdrop of a subsequent analysis of a short classroom episode from an on-going study of Anna, a young teacher at the lower secondary level. The intention is to analyse Anna's contributions to the interaction as her simultaneous engagement in a range of present and prior social practices, some of which are meta-mathematical, while others are mathematical, and still others relate to her broader tales of herself as a novice teacher.

PATTERNS-OF-PARTICIPATION

Students and teachers engage in multiple simultaneous practices in the classroom, some of which relate to the teaching and learning of mathematics and some of which do not. Some of them are discursive in an explicit verbal sense, while others are not;

and some of them are virtual in the sense that they relate to communities that are not physically present in the classroom or at the school (Skott, 2009). For example a teacher may at any instant engage in practices associated with e.g. her colleagues, the school management, the parents, and her pre-service teacher education programme. These activities may function as resources that asymmetrically structure the teacher's contribution to the practices that unfold in the situation (Lave, 1988).

Mathematics itself is a patterned activity on e.g. numbers, variables, and operations. Meta-mathematical patterns indicate what questions to ask, what answers to expect, and what quality is in relation to a solution or procedure. In the classroom the patterns also designate the relative responsibilities of teachers and students.

The teacher negotiates and contributes to the continuous (re-)generation of classroom practices. She becomes involved in actions as diverse as repeating procedural explanations, solving disciplinary problems, ensuring a student's position in the classroom community, and taking a child's problematic home situation into account. In all of this, patterns in the teacher's prior engagement in social practices are enacted and re-enacted, moulded, fused and sometimes changed beyond recognition as they confront, merge with, transform, substitute, subsume, are absorbed by, exist in parallel with, and further develop those that are related to the more immediate social situation.

From this perspective, teaching is not the enactment of pre-reified knowledge and beliefs. It is a meaning-making activity in which the teacher continuously manoeuvres between different forms of participation in different past and present practices. The research task is to outline the character of these practices, to disentangle the patterns in the teacher's participation in them, and to understand if and how they influence the learning opportunities that evolve. From the outset it should be made clear that this framework does not do away with individual meaning-making and returns to a behaviourist approach to professional activity. Instead it reformulates what such meaning-making is, by accepting and focusing on the floating character of human interaction and suggesting that meaning is made up of a continuous interpretation and reengagement in value-laden, prior social practices.

THE STUDY

This study is on teachers' professional development prior to and in the first few years after their graduation. It addresses the question of the relationship between their professional identity and their contributions to mathematics classroom practices, including issues related to what is traditionally described in terms of knowledge and beliefs. The study, however, does not focus on beliefs, knowledge, and identity as distinct, objectified, individual entities with an expected impact on practice. Instead it intends to develop processual interpretations of instructional practice by interpreting classroom interaction as emerging and teachers' acts and meaning-making activities as participation in meta-mathematical, mathematical, and broader social practices.

Methodology and methods

There are two main methodological challenges in the study. First, we need an approach that views (prospective) teachers' identity and involvement in educational activity as transformations in modes of participation in classroom practices in view of broader social processes at the institution in question and beyond. Second, an interpretive stance is needed that views institutional practices as well as shifts in the teachers' engagement in them from the perspective of the teachers in question.

To meet these challenges we use a qualitative approach inspired by grounded theory (GT) (e.g. Charmaz, 2006). We use the methods of GT (e.g. coding schemes, constant comparisons, memo writing) as flexible guidelines for theorising processes of teacher identity and classroom teaching. However, we do so without subscribing to the objectivist connotations sometimes associated with them.

The research participants are selected purposefully. One selection criterion is their commitment to current reform discourse; another is their mathematical and pedagogical self-confidence. If possible these criteria are supplemented with whether the schools have clear educational and/or school mathematical priorities. Between them the criteria are to ensure that the cases are critical (Flyvbjerg, 2006), and may bring to the fore aspects of conflict and congruence between the participants and the dominant practices at their schools. In turn this may allow interpretations and analytic generalisations about teachers' identity and their role for classroom practice.

The study is a multiple case study of app. 15 prospective and practising teachers at levels ranging from elementary to upper secondary school in Denmark and Sweden. The data reported in the following concern Anna, who teaches lower secondary mathematics at Northgate Primary and Lower Secondary School in Denmark.

