SCHRIFTENREIHE DES FACHBEREICHS MATHEMATIK

Current State of Research on Mathematical Beliefs

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edited by
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Editor’s Statement

We are proudly looking forward to presenting a first collection of papers prepared by the German-Finnish research group MAVI (MAthematical VIews on Beliefs and Mathematical Education) to you.

These papers contain the abstracts of talks given at the workshop on ”Current State of Research on Mathematical Beliefs”, which took place at the University of Duisburg on October 4-5, 1995. The aim of this research group, being the initiative of my colleague Erkki Pehkonen and myself, is to study and examine the mathematical-didactic questions that arise through research on mathematical beliefs and mathematics-education. There is a vivid movement around research on mathematical beliefs in Europe. At the last Conference on Mathematics-Education in Kassel (Germany) in March 1995, we established a working group of interested scientists called ‘Mathematical World View’ in order to build up a network of belief-researchers in Germany. The European Research Conference on Mathematics-Education took place in Osnabrück (Germany) a couple of days before our conference in Duisburg. In Osnabrück, we enlargened the network of belief-researchers into European dimensions.

Now, the initiators would like to encourage all interested colleagues to join our network and to participate in our activities.

Duisburg, October 1995 *Günter Törner*

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Erkki Pehkonen & Günter Törner

Mathematical Belief Systems and Their Meaning for the Teaching and Learning of Mathematics

Background

Mathematics teaching in schools has been subjected to change for a long time in most countries, and this change process is still continuing. Especially our concept of school mathematics and its teaching is changing. Mathematics is no longer being understood as a static system, which pupils are thought to be against, as such. Instead of that, the idea is that pupils are learning at their best when they are doing it actively (dynamic aspect).

Within school research, the understanding of learning has at first been measured by cognitive academic achievement. Affective by-results which are derived from with an individual's metacognitions, however, determine the quality of learning, but they are often left aside in studies. During the last decade, researchers around the world have paid more and more attention to mathematics learning from a metacognition point of view, especially with regard to pupils' and teachers' beliefs. Beliefs are situated in the ”twilight zone” between the cognitive and affective domain; they have a component within each domain.

Behind the mentioned active understanding of learning, one finds the view on learning which is compatible with the constructivism. According to this view it is essential that a learner is actively collaborating, in order to be able to elaborate on his knowledge structure (e.g. Davis & al. 1990; Ahtee & Pehkonen 1994). Thus, the meaning of pupils' own beliefs (subjective knowledge) concerning mathematics and its learning will be emphasized as being a regulating system of their knowledge structure. Since the teacher is a central influential factor, as being the organizer of learning environments, his beliefs are also essential.

1. The central concept: belief

Since in definitions for beliefs to be found in the educational literature, there is no common line to be seen (Pehkonen 1994b), we will say here: An individual's mathematical <u>beliefs</u> are compound of his subjective (experience-based) implicit knowledge on mathematics and its teaching /learning. Conceptions could be understood as conscious beliefs, and thus are separated from so-called basic beliefs which are often unconscious. We think that in the case of conceptions, the cognitive component will be stressed, whereas the affective component is being emphasized in basic beliefs.

The spectrum of an individual's beliefs is very large, and its components influence each other. For example, Green (1971) speaks about the quasi logical structure of beliefs. Thus, one may think that an individual's beliefs will be structured in what we will call his belief system. An individual's belief system is closely related to his knowledge system, in such a way that they together seem to form a ”bundle of spaghetti”. If you try to consider one aspect separately from the other (and take it away), almost the whole bundle will be sure to follow. A belief system will here also be referred to as a *view on mathematics* – which might be in the case of teaching mathematics more informative than beliefs. Schoenfeld used in his book the term ”mathematical world view” when speaking about a belief system (Schoenfeld 1985). Törner and Grigutsch have for example adapted this, and translated the term directly into the German term ”*mathematisches Weltbild*” (Törner & Grigutsch 1994). This concept was elaborated on in further research (Grigutsch & al. 1995), and was theoretically based upon the theory of attitudes. These are both closely related to the concept of mathematics-related belief systems.

Thus, an individual’s view on mathematics is the compound of a wide set of beliefs and conceptions which can be divided into four main categories: (1) beliefs about mathematics, (2) beliefs about oneself within mathematics, (3) beliefs about mathematics teaching, (4) beliefs about mathematics learning. These four main categories can be divided further into smaller pieces: For example, beliefs on the nature of mathematics as a school subject and as a scientific discipline pertain to group (1). Similarly, beliefs on the nature of mathematical tasks and beliefs on the origin of the mathematical knowledge, as well as beliefs concerning the relationships between mathematics and empirical world (the application and use of mathematics), belong to that group. The concept ”view on mathematics” has been discussed in detail in the published paper of Pehkonen (1995c).

With the view on mathematics thus being defined, it follows that it is (for an individual) a relational concept which is related to an individual and dependent on him. The concept has also explicitly an interactive component between the individual and the object of the belief. Psychologically, the concept ”view on mathematics” can be anchored into the theory of attitudes, since in many definitions of attitude, a three-component-model, where beliefs form one component, is used (e.g. Statt 1990).

2. The meaning of mathematical beliefs

During the last years, pupils’ thinking processes have been intensively studied. About ten years ago, researchers noticed that pupils’ beliefs seemed to form the key to the understanding of their behaviour, also in mathematics (Wittrock 1986). The central role of beliefs for the successful learning of mathematics has been pointed out again and again by several mathematics educators (e.g. Schoenfeld 1985, Silver 1985, Frank 1988, Garofalo 1989, Baroody & Ginsburg 1990, Borasi 1990, Schoenfeld 1992). In this regard, the following reasons are given as an explanation for these effects: Beliefs may have a powerful impact on how children will learn and use mathematics, and therefore, they may also form an obstacle for the effective learning of mathematics. Pupils who have rigid and negative beliefs of mathematics and its learning easily become passive learners, who emphasize more remembering than understanding in learning.

Beliefs and learning seem to form a circle: Pupils’ experiences in mathematics learning influence and form their beliefs. On the other hand, beliefs have a consequence on how pupils will behave in mathematical learning situations, and therefore, how they are able to learn mathematics (Spangler 1992). Thus, pupils’ beliefs revealed through research, reflect teaching practices in the classroom. The way mathematics are taught in the classroom will little by little form the pupils’ view on mathematics.

The latter is easily understood when we remember that an individual’s mathematical beliefs, his view on mathematics, form a regulating system for his knowledge structure.

Within this frame, the individual may act and think. On the other hand, this frame greatly influences his mathematical performance. Let's take an example: There is a pupil who understands mathematics merely as calculations. His understanding could often have resulted from a one-sided, calculations emphasizing teaching at the primary level. Then tasks which demand thinking, and where mere calculation do not lead to an answer, might be difficult or even impossible for him.

Mathematical beliefs as a regulating system

In her dissertation, Martha Frank (1985) introduced a schematic picture of some factors affecting pupils' problem solving behaviour. Since most of the factors act via pupils' belief systems, we have organized the components in the original scheme in another manner (Figure 1). This scheme, in fact, shows the regulating character of a pupil's view on mathematics (his mathematical belief system).

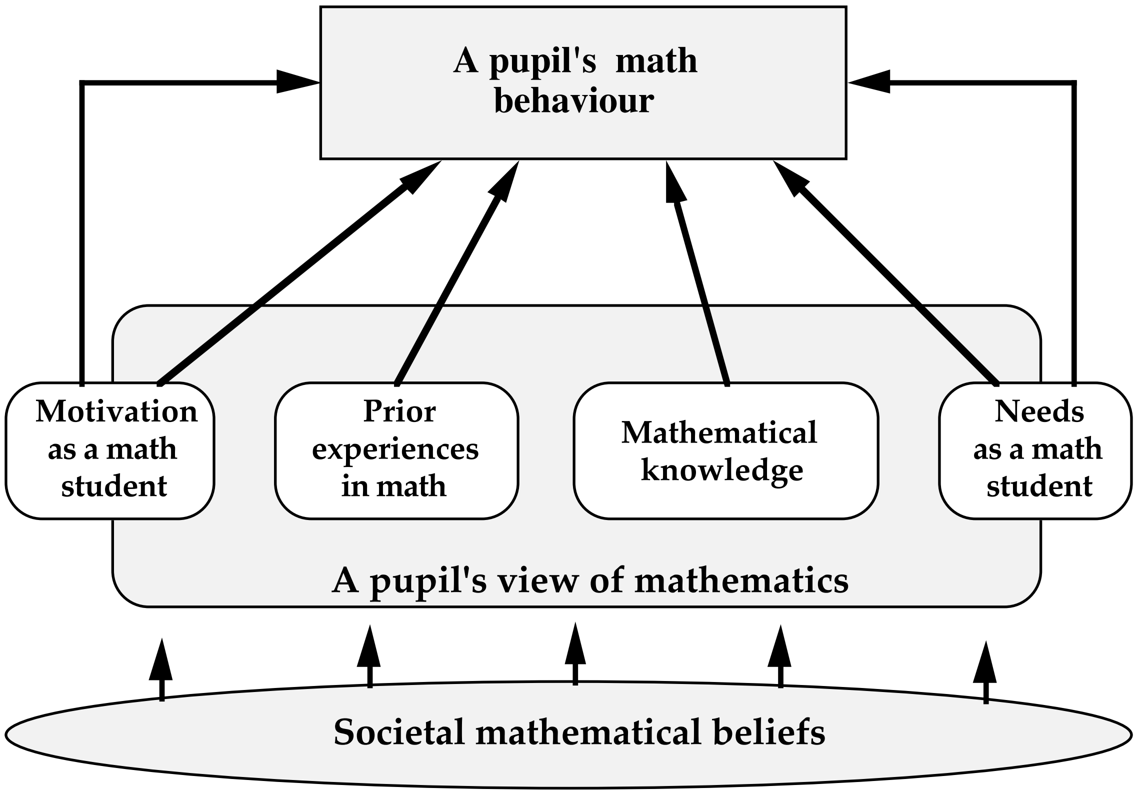

Figure 1. Factors affecting pupils' mathematical behaviour.

Beliefs play a central role as a background factor for pupils' thinking and acting. A pupil's mathematical beliefs act as a filter which influences almost all his thoughts and actions concerning mathematics. A pupil's prior experiences of mathematics fully affect at the level of his beliefs – usually unconsciously. When he uses his mathematical knowledge, his beliefs are highly involved, too.

In contrast to this, a pupil's motivation and needs as a learner of mathematics are not always connected with his mathematical beliefs. Additionally, there are many societal mathematical beliefs, perhaps myths, e.g. that mathematics is merely calculation (for more myths see Frank 1990 or Paulos 1992), which also influence a pupil's mathematical behaviour via his belief system.