At present the data on Anna include a questionnaire used to select participants in the study; three interviews, including one using stimulated recall; observations of eight lessons organised as four sessions of 90 minutes and of one team meeting between Anna and her three closest colleagues; analysis of students' work and of Anna's interpretations of it and of a few supplementary teaching-learning materials that she develops herself. The interviews and classroom observations are audio and video recorded, respectively, and transcribed. The data from the team meeting is a text that was written immediately after the meeting on the basis of comprehensive observation notes. This is so, as Anna's colleagues are opposed to recording team meetings, because they discuss confidential issues in relation to named students at the meetings, and they are worried about a possible breach of confidentiality. Also, they are explicit that they would be less relaxed at the meetings, if they were recorded.

Anna was interviewed immediately after the holiday following her graduation and again 9 months later, i.e. towards the end of her second term as a full-time teacher. The interviews were semi-structured (Kvale, 1996) and invited Anna to reflect on good and bad experiences with mathematics and its teaching and learning. For instance she was asked to elaborate on the roles of the teacher in specific episodes when instruction went well and to exemplify the qualities – and lack thereof – of her teacher education programme. The second interview focused on Anna's experiences with mathematics teaching at Northgate and on her relationships with the school, her colleagues and the students. The third interview was conducted after two weeks of classroom observations in Anna's third term of full-time teaching.

THE CASE OF ANNA AT NORTHGATE

Anna is 25 years old when she finishes her teacher education programme at a city college in Denmark. She has studied mathematics as one of four school subjects.

18 months before her graduation Anna gets a part-time position at Northgate Primary and Lower Secondary School. The school is located in a well-to-do area, few miles from the city centre. Anna teaches 6-8 lessons of physics a week in her first year at Northgate and 15-20 lessons, incl. mathematics in grade 8, in the following six months. Upon graduation she accepts the offer of a full-time position at the school.

In the first interview soon after her graduation, Anna considers it important, that she has taught before, and she is confident that she can manage the challenges of the profession better than most of peers. She does not think of her teaching as a site for the application of educational theory from college. Rather, she has used her teaching experience to prioritise and inform her interpretations of the theoretical contents of her pre-service education:

> I have been able to use really a lot of it [the experience], not only in relation to the school subjects [as taught at the college], but also in educational studies. You know, I have been able to take something with me [from the school to the college] every week. (Int. 1).

Anna always wanted to be a teacher, and mathematics was always her favourite subject. This is still the case, and she explicitly considers herself a *mathematics* teacher. She emphasises that a teacher needs to be mathematically creative in order to view the subject from the students' point of view and unobtrusively guide them so that they "develop their own ways of seeing things" (int. 1). However, she is also explicit that "the good mathematics teacher needs to be able to do and know so much more than mathematics" (int. 1). Especially, she emphasises the need for a trusting relationship with the students and that even though she is fond of mathematics, she hopes to become important for them as a role model and a person not only as a teacher of mathematics (int. 1).

Anna describes mathematics as "a multi-facetted subject" (int. 2). She is concerned with connections within the subject, but in particular with the ones between mathematics and the students' everyday lives. She refers to her own education both in lower and upper secondary school to make the point. In the latter she had a teacher "who was really good at jumping out of the book and let us do projects and things" and who "made mathematics part of everyday life", for example by having the students measure the school buildings and compound in trigonometry (int. 1).

Anna distances herself from an overemphasis on basic skills, and calls for a strong process orientation, for instance using the term of "landscapes of investigation" from her mathematics education course at college. Also drawing on that course she emphasises students' mathematical communication, even when the students' find it difficult. It allows the teacher some access to the students' thinking, and more importantly it is vital to the students' understanding and remembering:

> They simply find it *so* difficult to put it into words. […] 'Then you just do like this'. Yes, but why? […] I think that the communication part is so important, 'cause if they don't know […] *why* they can change between percentages and fractions the way they do, if they don't know *why*, I don't think they remember in six months. (Int. 3)

Anna also uses student communication to differentiate her teaching. If she just uses the textbook "some [students] rush right through […]; others get stuck in exercise 1" (int. 1). Students communicate differently, and Anna accepts that their "words may not be the final truth […]. But if there is a meaning in it for them […] I think there is a certain quality in it" (int. 3). This also implies that Anna accepts leaving part of the responsibility for the quality of the students' explanations to themselves. She wants the students to be able to do mathematics "without the teacher checking it" (int. 3).

Anna at Northgate

Anna enjoys Northgate, not least as the teachers are organised in teams, who teach (almost) all subjects to a year group. In the first year after her graduation Anna is in a team of four, who teach grade 9, the last year of lower secondary school. Now, in Anna's second year, the same team teaches grade 7, Anna teaching mathematics in all three classes.

Anna's team meets approximately every three weeks. They discuss issues related to individual students and social problems in the year group. They also plan for instance PTA-meetings, the school's sports day, and changes in the time table. They do not, however, jointly plan instruction or teach together. Anna, for instance, is trusted by her colleagues in the team with all responsibilities related to mathematics in the year group. The other side to this is that in practice she is very much on her own. She once asked an older colleague for suggestions, and she sometimes shares ideas with another novice "in the corridor" (int.2). These two teachers are not in Anna's team.

Anna does not consider the lack of cooperation about instruction a problem: "in mathematics I have been happy to paddle my own canoe" (int. 2). Her self-confidence is backed by good relationships with the students and frequent praise from the parents (int. 3). Also, she has a sense that the practices she wants to promote are not widely shared among the mathematics teachers at Northgate. For instance, it is evident to her that the students never spent time discussing mathematics before she came. For them "words do not belong in mathematics; that is only numbers" (int. 3).

Anna, then, is satisfied with her professional isolation as it relates to her teaching of mathematics. In contrast, she highly appreciates her collaboration with and assistance from her team, especially from her older colleague, Ian. She does not discuss instructional planning or classroom teaching with him, but she tries to "maybe copy a little of what [he] does" as it relates to other aspects of the profession, including how to prepare for meetings with the parents, how to keep your teaching-learning materials organized for the next time you need them, and how to maintain a high level of commitment to the job. Especially, Ian has "this way of being with the students" that Anna admires, and that she finds helpful for her own attempts to build a trusting and confident relationship with them (int. 2).

Anna in 7A

7A is a class of 8 girls and 13 boys. Anna is very fond of the class and vice versa. Some students have social and domestic problems, but not so to such an extent that Anna finds it difficult to cope. According to Anna the students' mathematical performance varies a lot. Some of them can work independently to a great extent, while others are very weak. Catherine, for one, has so many learning problems that "it is really a question of whether she should be here at all" (int. 3).

In the middle of Anna's first term with 7A, the class works with fractions, decimals, and percentages. They have worked on these concepts before, but the emphasis is now on the connections between them and how to transform one into another.

Introducing the topic, Anna asks the students what they did before on fractions, decimals and percentages. They agree to have discussed where the concepts are used in everyday life, and the students mention some examples of that. No other previous work is mentioned. Next Anna asks the students about the proportion of girls in the class, and this is used to exemplify the transitions between the three concepts.

After that the students begin to work on a set of textbook tasks. In some cases they are expected to make conversions between fractions, percentages and decimals themselves, while in others the book presents a conversion. In all cases, it is the task for the students to explain why or why not a certain conversion is correct, using oral and written language as well as drawings. Introducing these tasks Anna says:

> And the most important of it all here is that you say the words, that you also talk [about this] in pairs. The more you talk mathematics, the better you get at […] putting it into words, how you do the calculations.

The students begin, but many find it difficult and have questions to ask. Anna walks around from table to table to help them. Soon after she stays 3½ minutes with Debra and Annika, who are trying to explain why $20\% = 0{,}20 = \frac{1}{5}$. Annika got stuck in her attempt to reduce the fraction of $\frac{20}{100}$. Debra has written "$\frac{20}{100} = \frac{2}{10}$" in her notebook:

1 Anna: [To Annika] Try to look at what Debra has written.

[Annika looks in Debra's note book.]

2 Debra: And then you divide by 2 and then it becomes 1 and 5 [writes; it now says "$\frac{20}{100} = \frac{2}{10} = \frac{1}{5}$"]. That is also what it says here [points in the book].

3 Anna: But how did you go from here to here, Debra [points at the first equals sign]. Try to explain your steps so that Annika can see what you mean.

4 Debra: Really, I am not very good//

5 Anna: yes, you are, come on now//

6 Debra: so I just say that I take away the zero, and then I take half of that [points]. But I don't really know if that is the easiest way.

7 Annika: But that depends on what kind of decimal number you have.

8 Anna: Absolutely. It would be much more difficult, if it was thirty-seven, thirty-seven over one hundred.

[…]

9 Debra: But is it correct?

10 Anna: Completely correct. It is really good. Now you two talk about this. Make a drawing and write some text that explains why what Debra did is right.