Figure 1 shows such a situation in which a pupil's mathematical performance is influenced by several factors which affect through a system or a net of his own beliefs. However, this is only part of the truth, in fact, the situation is much more complex. Pupils act within a very complex net of influences – Underhill (1990) speaks about a web of beliefs. For example, their mathematics teacher, classmates, friends, parents, relatives and teachers of other subjects all have their own views on mathematics and its teaching and learning (Figure 2). These beliefs more or less affect learners' beliefs, and usually not in the same direction.

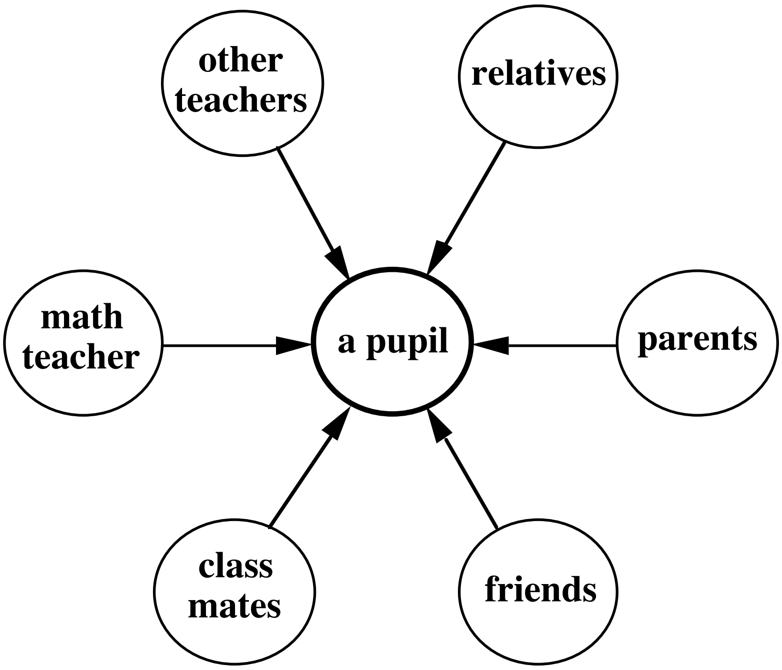

Figure 2. There is a variety of persons in a pupil's environment whose beliefs influence him.

If we were to condense an answer to the question "Why do research on mathematical beliefs?", we might stress at least two points: The knowledge of pupils' conceptions provides an opportunity for teachers, (1) to better understand pupils' thinking and actions, and (2) to support pupils' learning. Both these reasons will help teachers to better organize their teaching to correspond with pupils' view on mathematics.

Mathematical beliefs as an indicator

There is another practical meaning of beliefs: A view on mathematics (mathematical beliefs) may form a practical indicator in a situation which one is not otherwise able to observe. Since the view on mathematics transmitted through beliefs, expressed by an individual, gives a good estimation of his experiences within mathematics learning and teaching, we will thus have a method to indirectly evaluate the instruction he has received or has given: In the case of a teacher, the view on mathematics may act as an indicator

(1) of teachers' university studies,

(2) of teachers' professional view,

(3) of teachers' in-service training.

In the case of pupils and students, the view on mathematics could function as an indicator

(4) of students' experienced teaching (in schools and universities).

Generally, one may consider the view on mathematics as an indicator

(5) of the functioning of the whole school system.

Mathematics teaching forms part of the general education provided by the school, which will be realized within a societal context. In the research done, one may find connections to the change processes within the society, connections which have arisen outside the framework of mathematics education (Pehkonen & Törner 1994). Thus, the view on mathematics also has a role as an indicator

(6) of social sensitivity.

Mathematical beliefs as an inertia force

If we aim to develop mathematics teaching in schools, we are compelled to take into account teachers' beliefs (their view on mathematics) – and also pupils' beliefs. Usually, the question is of experienced teachers' rigid attitudes and their steady teaching styles, which will act as an inertia force to change. Experienced teachers know through their long practice what kind of mathematics teaching is (according to them) good, and this subjective knowledge (beliefs) is usually deeply rooted.

If a teacher thinks that mathematics learning occurs at its best by doing calculation tasks, his teaching will concentrate on doing as many of these tasks as possible. This phenomenon was already observed more than ten years ago: Teachers' different teaching philosophies (belief systems) will lead to different teaching practices in classrooms (Lerman 1983; also Ernest 1991).

Thus, the problem is how to help teachers to develop and extend their own pedagogical knowledge. Also, pupils' view on mathematics might be improper for the development, and thus the conditions of change should be considered. If pupils have a rigid view about how mathematics should be taught, e.g. in earlier grades they have only learned to work on text book tasks in mathematics, then they might get confused with other approaches. Therefore, beliefs have a central position when trying to change teaching.

The prognostic character of mathematical belief systems

As a consequence of the mentioned arguments, one should stress that mathematical belief systems also have a prognostic aspect. Pupils who consider mathematics only as a manipulative calculation system have an ignorant attitude toward problem solving, and therefore their opportunities to learn effectively in school are restricted, since studying mathematics on upper grade levels, e.g. in secondary schools or at university, demands also other components of mathematics than mere calculations.

3. Development and developing of beliefs

Research done on beliefs has been much concentrated on recognition and characterization of teachers' and pupils' views on mathematics (static level). There are also some research on the correlation and influential relationships between the components of a view on mathematics (dynamic level). But one of the remaining questions is how pupils' views on mathematics (mathematical beliefs) will develop within school instruction, and which are the most influential factors on this development.

Based on the ideas of Thompson (1991), a model of the development of an individual's mathematical beliefs can be constructed. According to her, the development of mathematical beliefs can be considered at least on three levels (level 0, 1, 2) with the following questions (which are in fact components of view on mathematics):

(a) What is mathematics?
(b) How is mathematics learned and taught?
(c) Which are a teacher's and pupils' roles?
(d) Which are the criteria for judging the correctness?

(This elaboration of Thompson's model has been done in a paper by Pehkonen 1994b.) Although Thompson (1991) speaks about levels of beliefs, they are not levels in the sense that they are excluding each other. The levels of beliefs should be understood as enlargements, i.e. the next level contains the earlier one.

The fitting of the model could be checked e.g. by investigating both its poles: On the one hand, the conceptions of school beginners about mathematics are clarified – they should be on the most elementary level (level 0). On the other hand, the most developed mathematical conceptions (level 2), e.g. those of professional mathematicians (at an university institute of mathematics), will be investigated. For example in question (1), the most elementary thinking (level 0) is represented by the view ”mathematics is calculating”. Whereas, level 2 could be characterized with the expression ”mathematics is understood as a complex system of different interconnected concepts, procedures and representations”.

Optimal teaching from the view point of beliefs

As a hypotheses one may set that optimal circumstances (i.e. instructional organization) for the development of beliefs demand from the teacher along mathematics knowledge and pedagogical skills also the most developed view on mathematics and flexibility in realization of instruction. Then the teacher may flexibly take account of pupils' needs and earlier beliefs when planning his teaching, and before of all, when realizing it.

This is closely related to an important aspect of instruction: sharing authority, i.e. how much freedom and responsibility of their own studying the teacher will give his pupils. This has been one keynote theme in the international discussion on developing teaching (e.g. Cooney 1993). The central idea is that when a pupil holds himself responsible for his own studying, his viewpoint with regard to studying and to mathematics itself will be changed.

From these ideas, one may derive some criteria for observation of classroom teaching: From the pupil's point of view, this change of focus means also his more personal relationship to mathematics and its learning: The learned mathematics is then meaningful for the pupil in question. According to the teaching and learning which is compatible with the developed view on mathematics, it will aim on the understanding of topics and of relationships between them, as far as possible. The third important criteria is the fostering of the mathematics using skills. One component that will surely promote learning is mathematical communication between pupils which may be fostered e.g. with different forms of co-operative learning.

Then the question arises how to realize these ideas in teaching. What kind of approach to mathematics is a proper one? One possible method to help a teacher to develop a suitable learning environment for his pupils is the internationally approved so-called ”open approach” which has been developed in the 1970's in Japan (e.g. Nohda 1991; Pehkonen 1995a). This method could have its roots in the use of investigations which was a key topic in the 1960s and 1970s in British educational journals and conferences (Jaworsky 1994). One could use open-ended problems which have been shown up to be a promising solution when creating suitable learning environments. They seem to offer an opportunity for more meaningful teaching and learning from the pupils' point of view. In the research project during the years

1989–92, Pehkonen tried to find out the influence of open-ended tasks on pupils' motivatedness (Pehkonen & Zimmermann 1990; Pehkonen 1993, 1995b). But in this study, the limitations set by beliefs as a regulating factor of knowledge system were not considered as a possible obstacle or help towards change.

4. The current interest of the theme in research

Mathematical belief systems as a basic theme for mathematics

The theme "mathematical belief systems", i.e. attitudes, leading ideas, background theories and paradigms to the question *What is mathematics?* is an ever young basic theme in mathematics, its philosophy and its history. Here we are not going so far as to discuss the contents of the classical or current mathematical monographs. One is not able to emphasize enough that mathematics must not be seen as monolithic or canonical (Davis 1994). The same idea is expressed in the book of Ewing (1994) who has investigated the development of mathematics in the published papers of the journal *American Mathematical Monthly* during the last hundred years. A large variety of material is contained in books by Moritz (1993) and Schmalz (1993). Mathematical belief systems have been developed and changed into different forms during the history of mathematics, and come out as mathematical background theories and paradigms.

Mathematical belief systems as a basic theme for mathematics education

Whereas for mathematicians mathematical belief systems served the function of *usually unreflected* background theories and attitudes which conduct their mathematical doing, they come alive in mathematics education as conceptions, schools and trends. For each person, especially for pupils and teachers, belief systems form the structure of attitudes about mathematics which mainly define their behaviour towards mathematics. The teacher's view on mathematics and his pedagogical knowledge are more essential for his teaching management, especially in general course of upper secondary school, than his mathematical knowledge (Tietze 1991).

The convictions and teaching principles of mathematics education and of the teacher influence the structuring of mathematics teaching, and affect thus the pupils' view on mathematics. Pupils' attitudes and convictions about mathematics and mathematics teaching influence how they do mathematics and learn. Therefore, the task of mathematics education is to find out what these mathematical belief systems are, to make them conscious, and to try to find out their working mechanisms. How current this question is, one could read e.g. in a quotation from the news letter of the German Mathematician Association: "It should be mentioned, as the first one, the one-sided view on mathematics which the teacher student adapts or is delivered" (Schmidt 1995, 22).

Belief systems as a highly current research task – a summary of reasons

The analysis of leading principles and paradigms of mathematics has become remarkably meaningful during last years, and recently gained high current interest. As reasons for this development, the following aspects are given:

(1) During the last years, an extending interest on the metatheory of mathematics and mathematics education could be noticed. An essential component of this interest is the analysis of paradigms and metatheories upon which the mathematics educational theories, leading principles and trends are based.