[Anna leaves them; many students are waiting for her help].

In the episode Annika has problems reducing a fraction. Anna asks her to look for help from Debra. Debra produces a procedural explanation that works for the specific numbers, but Annika realises that the procedure does not work in general. Anna supports her in this, but they do not follow up on Annika's comment. Anna leaves them, confirming Debra that her procedure is right.

Anna was asked to comment on the episode in the stimulated recall interview:

> What immediately comes to mind, which I am sure it didn't at the instant, is that 'take away the zero' [cf. '6'][…] there is no explanation […] she explains what she does, but not why she does it. You know, […] this is not a mathematical explanation. (Int. 3).

Anna also commented on her own reaction to Debra. She says that Debra did explain *something*, but Anna also noticed Debra's comment that she is not very good at this. It reflects that "Debra generally has the problem that she thinks she is the weakest in mathematics in the whole world, which is not the case at all".

DISCUSSION AND CONCLUSIONS

Anna enjoys taking on the responsibilities that follow from the functioning of her team at Northgate. She does so with confidence in 7A and engages in mathematical discourse, discussing and manipulating the concepts, the symbols and the operations of the subject in whole-class and small group settings. At the same time she re-engages in two distinct educational discourses. One of these relate to mathematics and addresses the issues of a process orientation of school mathematics and the use of the real world problems. This relates to theoretical parts of her mathematics education course at college, and is backed by her own school experiences. The other educational discourse concerns how to develop a trusting relationship with the students. This is connected to other theoretical aspects of her teacher education programme and to her imitation of Ian's ways of being with the students. Between them they suggest allowing the students significant degrees of freedom and responsibility, both in relation to their learning of mathematics and beyond.

In the transcript, Anna insists that the students explain a result given in the book. She also tries to convince Debra that she – Debra – is not so weak in mathematics as she thinks herself. This reflects her engagement in the two educational discourses mentioned above. In contrast, she does not engage in a genuinely mathematical discussion with the two students about the relationships between percentages, decimals and fractions or about the functioning of Debra's procedure, even when Annika questions its applicability. In turn this lack of attention to mathematics per se, leads to missed opportunities for meta-mathematical discussions about what constitutes a good mathematical explanation. In this episode, then, Anna's participation in mathematical discourse is overruled by a merger of her engagement in a more general educational one and her emphasis on reform issues in mathematics, especially student communication. This is noteworthy as Anna's comments in the subsequent interview indicate that the lack of mathematical emphasis is neither due to a general lack of command of the relevant mathematical discourse, nor to a general interpretation of Debra's comment as a satisfactory explanation.

In the terms of patterns-of-participation, Anna is renegotiating what it means to be teacher in her team at Northgate. She takes on the sole responsibility of mathematics in the year group, and doing so she positions herself within the team as well as in relation to the students and the parents. As part of this she participates in emerging classroom practices by re-engaging in theoretical discourses from her teacher education programme, in more immediate forms of social interaction from her own schooling, and in mental negotiations with Ian about ways of relating to the students. In the transcript, these prior practices dominate her contributions to the interaction to the extent that the mathematical potential of the task vanishes.

This is an alternative to interpreting the practices in 7A as a result of Anna's enactment of pre-reified mathematical knowledge and beliefs. In comparison it does away with the acquisitionist connotations of the latter, and it offers a view of classroom processes as indeed processual. It may be developed into a coherent approach to understanding the role of the teacher for emerging classroom practices.

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PROSPECTIVE AND PRACTICING PRESCHOOL TEACHERS' MATHEMATICS KNOWLEDGE AND SELF-EFFICACY: IDENTIFYING TWO AND THREE DIMENSIONAL FIGURES

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Recently, the importance of preschool mathematics education has come to the fore. One of the key domains to be taught in preschool is geometry, specifically, identifying two and three dimensional figures. This paper describes a framework for investigating teachers' knowledge and self-efficacy for teaching mathematics. Prospective and practicing preschool teachers' knowledge and self-efficacy regarding identifying two and three dimensional figures is examined. Results suggest that in general, practicing teachers are more knowledgeable and have a higher self-efficacy than prospective teachers. In addition, self-efficacy was more or less in line with knowledge scores.