(2) In the mathematics education and in mathematics, a restructuring of leading principles has taken place within the last decade. Summarizing, the change means to proportion of the formalist-positivist paradigm and the emphases of the constructivist paradigm.

(3) The research on problem solving has played an essential role in the research and discussion of mathematics education in the States since the 1980s, and later in Europe (also in Germany and in Finland), and is continuing. This has essentially stressed the discussion on attitudes, belief systems and background theories, and emphasized their importance as the focus of teaching and learning research in the field of mathematics education (e.g. Pehkonen 1991). The main theses and observation is that mathematical belief systems have an extraordinary importance for an individual doing mathematics. Attitudes toward mathematics act as determinants for cognition.

(4) When compared to American research, there are very few European research done on beliefs, and especially few in Germany and in Finland. There do not exist many international comparisons.

(5) The stressing of the constructivist paradigm in mathematics education is compatible with the development of the modern cognition psychology and learning psychology. Both are based on the epistemological paradigm of constructivism, and therefore are closely related to the developmental psychology of Jean Piaget and to the pedagogical psychology of his student Hans Aebli. When elaborating the constructivist paradigm within the pedagogical psychology, the reform pedagogy and the pedagogy of Pestalozzi has been revived.

(6) In the education of other natural sciences, content-related research questions are also being studied (e.g. Ahtee 1993, Ahtee & Pehkonen 1994, Gerhardt 1994, Rhöneck 1994, Pfundt & Duit 1994).

5. The meaning of international comparisons

All the important conditions for views on mathematics may be seen a priori as nationally defined. Thus, university schooling, curricular organizations, special kinds of school forms and organization will decide the extent to which results obtained can be transferred from one country to another country. Therefore, it is clear that an international comparison offers an opportunity to clarify the influence of these national factors. For example, comprehensive school using inner differentiation is characteristic for the Finnish schooling system, whereas in Germany, they are mainly following a parallel school system using outer differentiation. There are also differences between Finland and Germany in the amount of teacher training both in subject studies and in pedagogical studies. Therefore, comparative international analyses are to be seen as a central research object. The juxtaposition of international situations might just reveal hidden determinants.

Comparative studies generally pertain primarily to basic research, which usually does not stress an applicable point of view. But in this case, through a comparative study, we may be able to see our own system from the outside, which could help us to better determine its weaknesses and strengths. From the results of a comparative study, we might notice that pupils' conceptions in some countries are desirable, and then begin to think how to develop similar conceptions in our own country.

Another benefit regards the transferability of the research results obtained. Is it possible, for example, to tranfer the results obtained in the United States directly to the European situation? That is something we have been believing in, and as a consequence we have used the results in our own country without questioning them. Or is it true that pupils' conceptions are culture-bound? If this is generally valid, it might have interesting consequences on the interpretation of research results. Perhaps it is not possible to directly generalize the results from the United States to all industrial countries. This possibility was explored, in the case of teachers' beliefs, in the paper published by Pehkonen and Lepmann (1993). Correspondingly, preliminary results in comparing of pupils' beliefs show that differences between countries are larger than within a country itself (Pehkonen 1993).

Research results on international comparison

The question of the international comparison of pupils' and teachers' mathematical beliefs still seems to be an almost unexplored field. The main question is: "Are there essential differences in conceptions about mathematics teaching in different countries?" We know that mathematics can be understood as a universal discipline. So, the question arises whether pupils' and teachers' conceptions on mathematics and on mathematics teaching and learning are also universal, or whether they are, perhaps, culture-bound.

About five years ago, an international project on comparison of pupils' mathematical beliefs was started. Before the project which was named "International comparison on pupils' mathematics-related conceptions", and from which some preliminary results were published (Graumann & Pehkonen 1993, Pehkonen 1993, 1994a, Pehkonen & Tompa 1994, Pehkonen 1995), there almost no research into variations between pupils' beliefs on an international scale seems to have been done. Only in the Second International Mathematics Study (Kifer & Robitaille 1989) pupils' responses to some questions on the affective domain were dealt with in a background questionnaire. The study indicates that there are large differences between countries with regard to mathematical beliefs and attitudes. Today, there seems to be a growing interest in the international comparison of pupils' conceptions (e.g. Berry & Sahlberg 1994a, 1994b).

6. Central questions for research – an overview

In the domain of mathematical belief systems, there seem to exist four large theme fields of focus which are of special interest:

(1) Identifying and describing of beliefs in an individual's belief system.
(2) Influences of mathematical belief systems.
(3) Birth and development of mathematical belief systems.
(4) Conditions for the change of mathematical belief systems.

In Table 3 which represents only the first approximation and will in no way reveal the status of the final situation, we arranged some papers on beliefs which have been published during the last decade according to two criteria: On the one hand, they were arranged according to the theme field of focus just mentioned, and on the other hand, according to the deliverer of beliefs.

In Table 3, one could easily add more differential factors, e.g. comparison between countries, comparison between gender, and comparison between age groups ñ but then we would be compelled to use a three-dimensional (or four-dimensional, etc.) model.

Additionally, one should note that e.g. the column ”pupils” contains pupils from different school forms: pre-school, primary school, lower secondary school, upper secondary school, vocational schools, colleges, universities etc. Similar differentials are valid for other columns, too.

held by	**pupils**	**teachers**	**teaching materials**	**public opinion**	**school administration**	**teacher organizations**
1. What are beliefs?	Frank (85) Schoenfeld (85) Schoenfeld (89) Garofalo (89) Zimmermann (91) Pehkonen (92) Törner & Grigutsch (94) Pehkonen (95)	Thompson (84) Jones (90) Zimmermann (91) Mura (93) Pehkonen & Lepmann (95) Grigutsch, Raatz & Törner (95)		Jungwirth (94) Gruß (95) Weber (95)		
2. How do beliefs influence?	Malmivuori (95)	Lerman (83) Raymond (93) Kupari (95)				
3. How are beliefs born?						
4. How do beliefs change?	Simon & Schifter (93) Grigutsch (95)	Shaw & al. (91) Thompson (91) Pehkonen & Törner (94) Lindgren (95) Rahmann (95)		Stenmark & al. (86) Onslow (92)		

Table 3. Research done on central questions in mathematical beliefs according to variable groups.

As Table 3 shows, there exist a large field of questions which could only partly be answered. It is possible that the questions overlap with each other. There could also exist some dependence between separate problem fields. Keeping these aspects (and the ”white” fields of Table 3) in mind, we have established a few theme domains where, at least, more research is required:

(A) The birth and influence of beliefs.
(B) Teachers’ beliefs in different teaching institutes.
(C) Change in teachers’ beliefs.

To have research results in domain (A) is of paramount importance, since we know very little on the dynamic structure of beliefs systems. In domain (B), especially those teacher groups which are underrepresented in research, should be focused on (e.g. mathematics professors, mathematics educators, mathematics teachers in vocational schools). Domain (C) is central when aspiring to the development in mathematics teaching on all levels. Additionally, one should make serious attempts to establish the theory of belief research.

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Peter Berger

Teachers' Beliefs about Computers and Computer Science

A Study on Mathematics and Computer Science Teachers

The term 'computer science' corresponds to the German (and French) term 'Informatik' (informatics), which is formed as an analogue to 'mathematics', emphasizing a certain affinity between both fields. Although that slight but subtle distinction may be of some importance to our subject, being a revealing sign of immaturity in the evolutionary process of forming a new scientific discipline, for reasons of convenience we will use both terms as equivalent here.

The historical and substantial relationship of computer science and mathematics is present in the German classroom, too, as by far most computer science teachers originally are mathematics teachers who have extended their qualifications by doing in-service trainings of several years or university studies in computer science.

It should be mentioned that it is not the objective of computer science at German schools to establish the computer as a new technological medium. This objective is aimed at by the 'Informationstechnologische Grundbildung' (basic training in information technology). Here the use of computers is exemplarily taught with regard to the needs of different subjects (e.g. in a German class the layout of a newspaper, in a biology class access to databases on environmental issues, in foreign language classes communication on a world wide basis via networks etc.).

In contrast to this, computer science as a school subject has a science-propaedeutical role in the classical sense of the word and is therefore essentially taught at schools preparing for a university education – at 'Gymnasien' and to a lesser degree at comprehensive schools. In North Rhine-Westphalia, a federal state of Germany where computer science as a subject is highly promoted, there are 6190 qualified mathematics teachers and 1175 computer science teachers at 'Gymnasien', whereas at comprehensive schools the ratio is 3167 to 102.

Thus one could say that *computer science in the classroom* in Germany is mainly a matter that regards the 'Gymnasium' (grades 9 to 13), however, against a backdrop of mathematics.

1. On the study

1.1 The aim of the study

The aim of this study is to investigate the hypothesis that, by analogy with 'mathematical world views' there are also 'computer science world views' to be found among mathematics and computer science teachers and that these views are multi-dimensionally structured. The changing of those views during teaching practice, as well as the possible forming of 'informatics' views through corresponding mathematical views might be of further interest.

1.2 The methods of the study

The investigation is mainly based on qualitative methods. The empirical material derives from 30 in-depth video taped interviews (made with 28 teachers at ‘Gymnasien’ and at comprehensive schools in North Rhine-Westphalia, and a further 2 who are employed in the school administration as mathematicians and computer scientists).

The interviews could be characterized as focused (Merton & Kendall 1979), or ‘problem-zentriert’ (Witzel 1985), and according to the terminology of Mayring 1990 as open qualitative interviews with different grades of standardizing. In a preliminary study, 9 unstandardized interviews were made, analysed according to the methods ‘generalization’, ‘paraphrasing’ and ‘reduction’ described by Ballstädt, Mandl, Schnotz & Tergan 1981 and Mayring 1985.

Based upon these empirical results, a questionnaire was developed to be filled in by the remaining 21 interview partners, as a preparatory and structuring tool for the standardized interviews of the main study (the previously interviewed persons were asked to fill in the questionnaire afterwards, too).

The interviews cover the three fields *views on the computer*, *views on computer science*, and *views on the learning and teaching of computer science*.

2. Some results

As the analysing of the interviews is not yet completed, there will be mainly referred to some results based on the questionnaires filled in by the 28 teachers. It should be mentioned that the questionnaire does not serve as a data gathering for a quantitative analysis, but as a interviewing instrument within a framework of qualitative research. The precise interpretation of the data from the questionnaire will require a thorough qualitative evaluation of the interview-material.

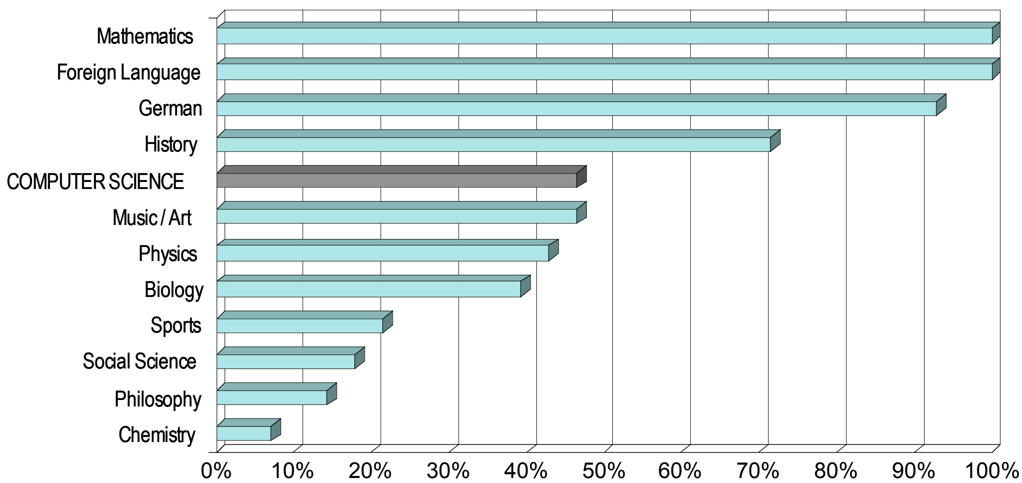

Diagram 1. The rank of computer science as a school subject.

2.1 The view on the importance of computer science as a school subject

On the one hand the majority of the interviewed teachers expressed that in future computer science ought to become a fundamental subject at schools ('Grundlagenfach'). The reason mostly given is that computer science is able to improve the pupils' skills of solving complex problems better than other subjects (even better than mathematics).

On the other hand they answered rather conservatively the hypothetical question of which six compulsory subjects they would stipulate for 'Sekundarstufe II' pupils (grade 11 to 13) in the case of a school reform. Everybody chose mathematics and a foreign language, almost everyone included German and 70% chose history, but only half of them chose computer science (Diagram 1). Computer Science, however, as music and art, is being ranked higher than science.

Although computer science teachers are for the most part highly committed to their subject (without this commitment they would not have gone to so much trouble as to study computer science while already being employed as teachers), they do not seem to overestimate the rank of their subject within the scope of the other school subjects.

2.2 The view on the main fields of computer science as a school subject

In the interviews of the preliminary study, six main fields were mentioned, namely *user software, technical computer science, algorithms, programming language, theoretical computer science* and *social aspects (computers in every day life).* According to these fields, the assessments of all 28 teachers interviewed are shown in Diagram 2.

Diagram 2. The assessments of the main fields of computer science.

Each of the teachers had strong reservations against computer science classes in the form of a mere programming course, which has been the custom in the beginning of computer science as a school subject. Keeping this in mind, it is noticeable that the field *algorithms and programming language* is still the dominant part (> 50%).

Compared with the estimation in the beginning of the interview partners' practice as computer science teachers, the importance of this field has slightly declined. But the 'hard' technical and theoretical fields still make out around three quarters of the teachers' assessments. At the same time most of them said in the interviews that they feel increasingly drawn towards the 'soft' fields *user software* and *social aspects*.

2.3 The view on the central concepts of computer science

According to the results of the preliminary study, three concepts were in general considered to form the central ideas of computer science, namely *computer*, *programming language* and *algorithm*. The results to the question of just 'how central' the teachers regard these concepts are given in Diagram 3. The interview partners were asked to specify their assessments by dividing 100 points between the three items, with regard to both their present position and their position at the beginning of their careers as computer science teachers.

Diagram 3. The view on the central concepts of computer science.

The answers are represented by barycentrical coordinates, with the points in bold face depicting the present assessments, the lines the changes, and the 'needle points' representing the former positions. Person A, for example, originally considered only *computer* and *programming language* as being of (equal) importance, and today he considers all three items as being of equal importance: the point in bold face is situated exactly in the centre of the triangle.

In general, a distinct orientation towards *algorithm* can be observed, and except for person g, there is no change towards *programming language*. A closer analysis of the interviews reveals that the reasons for this development can be described by the following two models:

Model A: Concentration on didactic essentials.

In the course of the time the person step by step enlarged his professional qualification as a computer science teacher. While in the beginning he had to occupy himself with the basic aspects of computers and programming, he gradually obtained a more general theoretical view, that made him realize the algorithm concept to be of fundamental importance and a pre-eminent factor especially with regard to general education.

Model B: Personal retreat from the 'race for innovations' – withdrawal into 'lasting values'.

This person, too, started by learning the innovative skills in computer programming. He shared his new knowledge with his pupils. However, over the years fast technological advance took place and this knowledge became outdated. It often happened that pupils were more informed than the teacher. Taking a dislike to going the trouble of keeping up-to-date, this person more and more resorted to the 'lasting values' of his subject.

2.4 The view on pupils' solutions

To the question of how they wanted their pupils to solve a programming-task, the teachers interviewed in the preliminary study gave a range of answers, that can be summarized into two main aspects.

Diagram 4. The view on the form of pupils' solutions.

With regard to the form the pupils' solutions could be a *program listing*, a *program demonstration* at the computer, or an *informal description* of an algorithm. These three forms of presenting a solution to a programming-task correspond with the three central concepts *programming language* (program listing), *computer* (program demonstration), and *algorithm* (informal description). So here again, from the teacher's reactions in the preliminary study there arose a trivalent question, and it is interesting to see how the interview partners in all rate these factors (Diagram 4).

A first glance reveals a picture similar to Diagram 3. The tendency of changes also points to the *algorithm* aspect (here*: informal description*). There are, however, considerable differences to be noticed with regard to certain individuals, for example persons H and M.

Person H almost exclusively emphasizes the factor *algorithm* (Diagram 3), whereas he essentially requires *program listings* as pupils' solutions (Diagram 4), emphasizing here the *programming language* aspect (similar to Person M).

With regard to the desired substance of the pupils' solutions, the statements of the teachers interviewed can be summarized into the following aspects: the solutions should show *creative ideas*, the mastering of *standard techniques*, and the *reflections of obstacles* and difficulties encountered in the solution process.

Diagram 5. The view on the substance of pupils' solutions.

As Diagram 5 shows, the field of the answers displays a coherent placement between *creative ideas* and *standard techniques*. Changes do not seem to be of significance, and the factor *reflection of obstacles* seems to be of remarkably small importance.

2.5 The teachers' self-assessments as computer-users

It was noticeable during the preliminary study that the interview partners for the most part did not bring up their personal attitudes towards computers of their own accord. On inquiry multi-faceted and partly contradictory statements were given, which have been shaped to three 'typical statements' forming another question of the questionnaire. (While the former trivalent questions immediately arose from the empirical material, this question resulted from construction on the basis of this material.) These statements were:

- 'I like to work with a computer. I am able to use it creatively. It saves time and work, and where it does not, I enjoy doing the work with a computer in the same time that it would take me without it – but with a much better result.'
- 'I regard the computer as a necessary (and sometimes troublesome) evil. It is relevant, and as a teacher I therefore feel it important to concern myself with computers. I regard their role with some scepticism.'
- 'I think the computer is a means to an end. I do work with it that I previously have done without it. Sometimes it bothers me when I realize that the computer costs more time than it saves.'

According to Diagram 6 the teachers interviewed are open-minded with regard to computers, with a change tendency of higher rank given to their practical value.

Diagram 6. The teachers' self-assessments as computer-users.

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Stefan Grigutsch

On Pupils' Self-Concepts as Learners of Mathematics

Developments, Reciprocal Effects and Factors of Influence in the Estimation of Pleasure, Diligence and Achievements

In May and June 1994, 1650 pupils in 20 secondary schools (Gymnasium) in a local governmental district around Düsseldorf (Regierungsbezirk Düsseldorf) have filled in a closed questionnaire about their view on mathematics. In each of the 20 schools, one grade 6, one grade 9, one basic course (3 hours weekly) and one high-performance course (6 hours) in grade 12 have been explored. The results about the view on mathematics have been reported earlier (see "Beiträge zum Mathematikunterricht" 1995), and is the central topic of a doctoral dissertation.

The questionnaire also contained three items which globally registered isolated aspects of the self-concept as a learner of mathematics: the pleasure associated with mathematics education, the estimation of achievements and the estimation of diligence. Furthermore, the pupils gave their mark in mathematics (1 = very good, 6 = very bad).

1. About the term "self-concept as a learner of mathematics"

The basic term is "mathematical world view". This is a structure of attitudes which contains a wide spectrum of beliefs (cognitions), emotions and intentions of action (conations) concerning mathematics. There are four main components: attitudes towards (i) mathematics (view on mathematics), (ii) mathematics teaching, (iii) mathematics learning and (iv) oneself as a learner and user of mathematics (self-concept).

The self-concept as a learner of mathematics is a structure of attitudes and a substructure of the mathematical world view. This self-concept consists of the subjective knowledge (beliefs, cognitions), the emotions and the intentions of action about oneself related to mathematics and mathematics education. The most important elements are the subjective knowledge and the emotions concerning

- the interest in mathematics and the interests (aims, motives) with mathematics,
- the motivation and the pleasure associated with mathematics as well as their reasons,
- the efficiency in mathematics, the strength and the weak themes and topics,
- the causal attributions for ones success and failure.

The items "mark", "estimation of mark/achievements", "estimation of diligence" and "pleasure" only measure two elements of the self-concept, that is the pleasure associated with mathematics and the estimation of one's efficiency. Above all, the measurement is very global and only in single, isolated items.

2. The attitudes in the items of the self-concept

In grade 6, most of the pupils (60%) like mathematics education well. They estimate their diligence mediocre (average 2,9) and are in correspondence with the mark of the teacher (3,0). They judge their achievements relatively good, with an average of about 2,7 , higher than the teacher and higher than their diligence. Thus, the pupils in grade 6 have a positive view on themselves in their relation to mathematics.

In grade 9, the opinions about the pleasure in mathematics education are split (distribution bimodal). There is a group of pupils, who dislike math education, and a larger group who like it. The mark tends to be under-average (3,1), what might have negative effects on the self-concept. The pupils judge their achievements with an average of 2,9 , higher than the teacher and therefore tendencielly over-average. They estimate their diligence rather low (3,2), especially in comparison to their achievements.

In the basic course, there is a group of pupils which derive pleasure from math education, and a larger group without pleasure (bimodality). The pupils obtain marks that are slightly under-average (3,2), what might have negative effects on their self-concept. They judge their achievements higher than the teacher, but mediocre (3,0). In comparison to this, they take themselves to be rather ”idle” (3,5).

In the high-performance course, a great part (70%) of the pupils likes math education. They obtain slightly upper-average marks (2,9), what might have positive effects on their self-judgement. Indeed, the pupils estimate their achievements even higher (2,6), while they estimate their diligence rather low (3,3).