INTRODUCTION

Recently, there has been increased interest in preschool mathematics education. For example, in the US, a joint position paper published by the National Association for the Education of Young Children (NAEYC) and the National Council for Teachers of Mathematics (NCTM) stated that "high quality, challenging, and accessible mathematics education for 3- to 6-year-old children is a vital foundation for future mathematics learning" (NAEYC & NCTM, 2002, p.1). In England, the non-statutory Practice Guidance for the Early Years Foundation Stage (2008) suggests ways of fostering children's mathematical knowledge from 0–5 years. In Israel, the preschool mathematics curriculum specifically states that "the preschool teacher has an important role in fostering children's mathematical abilities. It is up to her to devote attention both to planned mathematical activities as well as mathematical activities which may spontaneously arise in the class and to pay attention to the mathematical development of the children" (Israel National Mathematics Preschool Curriculum, 2008, p. 8). Yet, in Israel, as in many countries, attention to mathematics teacher education is mostly given at the elementary and secondary levels (Kaiser, 2002). All too often, preschool teachers receive little or no preparation for teaching mathematics to young children (Ginsburg, Lee, & Boyd, 2008).

In order to provide appropriate preparation and professional development it is important to investigate preschool teachers' mathematics knowledge necessary for teaching (Ruthven & Goodchild, 2002). Yet, research on preschool teachers' mathematics knowledge is limited. This study focuses on prospective and practicing teachers' knowledge of geometry. Geometry is a key domain of knowledge mentioned is several preschool curricula. Yet, research has shown that geometry is only superficially discussed in kindergarten (Clements, 2003). In addition to teachers' knowledge, teacher self-efficacy is also related to classroom practice (Ashton & Webb, 1986). Thus, this study also investigates prospective and practicing teachers' self-efficacy related to geometry.

THEORETICAL FRAMEWORK

In this section we present the theoretical framework which guided this study. The section begins with a discussion of teachers' knowledge for teaching and continues with a review of self-efficacy. Finally, we present a model of the framework and the specific research questions related to in this paper.

Teachers' knowledge for teaching

In framing the mathematical knowledge preschool teachers need for teaching, we draw on Shulman (1986) who identified subject-matter knowledge (SMK) and pedagogical content knowledge (PCK) as two major components of teachers' knowledge necessary for teaching. In our previous work (Tabach, et al., 2010), we found it useful to differentiate between two components of teachers' SMK: being able to produce solutions, strategies, and explanations and being able to evaluate given solutions, strategies, and explanations. Thus our framework takes into consideration both of these aspects of SMK. Regarding PCK, we draw on the works of Ball and her colleagues (Ball, Thames, & Phelps, 2008) who refined Shulman's theory and differentiated between two aspects of PCK: knowledge of content and students (KCS) and knowledge of content and teaching (KCT). KCS is "knowledge that combines knowing about students and knowing about mathematics" whereas KCT "combines knowing about teaching and knowing about mathematics" (Ball, Thames, & Phelps, 2008, p. 401).

Within geometry, preschool teachers' SMK includes knowledge of defining geometrical concepts and identifying various examples and nonexamples of two and three-dimensional figures (solids) as well as ways of justifying this identification. Teachers' KCS includes knowledge of which examples and nonexamples children intuitively recognize as such (Tsamir, Tirosh, & Levenson, 2008a) as well as knowledge of children's commonly held concept images and concept definitions for geometrical figures (Tall & Vinner, 1981). In both domains, KCT includes knowledge of designing and assessing different tasks, affording students multiple paths to understanding.

Self-efficacy

This framework also draws on Bandura's (1986) social cognitive theory which takes into consideration the relationship between psychodynamic and behaviouristic influences, as well as personal beliefs and self-perception, when explaining human behaviour. Thus, besides investigating preschool teacher's knowledge it is important to also relate to their self-efficacy. Bandura defined self-efficacy as "people's judgments of their capabilities to organize and execute a course of action required to attain designated types of performances" (1986, p. 391). Hackett and Betz (1989) defined mathematics self-efficacy as, “a situational or problem-specific assessment of an individual’s confidence in her or his ability to successfully perform or accomplish a particular [mathematics] task or problem” (p.262). This framework takes into consideration teachers' mathematics self-efficacy as well as their pedagogical-mathematics self-efficacy, i.e. their self-efficacy related to the pedagogy of teaching mathematics. Teacher self-efficacy has been related to a variety of teacher classroom behaviors that affect the teacher’s effort in teaching, and his or her persistence and resilience in the face of difficulties with students (Ashton & Webb, 1986; Meijer & Foster, 1988). Studies report that teachers with a high sense of self-efficacy are more enthusiastic in teaching (Allinder, 1994; Guskey, 1986) and are more committed to teaching (Coladarci, 1992; Evans & Tribble, 1986).