3. The development in the items of the self-concept

Mark: In grade 6, the pupils obtain an average mark (3,0). In grade 9, their achievements are judged a little worse (3,1). In the basic course, the mark (3,2) remains on a similar level as in grade 9, while in the high-performance course, the pupils receive a better mark (2,9) and reach the level of grade 6 again.

Estimated Mark: The development in the estimation of the achievements follows the development in the marks, but lies on a higher level. Pupils in grade 6 judge their abilities and achievements very positively with an average of 2,7. The self-judgement gets worse to grade 9 (on the mark 2,9) and remains in the basic course on a similar, mediocre and relatively bad level (3,0). On the contrary, the pupils in the high-performance course estimate their achievements better (2,6) and judge themselves similarly above-mediocre and good as in grade 6.

Estimated Diligence: Pupils in grade 6 think that they have an average diligence (2,9). In contrast to this, older pupils take themselves to be rather ”idle”. The diligence of the pupils decreases continually – in their own estimation – over grade 9 (3,2) to the basic course in grade 12 to the relatively low value of 3,5. In the high-performance course in grade 12, the diligence remains about the level of grade 9, but with an average of 3,3 it is rather bad, too.

Pleasure: The pleasure derived from math education changes very clearly in the course of the schooltime. In grade 6, the pleasure factor is still relatively high – the average lies between pleasure and indifference. To grade 9, the pleasure decreases and lies in the area of indifference. The pleasure decreases further to the basic course, while in the high-performance course, it increases again to the area of pleasure, higher than the value of grade 6.

All developments of the pleasure, mark, estimation of mark and estimation of diligence have the same direction: From grade 6 to grade 9, the self-concept in these attributes gets worse, and from grade 9 to the basic course it remains on this bad level or even decreases further. In the high-performance course, there is an improvement at least on the level of grade 6, or in the estimation of diligence no further change for the worse (in contrast to the basic course). If one takes into consideration that the estimation of diligence is discoupled (untied) from the other variables from grade 9 onwards, and that therefore it delivers a smaller contribution to the character of the self-concept, one can resume the following: At the end of their schooltime, only the pupils in the high-performance course have a positive self-concept. For the pupils in the basic course, the development of the self-concept is characterized by a process of decrease and change for the worse. The pupils decreasingly experience less pleasure, they recognize less and less commitment and diligence, and they attest themselves of having less knowledge and abilities; they see their own person in a less and less close position toward mathematics. Therefore, the development in the basic course can be interpreted as a dissociation from mathematics.

Normally, there are mechanisms which inhibit a change of the self-concept; self-concepts tend to be constant over longer periods of time. Thus, it is very important that developments could have been observed. The pupils must have made weighty and incisive experiences over longer periods of time.

4. The relations between the estimations of pleasure, diligence and achievements and the marks

The starting-hypothesis about the relations between the elements of the self-concept can be represented by a process in a circle: The pleasure in math education influences the diligence and the commitment, which have effects on the achievements in mathematics. Vice versa, good resp. bad achievements have effects via a confirmation (encouragement) resp. discouragement on the pleasure. This mechanism forms a circle, that generates itself and cumulatively increases. It appears to be a very simple and naive theory, which describes and explains how achievements and motivation arise and become stabilized. But it can very often be found in the thinking of pupils and teachers, especially to explain decreasing motivation and achievements.

According to the results of this inquiry, the starting-hypothesis can not be kept. Ultimately, this model describes the reciprocal effects between pleasure and achievements. If we only regard the relation between pleasure and marks without controlling the influence of the other variables, then the results show, that pleasure and marks are closely connected. On one hand, pleasure has an influence on the marks, on the other hand, marks re-influence back upon the pleasure. If this reciprocal effect is described in detail – as in the model of the starting-hypothesis – using the variables diligence resp. estimation of diligence and estimation of achievements, then the results show, that there is no direct reciprocal effect between pleasure and achievements.

Concerning to this model, it could not be proved that

(1) the pleasure with math education has an effect via the estimation of diligence upon the mark as a measure of achievements. Instead, the estimation of diligence rather contributes nothing to the effect of the pleasure upon the achievements, so it rather contributes nothing to describe and explain the arise of mathematical achievements. In particular, the estimation of diligence is discoupled (untied) from pleasure and achievements at the latest from grade 9 onwards. – The assumption that the effect of the pleasure upon the marks is mediated by the diligence resp. the estimation of diligence, must be refused. There possibly are many factors, which are influenced by pleasure and which cause achievements.

(2) the marks have an immediate (direct) effect upon the pleasure in math education. Instead, the effect is mediated by the estimation of achievements. According to the inquiry, this estimation of achievements takes a central position in the self-concept and can probably be interpreted as ”self-satisfaction” or ”confirmation”. – The assumption that the effect of the mathematical achievements upon the pleasure are mediated by a variable ”self-confirmation” or ”self-satisfaction” can not be refused.

In addition to (2). The relations that have been observed can appropriately be described and explained in the following model: The process of effect of the achievements upon the pleasure is not direct. It consists of partial processes which mediate this effect.

First, the achievements, which are as the mark part of the teacher’s pupil-concept or which are as the objective achievements part of the pupil’s environmental world, are integrated in the self-concept of the pupil. Second, a transition (passage) from the factual and objective sphere of achievements and judgement into the affective sphere of pleasure takes place.

This double ”passage of borders” lets it seem plausible that there are several partial processes which participate in the process of effect of the achievements upon the pleasure. One partial process surely is the effect of the achievements upon the self-judgement and its effect upon the pleasure. Seen as a whole, the achievements resp. the marks start and involve judging and emotional processes which can be described as confirmation (encouragement) resp. as discouragement.

After the results of the inquiry and in the framework of the measured variables, the estimation of achievements takes in the central position in pupils’ self-concepts. It mediates in the process of effect of the achievements upon the pleasure and thus lies at the cutting-point resp. the diffuse domain of borders between the factual and objective sphere of achievements and the affective sphere of pleasure. On one hand, the objective achievements resp. the objective and factual marks of the teacher have an essential influence on the estimation of achievements, on the other hand the emotional pleasure has an influence. Therefore, the estimation of achievements contains both an objective and factual as well as an affective component. In my opinion, it can be well interpreted as ”pupils’ self-satisfaction with their achievements”.

5. The influence of gender on the self-concept

There are gender-specific self-concepts as learners of mathematics. Boys and girls have different though not opposite self-concepts. Boys in all ages feel a higher pleasure and estimate their achievements in mathematics higher than girls. Thus, boys have a better concept (image) of themselves and their achievements and a higher self-satisfaction. On the contrary, girls from grade 9 onwards estimate their diligence higher than boys, and this difference still increases to grade 12. The gender-specific difference in the attribution of diligence probably develops only in the secondary school mathematical socialization.

Boys and girls have gender-specific self-concepts, especially in the estimation of their mathematical achievements, although there is no objective reason. Because boys and girls get (in grade 6, grade 9 and basic course) similar marks and therefore probably have similar mathematical achievements. Obviously, they have gender-specific different mechanisms (or patterns) of interpretation, which cause, that objectively similar mathematical achievements are subjectively different interpreted and then integrated in the self-concept. These different patterns of interpretation already exist in grade 6, so they are probably internalized in earlier socialization.

6. The relations between the view on mathematics and the self-concept

The pupils' view on mathematics contains – according to the results of my inquiry – the dimensions algorithm-, rigid algorithm-, process- (problem-) and formalism-orientation as well as an estimation of the usefulness of mathematics.

Certain views on mathematics are connected with certain self-concepts and certain achievements. A (higher) algorithm- and rigid algorithm-orientation rather corresponds with a low pleasure factor, bad mathematical achievements and a low estimation of achievements, while a process orientation and a conviction in the usefulness of mathematics are rather connected with high pleasure, good mathematical achievements and high self-judgements. The formalism-orientation has in principle no relations to the self-concept. The estimation of diligence in the self-concept is largely independent of the view on mathematics, too.

From the viewpoint of the self-concept, we can resume the results with the following statements: Objectively or subjectively bad pupils resp. pupils without pleasure are

- more algorithm-orientated,
- more orientated in rigid algorithms,
- less process-orientated and
- less convinced of the usefulness of mathematics

as objectively or subjectively good pupils resp. pupils with pleasure in mathematics education.

An algorithm-orientated view on mathematics corresponds with a negative self-concept and bad mathematical achievements, while a processual and application-orientated view on mathematics rather is connected with a positive self-concept and good mathematical achievements.

The results that there are significant and systematic relations between the view on mathematics on the one hand and the mark and the self-concept on the other hand, is important for the theory of attitudes. It supports the assumption that attitudes have a relevance for action. These effect – in both directions – between the view on mathematics and the self-concept can hardly be explained with internal cognitive processes. It is more plausible that the view on mathematics influences the behaviour of learning and doing mathematics, and the experiences influence the success in math education, the self-concept and the self-satisfaction, or vice versa, that the self-concept influences the behaviour in math education and the experiences influence the view on mathematics.

The relations between the view on mathematics and the self-concept make it possible to set up founded hypotheses about an evaluation of views on mathematics as positive or negative. This evaluation does not depend on superior and global aims (e.g. preparation for science), conceptions and schools in the didactics of mathematics education, pedagogical aims (e.g. emancipation), which are often value-bounded. In contrast to this, the aim to improve pupils' mathematical achievements and to turn their self-concept to a positive one is very "natural" and evident, largely free from values and free from superior pedagogical and didactical aims.

If mathematical education has the aim to support pupils' mathematical achievements and to turn their self-concepts to a positive one, then the results allow to make the following hypotheses:

- It is relevant, what view on mathematics is mediated to pupils.
- A view on mathematics, which strongly emphasizes the algorithmic aspect of mathematics and which neglects (or emphasizes too weak) the process- and application-orientated character of mathematics, should be evaluated as negative. It should not be mediated in math education.
- A view on mathematics, which doesn't emphasize the algorithmic aspect too strong, but which emphasizes the character of mathematics as an application-orientated process of understanding and finding, should be evaluated as positive. This view on mathematics should be mediated in math education.

Dietlinde Gruß

How Do Adults Experience their Mathematical Engagement?

Adults' beliefs about mathematics in daily life are investigated in a case study. The method of the study is based on ethnomethodology. For this purpose an approach as "naturalistic" as possible has been selected. In the sense of Glaser/Strauß 1967 the aim is to discover "a grounded theory" on the base of empirical data in form of qualitative interviews, i.e. to explore hypotheses and to get an idea about different types of people's beliefs. The evaluation is based on interviews with 16 persons of different profession and education, different sex and age. There is only few literature about adults' beliefs about mathematics in daily life, in particular the literature of Sewell 1981, Carraher & Schliemann 1985 and Lave 1988.

In this case study adults' beliefs about mathematics are divided into two parts. The subject of the first part is the approach of an everyday situation which contains mathematics. The second part associates the meaning of mathematics within different areas.

As part of the whole study in this article the following aspect is presented: "How do adults experience their mathematical engagement?" Excerpts from two interviews are analysed exemplarily. Different possibilities of interpretations for the used metaphors to do mathematics are introduced. At the same time the approach to the evaluation of the interviews can be seen.

Mathematical engagement as "digging through"

The first excerpt is taken from an interview with A., who studies theology to become a social worker. Here she answers to the question: "When and where do you use mathematics in your profession?" A. mentions that among others she needs mathematics to interpret statistical diagrams.

> "...Wenn, setz ich mich meist <u>mit Mitstudenten</u> dran, ne? Ich merk, ich find das <u>sehr sehr schwierig</u> so was so anzugehn, so ne? Da ist vielleicht auch immer noch von der Schule drin, es einfach <u>nicht gern</u> zu machen, <u>Mathematik</u>. Also, das kann ich für mich schon so sagen, daß ich so dann denke: <u>Oh, mein Gott</u>, das ist so <u>trocken</u>, denk ich dann immer, sich so <u>da durch zu buddeln</u>. Das ist alles so allgemein. Dann denk ich immer, was sagt mir jetzt die Statistik für diesen <u>Einzelfall</u>? Klar, ist sie wichtig. Dann hat man immer so das Gefühl, der Einzelfall, den man jetzt so vor Augen hat, daß der jetzt gar <u>nicht</u> in die Statistik <u>reinpaßt</u>. (A. lacht) (I: hm) Das <u>klafft</u> manchmal so n bißchen <u>auseinander</u>."

(A., 24 J., w, Theologiestudentin)

English translation:

”... If so, I usually set to work with fellow students, right? I notice, I find it very very difficult to approach it in such a way, right? It is probably still caused by school that I simply don’t like to do it, mathematics. Therefore, as for me I can say, that I think then: Oh my God, it is so dry**,** I always think dig through it. All this is so general. Then I ask myself, what message does this statistical diagram have for me in this isolated case? Of cause, it is important. Then one always gets the impression that the isolated case which one now has in mind, that this now does not fit into the statistical diagram. (A. laughs) (I: hm) This is sometimes a little bit contradictory.”

(A., 24 years, female, student of theology)

Starting with the first sentence it becomes evident that A. prefers teamwork. She assesses her personal procedure, which is ”very very difficult”. Her words ”to approach it in such a way” can show a barrier, which she has to overcome. She assumes that the reason for her negative beliefs goes back to school-times as she ”really doesn’t like to do it”. ”Simply” in this context may indicate that this is an experience for which she can not give a direct explanation. Separating ”mathematics”, thus, isolating it in the interview, could express her emotional distance to it. She attributes mathematics as ”dry”, i.e. bloodless and not interesting.

As a metaphor for her mathematical engagement A. uses the words ”to dig through”. This metaphor is now more closely investigated. Keeping with the metaphor, ”digging” is a physically exhausting act. With the hands or with some tools sand or earth is moved. When digging, time plays an important role. It has to be fast. It is a monotonous movement without big reflections – if we do not think of children who usually have fun while digging in a sand-pit. If we extend ”digging” to ”digging through”, this term is used e.g. in the context of tunnel construction. A hole is shovelled over a long distance. The act is performed in darkness, the objective is invisible. It is only known that it exists. Only the objective counts which has to be reached. The way towards it is troublesome, but without a value of its own.

This picture can now be transferred to mathematical engagement. Mathematical engagement is very exhausting. It requires the whole human effort. One assumes or knows that there is a solution. The solution itself is unknown. The steps of the solution are intricate. After the solution has been found, the steps which led to it often become unimportant. This can be a signal that the results are significantly more important than the solution strategies. As A. puts it in another part of the interview: ”I am not interested in the way but only in the results.” While it can be assumed that A. has discovered this distinction by herself, it is expressed in mathematical education with exactly the same words.

While A. is digging through she seems to reflect the sense of her engagement: ”Oh my God”. Her conclusion is, ”this is all so general”. In a statistic she cannot find the human aspect or life itself. As a future social worker she has to deal with individual human problems. Therefore, statistical diagrams based on a large number of cases and containing mean values seem to be useless for her work with single persons.

In another excerpt A. uses a metaphor for mathematical engagement, namely ”getting down to something”, which, like ”digging through”, also originates from the area of hard manual labour. In keeping with the metaphor, ”getting down to something” is a kind of physical work where one gets dirty. One submerges beneath the ground in order to transport something to daylight. There is something which cannot be accessed yet, but which one wants to grasp. ”Getting down to something” and ”digging through” require endurance, sweat, and accepting disappointment. Both metaphors are idioms (in German), which are of importance also in other areas of life. Nevertheless it becomes visible how mathematical engagement is perceived.

Learning mathematics as ”hammering into the head”

The following is an excerpt from an interview with B, who is a clerk. He answers to the question: ”When and where are you using mathematics outside the scope of your profession?” According to Schütz & Luckmann 1975 daily life is separated into situations. Therefore, a number of cards are presented with variables like apartment, financial matters, games, homework to propose possible situations. B. gives his opinion on the card ”homework”.

> ”...aber ich meine, bei den <u>Schularbeiten</u> eigentlich dies <u>Rechnen</u> sich mehr darauf bezieht, <u>was brauch ich</u> für die <u>Aufgabe</u> (I: mhm), was brauch ich für, für dies und dies, daß da wohl das im Vordergrund steht (I: hm) und daß dabei bei den Schularbeiten gerade bei den <u>Rechenfächern</u>, daß da die <u>Mathematik</u> vornean steht (I: ja), das sagt sich <u>von alleine</u>. Das ist ja <u>Grundlage</u>, wenn man es da <u>richtig in den Kopf reinhämmert</u>, kommt es einem <u>später</u> bei <u>all diesen Sachen</u>, wo wir eben drüber gesprochen haben, <u>zugute</u>.”
>
> *(B., 56 J., m, Bankkaufmann)*
>
> English translation:
>
> ”... but I think when it comes to <u>homework</u> these <u>calculations</u> relate more to <u>what I need</u> for the <u>exercise</u> (I: mhm), what I need for, for this and this, this is obviously in the foreground (I: hm) and for homework particularly <u>lessons about calculations</u>, there <u>mathematics</u> is coming first, this is a <u>matter of fact</u>. This is the <u>basis</u>, if you <u>really</u> <u>hammer it into the head</u>, you <u>later</u> have a <u>benefit</u> for <u>all these things</u> we have just talked about.”
>
> *(B., 56 years, male, clerk)*

When B. comes to homework he associates calculations with it, not in the narrow sense of solving exercises with numbers, but in the sense of logical planning to accomplish a task. One has to meet special requirements – ”what I need for the exercise” – to begin with the task. The planning may be related to expenditures of time and effort, e.g. to the time one needs to solve the exercise. Mathematical aspects are incorporated, because they help structuring the solving process. On the other hand in ”lessons about calculations” mathematics stands in the foreground in a narrow sense. It seems that B. uses the words ”calculations” and ”mathematics” as synonyms: ”lessons about calculations, there mathematics is coming first”. ”This is a matter of fact” for him, which he does not question. In this phrase the question remains why B. pluralizes ”lessons about calculations”. He may have all subjects in mind which deal with numbers, e.g. economics, statistics. This sentence contains an esteem for mathematics: ”this is the basis”, which is useful in many situations in life after school-time.

B. uses ”really hammering it into the head” as a metaphor in order to describe how a mathematical process of learning looks like and how it should look like according to his opinion. The metaphor derives from the same area like ”digging through” and ”getting down to something”, namely hard physical labour. The metaphor describes a mechanical work process. Something is processed using repeating monotonous strokes. This process is perspiratous and troublesome for the one who is hammering. Tools are required and wood and stone are used as materials.

Now the metaphor is transferred to learning mathematics. Tools for the teacher or the student at home are school-books and hand-books. The word ”really” implies a thorough process. The head of the pupil has to keep still and to passively endure the exertion of power. ”Hammering in” is related to pain for the student. Activities of the scholar constitute a disturbance and hinder one’s work. This hard process of learning mathematics happens in school-time. The profit will come in later days in life.

Comparison of the metaphors

”Really hammering it into the head” is a passive or reflexive process. It happens something to the head of the scholar. In contrast to this, ”digging through” and ”getting down to something” are active processes. They require the whole human effort. The pictures are obviously related to different contents and activities. ”Really hammering it into the head” has reference to skills, which are learned in practice. They can be strengthened by repetition. On the other hand ”digging through” and ”getting down to something” are more related to problem solving. A larger task is to be solved. The way to the solution is not necessarily known.

Are these metaphors desirable from an educational position? Is doing mathematics only labour? If one attributes labour as a necessary precondition to a successful solution strategy, one might say, the way to a solution is indeed difficult and troublesome. It requires full concentration and one has to go down to the last detail.

For the currently propagated constructivist view on learning mathematics in the mathematical education B.’s metaphor ”really hammering it into the head” is appalling. There is no space for own thoughts or for building up own knowledge. Yet in a retrospective view B. seems to evaluate his process of learning mathematics rather positive. This corresponds to the normal colloquial usage of the metaphor in his school-days.

Leaving the reflection of a desirable or non desirable metaphor it can generally be observed that persons had thorough experiences in learning mathematics in school-days. They are still present. The cognitive process of learning mathematics is anchored in their consciousness.

The above mentioned excerpts are only limited parts of two interviews. Many excerpts of interviews are interpreted in the same way. Persons tell about teamwork and why they prefer it. It is interesting that they do not mention social advantages of teamwork. It seems that they use teamwork to compensate a deficiency in solving a mathematical problem alone. Other persons describe how exciting doing mathematics is for them with words like ”fascinating” or ”being independent”. In other excerpts they express with ”to run away” how they avoid doing mathematics or how they try to disguise their incapability. Taking the whole case study into account and interpreting all the interviews about doing mathematics one can answer the question of how adults experience their own mathematical engagements in the following way:

- With every mathematical content people learn an affective content.
- Doing mathematics is always connected with emotions.
- Persons say, that the basis of these emotions originates back in school-time.
- Mathematical engagement is a continuation or a consequence of school-time.

If adults talk about their experience with mathematical engagement they affectively evaluate it with phrases like ”I always liked to do it” or ”I hate to do it”. In the latter case they mention a concrete situation in the past, where something terrible happened they still remember in all details. But it cannot be analysed whether this situation plays the only key role. This situation is mostly described in the context of school-time.

In the linguistic use of Bauersfeld 1983 this means: The cognitive ”microworld” doing mathematics is related to an affective one and through this it is in relation to many other ”microworlds”, e.g. physical labour or use for the profession.