RESEARCH DESIGN AND QUESTIONS

The research design of this study was based on the framework presented in the following 8-cell knowledge and self-efficacy matrix (see Table 1). In cells 1-4, and in cells 5-8, we address teachers' knowledge and self-efficacy respectively.

	Subject-matter		Pedagogical-content	
	Solving	Evaluating	Students	Tasks
Knowledge	Cell 1	Cell 2	Cell 3	Cell 4
Self-efficacy	Cell 5	Cell 6	Cell 7	Cell 8

Table 1: 8-cell knowledge and self-efficacy matrix

In this paper we report on Cells 1 and 5. The questions which guided this part of the study were: (1) Are prospective and practicing preschool teachers able to identify various examples and nonexamples of two and three-dimensional figures? (2) What are prospective and practicing preschool teachers' self-efficacy regarding their ability to identify various two and three-dimensional figures?

METHOD

Participants and procedure

Two groups participated in this study. The first group included 18 prospective preschool teachers in their second of four years of study. The study took place in the beginning of the year before they had attended any courses which specifically addressed teaching mathematics in preschool. The second group included 21 practicing preschool teachers all currently teaching 4-6 year old children in municipal preschools. The practicing teachers were enrolled in a professional development course aimed at promoting their knowledge for teaching mathematics in preschool.

Two questionnaires were handed out. The first questionnaire, which we will call the 2-D questionnaire, investigated teachers' self-efficacy and knowledge regarding triangles, pentagons, and circles. The second questionnaire, called the 3-D questionnaire, investigated teachers' self-efficacy and knowledge regarding cones and cylinders. The first questionnaire was handed out during the first meeting with participants; the second was handed out during the second meeting. It was explained to the participants in both groups that these questionnaires would serve as a basis for their studies.

Tools

Each questionnaire consisted of two parts. The first part of the 2-D questionnaire began with the following self-efficacy related questions: If I am shown a triangle, I will be able to identify it as a triangle. If I am shown a figure which is *not* a triangle, I will be able to identify it as *not* being a triangle. This was repeated for pentagons and circles. A four-point Likert scale was used for these questions, 1 meaning the teacher did not believe at all that she could identify the figure and 4 meaning that she was very sure that she could identify the figure. Similarly, the 3-D questionnaire inquired about teachers' ability to identify cones and cylinders as well as their ability to identify nonexample of cones and nonexamples of cylinders.

The second part of each questionnaire consisted of a series of examples and nonexamples of different figures. Each figure was accompanied by a question: Is this a triangle (or pentagon or cylinder)? Yes/No. Figures 1-3 present the figures used when investigating triangles, pentagons, and circles.

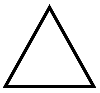
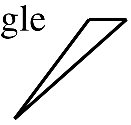
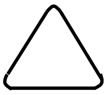
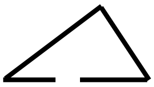
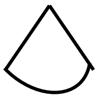
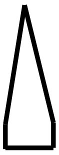
Is this a triangle?	Intuitive	Non-intuitive	
Examples	Equilateral triangle 	Scalene triangle 	
Non-examples		Rounded-corner “triangle”  Open "triangle" 	Pizza  Long "triangle" 

Figure 1: Is this a triangle?

In choosing the figures, both mathematical and psycho-didactical dimensions were considered. That is, we not only consider whether the figure is an example or a nonexample of the target geometrical shape but what developmental and cognitive issues might arise as children identify geometrical figures. Specifically we consider whether or not the figure, be it an example or nonexample would intuitively be recognized as such.

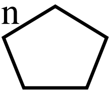
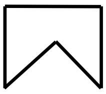
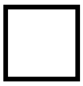
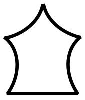
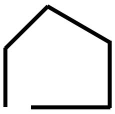
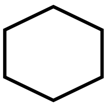
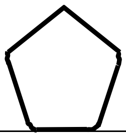
Is this a pentagon?	Intuitive	Non-intuitive	
Examples	Regular pentagon 	Concave pentagon 	
Non-examples	Square 	Curved-sides “pentagon”  Open "pentagon" 	Hexagon  Rounded-corner "pentagon" 

Figure 2: Is this a pentagon?