This is an observation which is valid for all areas of the interview, e.g. in answers to questions like ”What do you associate with the word mathematics?” or ”What is the use of mathematics in daily life?” Related to school this means that during mathematical lessons scholars learn in cognitive processes the whole context, i.e. the content, social aspects, the educational approach, evaluation and characteristics of the teacher.

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Sinikka Lindgren

Is it Possible to Attain Change in Pre-Service Teachers' Beliefs about Mathematics?

Theoretical Framework

Presumably we all admit that change is necessary but, paradoxically, most difficult to attain in education. There seems to be a magic teaching-learning-teaching cycle: teachers tend to teach as they were taught and expect students to learn as they learned. Thus the student teachers' own experiences as learners are re-enacted in their own teaching practices in class. (Stoddart et al. 1993, 238.) Alba Thompson (1991) has done an investigation for developing a hypothetical model for the development of teachers' conceptions of mathematics teaching. The framework consists of three levels in the progress of development.

At *Level 0* the instructional practice focuses on facts, rules, formulas, and procedures. The role of the teacher is perceived as a demonstrator, and the students' role is to imitate the demonstrated procedures and to practice them diligently. Obtaining accurate answers is viewed as the goal of mathematics instruction. The assessment of getting the right answers – usually with using prescribed techniques – to ”story problems”. At *Level 1* the conception of mathematics is broadened to include an understanding of the principles ”behind the rules”. Rules, however, still play a basic part in mathematics. The role of the teacher is perceived somewhat as in Level 0. Views on the role of the students are broadened to include some understanding. Problem solving is regarded as important in the mathematics curriculum, but it is viewed as a separate strand form the ”traditional content”. At *Level 2* the conception of how mathematics should be taught is characterized by a view that the student himself must engage in mathematical investigation. The role of the teacher is to steer the students' thinking in mathematically productive ways. The students are given opportunities to express their ideas and the teacher assesses their reasoning. (Thompson 1991, 9-13.) Thus the hypothetical model by Thompson suggests a hierarchy for the development of teachers' conceptions of mathematics teaching.

Paul Ernest distinguished three conception of the nature of mathematics. These psychological systems of beliefs can also be seen to form a hierarchy: 1) There is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules, and skills. Thus mathematics is a set of unrelated but utilitarian rules and facts (the instrumentalist view). 2) There is a view on mathematics as a static and unified body of knowledge, a crystalline realm of interconnecting structures and truths. Thus mathematics is discovered, not created. (the Platonic view). 3) There is a dynamic, problem-driven view on mathematics as a continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. Mathematics is not a finished product, for its results remain open to revision (the problem-solving view). (Ernest 1989, 250.) The following is the author's illustration of Ernest's view on the nature of mathematics (Figure 1).

Figure 1. Ernest's view on the nature of mathematics.

From the viewpoint of change, the theory of Thomas Green is also notable. He gives a metaphor in which belief systems are formed as a structure of concentric circles. In the core of the circle are those beliefs which are held with greatest psychological strength, those we hold most dearly and which are most difficult to change. Moving from circle to circle toward the perimeter, we find those beliefs we hold with less strength and which, therefore, are easier to change. Green explains man's incredible ability to hold fast to beliefs that are inconsistent with the third feature of this metaphor: beliefs are held in clusters, with shields that protect them. (Green 1978, 44-48.)

Also Theo Wubbels is concerned of the possibility to attain change in teachers' preconceptions. He refers to these conceptions as "world images". Building on Watzlawick's work he speaks of the two hemispheres of the brain and the two types of languages that apparently correspond to these hemispheres. The language that is processed in the left hemisphere is that of definition, the logical, and analytical language of reason, science and explanation. The right hemisphere language is the language of imagery, metaphor, and symbols. It represents synthesis and totality. According to Watzlawick it seems plausible from the synthesis of experience that the world image is mainly the function of the right hemisphere of the brain. Thus we have to use right hemisphere language in order to reach these world images. (Wubbels 1992, 143.)

Methodology

Subjects and instruments

In my research I want to follow the development of the pre-service teachers' beliefs and conceptions about mathematics. The main target group is 72 prospective elementary school teachers who began their study at the University of Tampere in the fall of 1993. They were given a Likert-type questionnaire in the beginning and in the end of their first year of university studies. Also 33 third or fourth year students, specializing in mathematics or early childhood education, took part in the first part of the study. The questionnaire included 56 items concerning the students' beliefs and conceptions about mathematics and teaching mathematics. On the basis of the results and a math exam, 12 first year students were selected for closer follow-up. They have been interviewed, and lessons, including their teaching practicum, have been videotaped. As this study has a longitudinal research design, the process still continues.

Procedure and Data Analyses

From a factoral analysis of the answers of the whole target group (105), and in accordance with the theory of Thompson, two new variables, Level 0 and Level 2, were formed. *Level 0* which I also call *rules & routines* method, was the mean of items including the importance of teaching facts, rules, and statements, the extensive practice of mathematics in class, the importance of getting there right answers, and the importance of mastering basic calculation, as well as good order in class. *Level 2*, which I also call *open-approach* method, was the mean of items stressing the importance of the students' understanding that the same result can be achieved in different ways, of formulating problems by themselves, and of the use of concrete material, and the variety of strategies for solving problems. Variable Level 2 also includes that the teacher should emphasize the importance of mathematical thinking and take into account the strategies the student uses.

Results

In the factor analyses two types of factors were clearly discernible: those which deal with the teaching of mathematics and those which deal with mathematics as a subject. The latter correspond closely to the views of Ernest which are illustrated above. When the distribution of Level 0 over Level 2 is studied of the whole data, no correlation can be seen. The scatterplod diagram is presented below (Diagram 2). The scale in the answering the items ranged from 1 to 5, where 5 corresponds to ”I fully agree”.

It can be concluded that those who got high scores on Level 2 do not clearly reject the status of right answers, and the teaching of mathematical knowledge, facts, and rules. When the diagram above is assessed from the view on Thompson's theory, one can see that there are no groups of teachers who stand at a certain hierarchical level. The author has gathered data using the same questionnaire for 101 elementary school teachers in Tokyo, 1991. Also this data gives no support to the theory of Thompson (Lindgren 1994).

Diagram 2.

The time between the two measurings for first year students was only well over seven months. However, some very interesting results were observed in relation to change. In the diagram above the data from the second measuring was used for the first year students. In the first measuring the correlation between Levels 0 and 2 in the data was negative and statistically significant (r = -40), which means that when entering the university there was some kind of grouping for those who agree on Level 0 and for those on Level 2. The correlation of the data of the second measurement was close to zero ant .06. This clearly means that while entering teacher training, the prospective teachers stand either for or against the open-approach method, or the rules & routines method. After their freshman year studies they do not know what to believe. Or rather, they have learned that a modern approach to teaching imperatively includes the activation of students and taking into account their proposals and own strategies in problem solving and other tasks. On the other hand they have not changed their beliefs of the effective outcome of the rules & routines method. A development in the direction of the theory of Thompson can not be seen.

Conclusion

These results support Green's theory of the possibility of the simultaneous existence of conflicting belief systems. In order to attain change a quite radical restructuring of teachers' conceptions is needed. Wubbels (1992, 143) refers to the theory of Watzlawick in psychotherapy that the students' preconceptions can be considered as ”world images” which cannot very well be influenced by logical reasoning. Wubbels suggests the use of ”right hemisphere strategies” in order to reach these world images. In addition to using figurative language patterns, and the reflecting of emotions rather than cognitions, he suggests for teacher educators the use of direct behaviour prescriptions. The student teacher needs to experience that the things will work ”the other way”.

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Christoph Oster

The Problem of Coaching Mathematics as Seen by Pupils and Active Teachers Involved

The problem of coaching mathematics

Private coaching is an international educational phenomenon which has been recognized for decades. It is not limited to certain kinds of schools or social sections of the population. There have been scientific studies of it in Germany from the mid-fifties onwards. In his book ”Nachhilfe – Erhebungen in einer Grauzone pädagogischer Alltagsrealität”, Darmstadt 1990, M. Behr describes global aspects of coaching with empirical methods and compares his results with older studies. The studies have shown that in Germany nearly every second pupil needs some coaching during his time at school. In the last few years more and more pupils of the Grundschule, attaining grades one to four, have private lessons.

In Germany (Verband Bildung und Erziehung in WAZ, 6.9.1995) the problem is called a ”pädagogische Privat-Industrie”, engl.: private educational industry, connected with some ”Nachhilfe-Boom”, engl.: increase in private coaching, in Germany at present. Hurrelmann (Hurrelmann, Kloke; University of Bielefeld 1995) has conducted own studies and he is more critical. He acknowledged that coaching is an educational, social and political problem. Quotation: ”In einem demokratischen Staat darf nicht die Dicke des Portemonnaies von Vater und Mutter den ‘feinen Unterschied’ machen, wenn ein Jugendlicher in einer schulischen Leistungkrise steckt. (...) Das öffentliche Schulsystem (...) darf der Etablierung eines politisch unkontrollierten privaten ‘Bildungs-Reparaturdienstes’ nicht länger Vorschub leisten.” He calls the money parents have to pay ”heimliches Schulgeld”, engl.: hidden school fees. It is possible that 30 million DM is paid out for private lessons each week in the older German federal states. In public the problem is increasingly seen to be important and discussed. There are articles in magazines like ”Stern” or ”Der Spiegel” and there are more often advertisements for established coaching institutes in newspapers.

All empirical studies prove that ”Mathematik” and ”Englisch” are the most common school subjects for coaching in Germany. Nearly 30% of all private lessons are lessons in mathematics. Therefore it seems necessary to conduct more research to describe and understand the global concepts of coaching mathematics more precisely. One method is to do case-studies using questionnaires and interviews. These are the subjects of my recently begun studies.

How coaching (‘Nachhilfe’) is done in Germany

Coaching means taking private lessons in addition to school lessons. There are two main kinds of private lessons:

1. Private lessons given by friends, older pupils or student teachers and also by qualified teachers although this is quite rare: This lessons are very individual and consist normally of one-on-one tuition which takes place in private flats.
2. Private lessons organized by private coaching institutes: These lessons take place in rooms at these institutes and consist mainly of small groups of pupils .

I think it is important to see that in the case of private tuition there is a second person teaching the pupils, and that this teacher does not necessarily have any mathematics qualifications. Although intensive help from parents might be in essence the same as ”Nachhilfe”, it is not recognized as such because it is not paid for and ”Nachhilfe” is seen as expensive private tuition.