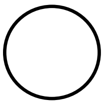
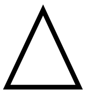
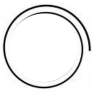
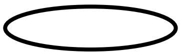
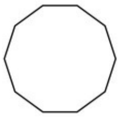
Is this a circle?	Intuitive	Non-intuitive	
Examples	Circle 		
Non-examples	Triangle 	Spiral  Ellipse 	Decagon 

Figure 3: Is this a circle?

When considering triangles, for example, the equilateral triangle may be considered a prototypical triangle and thus intuitively recognized as a triangle, accepted immediately without the feeling that justification is required (Hershkowitz, 1990; Tsamir, Tirosh, & Levenson, 2008a). The scalene triangle may be considered a non-intuitive example because of its “skinniness”. Whereas a circle may be considered an intuitive non-example of a triangle, the pizza-like “triangle” may be considered a non-intuitive nonexample because of visual similarity to a prototypical triangle (Tsamir, Tirosh, & Levenson, 2008a). Similarly, the regular pentagon was thought to be easily recognized by children who had been introduced to pentagons whereas studies have shown that even among children who had been introduced to pentagons, the concave pentagon is more difficult to identify (Tsamir, Tirosh, & Levenson, 2008b). Triangles and pentagons may vary in the degree of their angles providing a wide variety of examples. In contrast, the circle's symmetry limits the variability of its characteristic features. Thus, only one example of a circle was given. The nonexamples of each shape were also chosen in order to negate different critical attributes.

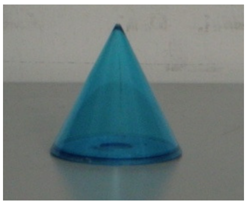
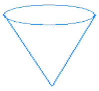
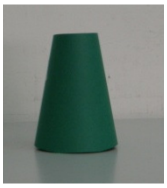
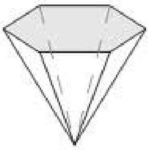
Is this a cone?	Intuitive	Non-intuitive	
Examples	Cone 	Up-side down cone 	Cone lying down
Non-examples	Sphere	Cone with its top cut off 	Up-side down pyramid 

Figure 4: Is this a cone?

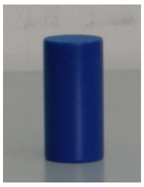
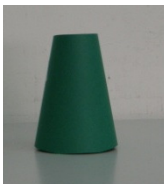
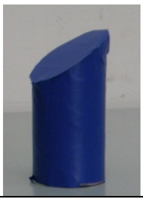
Is this a cylinder?	Intuitive	Non-intuitive	
Examples	Cylinder 	"Coin-like" cylinder	Cylinder lying down
Non-examples	Sphere	Cone with its top cut off 	Cylinder cut on a slant 

Figure 5: Is this a cylinder?

Figures 4-5 present the pictures and drawn figures of solids used when investigating cones and cylinders. As few studies have investigated young children's knowledge of solids, our differentiation between intuitive and nonintuitive solids is based on our hypothesis regarding how children might identify them.

Analyzing the data

For each geometrical figure, two self-efficacy questions were asked. The mean of these scores (1-4) was configured which resulted in the participant's self-efficacy score for that figure. Similarly, a mean score was configured for identifying the different figures. For example, when investigating identification of a cone, six figures were presented. Thus, a participant who correctly identified (either as an example or as a nonexample) three out of the six figures, received a score of 50%. Finally, the mean score of each group was configured per figure for both the self-efficacy and knowledge part of the questionnaires and likewise a general score resulted for each group.

RESULTS

In this section we describe the results of both questionnaires. We begin by presenting overall results of participants' self-efficacy and knowledge for both groups and then describe some results regarding specific figures.

Recall that self-efficacy was rated on a scale of 1-4, 1 being very low and 4 being very high. Results (see Table 2) indicated that participants' self-efficacy regarding their ability to identify triangles, pentagons, and circles were quite high. Regarding knowledge, in general, participants of both groups were able to identify correctly most of the figures presented to them. When identifying pentagons, 10 out of 18 practicing teachers incorrectly identified the curved-sides "pentagon" as a pentagon. Another unexpected result was that the mean score for prospective teachers when identifying circles was only 78%. This exception was a result of incorrect identification of the ellipse as a circle. Out of 18 prospective teachers, 16 incorrectly claimed that an ellipse is a type of a circle.

	Prospective teachers N=18		Practicing teachers N=18	
	Self efficacy	Correct identification	Self efficacy	Correct identification
Triangle	3.6	91	3.8	94
Pentagon	3.5	90	3.3	85
Circle	3.1	78	3.6	91
General	3.4	86	3.6	90

Table 2: Mean self-efficacy scores per 2-D figure per group and mean knowledge scores per 2-D figure per group

Results of the 3-D questionnaire are presented in Table 3. First we note that while most of the self-efficacy was quite high, prospective teachers' self-efficacy regarding their ability to identify cones was markedly lower. Overall, practicing teachers' had a higher self-efficacy regarding their ability to identify three-dimensional figures than did prospective teachers. Regarding knowledge, a marked difference was noted between practicing and prospective teachers' knowledge. When it came to identifying examples and nonexamples of cones, four prospective teachers simply stated on their questionnaires that they did not know what a cone is and therefore could not identify any of the figures. This was not the case when it came to identifying cylinders. None of the participants stated that they did not know what a cylinder is. Instead, when it came to identifying the "coin-like" cylinder, 11 out of 17 prospective teachers incorrectly claimed that it was not a cylinder.

	Prospective teachers N=17		Practicing teachers N=21	
	Self efficacy	Correct identification	Self efficacy	Correct identification
Cones	1.9	73	3.3	90
Cylinder	3.1	75	3.4	89
General	2.5	74	3.4	90

Table 3: Mean self-efficacy scores per 3-D figure per group and mean knowledge scores per 3-D figure per group

DISCUSSION

The aim of this paper was to investigate prospective and practicing teachers' ability to identify examples and nonexamples of various two and three dimensional figures as well as to investigate their related self-efficacy. In general, practicing teachers' had a higher self-efficacy than prospective teachers. Practicing teachers were also more knowledgeable than prospective teachers. This makes sense. After all, the practicing teachers have experience with the preschool mathematics curriculum while the prospective teachers are just beginning to learn the relevant content knowledge.

In addition, for both groups, self-efficacy was more or less in line with actual knowledge. This is important. While in general, it is advantageous to have a high self-efficacy, if that high self-efficacy does not reflect true knowledge, then it may actually hinder the promotion of knowledge. If a person believes he is capable of performing some task, then that person will not feel the need to learn how to perform the task. The opposite also holds true. As stated in the beginning, even if a person in actuality has the knowledge necessary to perform some task, if that person has a low self-efficacy, then the low self-efficacy may impede on that person's motivation and enthusiasm to carry out that task (Allinder, 1994). In the case of teachers, a low self-efficacy with regard to carrying out geometrical tasks, may affect the teacher's choice of geometrical activities. Looking back at the results, while it may have been surprising that prospective teachers' self-efficacy related to circles was lower than their self-efficacy related to pentagons, this actually went hand in hand with their knowledge, indicating a rather accurate sense of self-awareness. Likewise, prospective teachers' self-efficacy related to 3-D figures was lower than their self-efficacy related to 2-D figures. This was also in line with the difference between their knowledge of 2-D figures and 3-D figures. Regarding practicing teachers, a lower self-efficacy in identifying pentagons corresponded to a lower score when actually identifying pentagons. The one exception in correspondence between self-efficacy and knowledge occurred with prospective teachers. Interestingly, this group assessed their ability to identify cylinders higher than their ability to identify cones. However, for both cylinders and cones, only about 74% identified correctly the figures. This result indicates that this group was unaware of their lack of knowledge with regard to cylinders but was aware, as was indicated above, of their lack of knowledge regarding cones. This dissonance between self-efficacy and knowledge must be taken into consideration when planning professional development.

As mentioned in the beginning, this paper reported on initial results of a larger project. Referring back to the 8-cell knowledge and self-efficacy matrix, the next step will be to evaluate prospective and practicing preschool teachers' knowledge and self-efficacy related to evaluating tasks as well as their pedagogical-content knowledge and pedagogical-content self-efficacy. Finally, we note two limitations of this study. First, approximately 20 prospective and practicing teachers participated in this study. An additional study involving more participants might yield further information regarding preschool teachers' geometric knowledge and self-efficacy. Second, qualitative instruments, such as interviews might be added to understand more fully the sources of teachers' self-efficacy.

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