Details about my current research into coaching mathematics

In my studies I concern myself with the following questions:

- What are the mechanisms behind coaching mathematics?
- What decisions need to be taken before coaching begins?
- What experiences have children had with coaching?
- How do private tutors assess their pupils situation ?
- How is mathematics coaching carried out?
- What are pupils, teachers and parents concepts of mathematics that influence the learning of mathematics in school or in private lessons?
- Is it possible to find reasons or factors for this coaching problem?
- How are causes seen by the people involved ?

Primary Findings

A questionnaire

In January and February of 1995 I asked about 100 pupils of the higher grades of a Gymnasium (engl.: Grammar school) what they thought about the reasons any people need extra tuition in mathematics. In order of frequency their reasons relate to:

- the pupils and the way in which they learn
- the teachers and their way of teaching
- the subjects and the characteristics of these subjects
- other aspects.

About an interview with a coaching teacher

This teacher has given private lessons for more then 10 years and teaching private lessons has become his main profession. He is a qualified teacher, but he has not done any examinations in mathematics, neither at school nor at university. I want to summarize his own concept of teaching private lessons:

My opposition to traditional methods of teaching, my very pragmatical treatment of the mathematical subjects and an atmosphere conducive to learning are the guarantees of successful coaching.

About two interviews with pupils involved in private tuition

Two completely different kinds of self-evaluation are shown in two interviews with pupils from grade 8 at Gymnasium who have had private lessons.

(1) <u>A girl’s conception of how she came to need coaching</u>: *I think I am quite mediocre in maths. The only reason for my difficulties was my new teacher.*

During the interview I asked the pupil, what she thought the distinguishing features of good mathematics lessons were. The pupil only described the teacher himself and the speed of teaching mathematics, nothing else. Table 1 explains this reaction.

	History of coaching	Pupils' conceptions	Other aspects
Phase of decisions	Teacher had changed Pupil's marks slipped (2/3 to 4) less time for going to training in the afternoons Trainer organized a mathematics teacher for private lessons who himself belonged to the same club as the pupil.	New T. was very fast. T. was not able to expl.	Many pupils got some difficulties. Pupil thought that she was 'mittelmäßig' in maths.
Phase of coaching	coaching mathematics as necessary 1-1,5 h per week - doing exercises, also taken from other books	"The private teacher set some traps for me."	
Phase of ending coaching_	second change of teacher Pupil's marks improved to previous standard (3) private tuition ended Pupil has now fun in doing mathematics	 T. is the only reason for my difficulties in mathematics	 T. is the reason for my school friends' difficulties

Table 1.

(2) A boy's conception of learning by coaching: *I think I'm quite lazy and during mathematics lessons I'm not attentive all the time but I think that I'm intelligent, my parents also think so. So I've had more and more problems since grade 6. Therefore I've had private lessons from a lot of teachers but now I think my brother has developed the best system of teaching mathematics in private lessons.*

The system of this coaching follows (Figure 2). Because of the pupil spoke in slang terms, this chart shows the original quotations in the three rectangles.

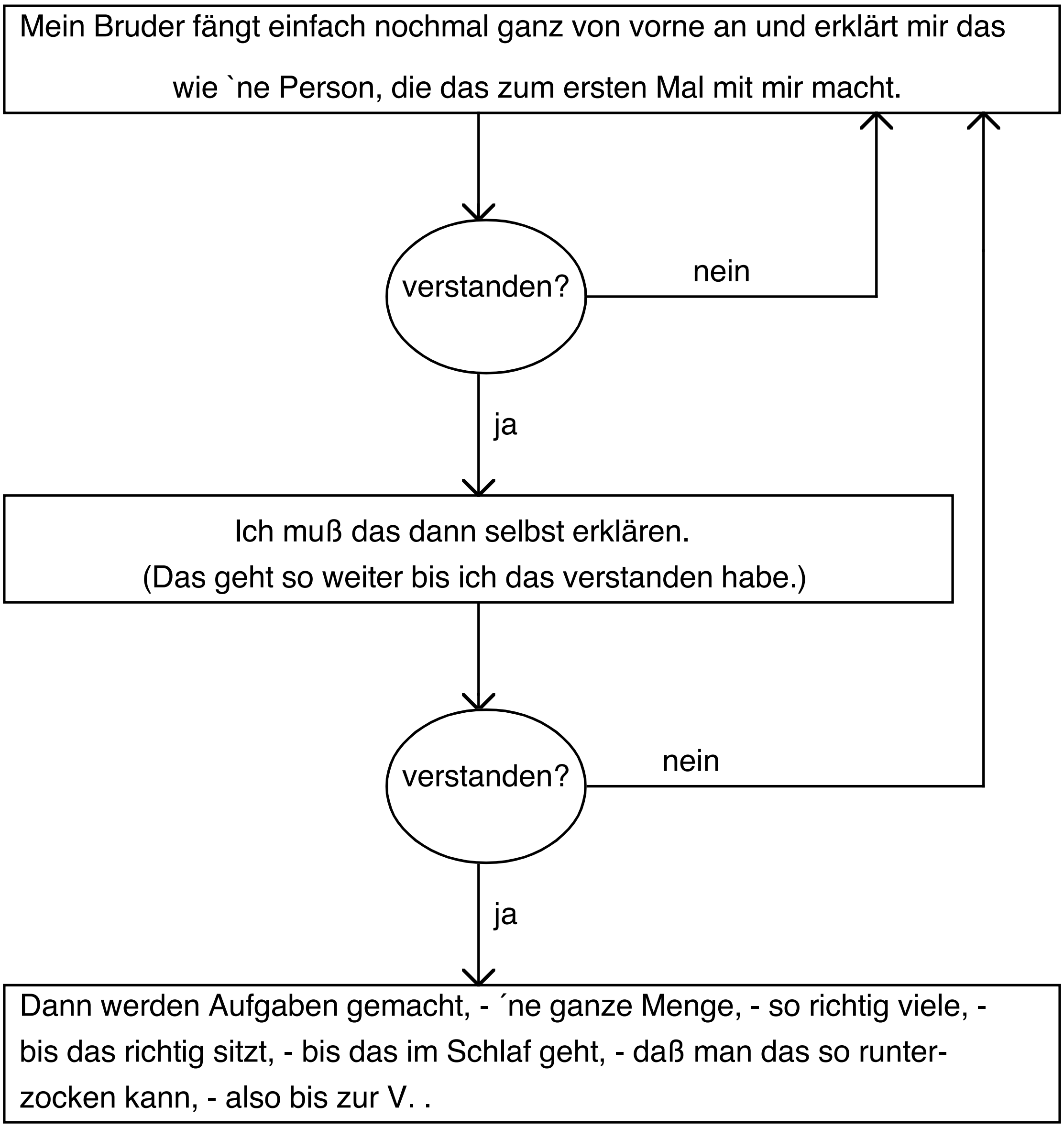

Figure 2. The boy's system of coaching.

Obviously this pupil likes a kind of systematic mechanical lesson to make sure nothing is forgotten. It seems that this pupil wants to be a good mathematics pupil. During the interview he said that he was quite interested in mathematics and he would choose to study mathematics until the end of school if he was able to learn mathematics successfully without any help.

All the pupils I have interviewed emphasized their aim to end their mathematics coaching soon and to go on learning successfully without help.

Erkki Pehkonen

Children’s Responses to the Question: What is Mathematics?

In research concerning beliefs, one of the first topics is to chart pupils’ or teachers’ beliefs about mathematics. But the very small children are usually set aside, since they are not seen to be able to fill in questionnaires or to possess of such complex conceptions as mathematics and mathematics teaching/learning. However, pupils’ mathematical beliefs have their roots in experiences of the childhood, and most of them are formed at preschool age. Therefore, the views on mathematics of small children, before their schooling starts, are of interest in trying to understand the birth and development of pupils’ beliefs.

1. On the study

1.1. Theoretical background

Since in definitions of beliefs to be found in the educational literature, there is no common guidelines to be followed (Pehkonen 1995), we will say here: An individual’s mathematical beliefs are compound of his subjective (experience-based) implicit knowledge on mathematics and its teaching and learning. Conceptions could be understood as conscious beliefs, and thus are separated from so-called basic beliefs which are often unconscious. We think that in the case of conceptions, the cognitive component will be stressed, whereas the affective component is emphasized in basic beliefs. Views are very conceptions, but views are formed more spontaneously than conceptions.

1.2. The aim of the research

Within the framework of the constructivist understanding of learning and teaching, it is essential in the learning process that the learner is actively working along, in order to be able to broaden his knowledge structure (e.g. Davis & al. 1990; Ahtee & Pehkonen 1994). Thus, the meaning of pupils’ own beliefs (subjective knowledge) concerning mathematics and its learning will be emphasized as a regulating system within their knowledge structure. Therefore, the understanding and taking account of pupils’ mathematics-related beliefs are essential for the teacher to be successful in his teaching.

The aim of this study was to find out, *what children’s conceptions about mathematics are, as well as the learning and teaching of mathematics before they will go to school*. In order to realise this aim, the following research problems were developed:

(A) What is mathematics according to children?

(B) What kind of views do children have of themselves with regard to calculation?
(C) What is a child's attitude toward mathematics?

1.3. Data gathering

The idea to gather children's mathematics-related views before their schooling starts, is due to Ms. Monika Riihelä who also developed such a test in co-operation with Ms. Riitta Kauppinen (both from STAKES). The test subjects were twenty children (age 6 years) from two different preschool institutions in Helsinki.

The method used to gather data was a structured interview. The interview was based on a questionnaire formula (written on a DIN A4 page) consisting of four open questions. The test was labelled *Mathematics and Me*, and the four questions were the following:

(1) What do I think about mathematics and/or calculation, what is mathematics or calculation?

(2) What kind of calculating person or mathematician am I?

(3) How could I learn to calculate and/or to do mathematics?

(4) How could somebody teach me mathematics?

On a DIN A4 page, there were empty spaces between the questions for the interviewer to fill in the children's responses.

Two prospective preschool teachers interviewed the children during the spring of 1995, by asking the children one question at a time, and writing down their answers as exactly as possible.

2. Some results

The results given here are based only on the children's answers in the interviews. Each of the three research questions will be dealt with in a separate section.

2.1. What is mathematics?

The answer to the research problem (A) is gathered from the children's responses to Question (1). In Table 1, the children's responses are grouped into six categories which form a kind of hierarchy, where the knowledge is structured on different levels.

	categories	test subjects	frequency
1.	I don't know	–	
2.	numbers	11, 12, 13, 15, 16, 18, 20	7
3.	counting of numbers	1, 2, 9, 10, 19	5
4.	plus-calculations (tasks)	3, 4, 5, 7, 17	5
5.	minus-calculations, times-calculations	6, 8	2
6.	special answers	14	1

Table 1. The frequency distribution of the responses to the question
"What do I think about mathematics and/or calculation, what is mathematics or calculation?"

Most of the twenty responses (35%) were on the level of mentioning that there are numbers involved. The typical answers are the following: (the number in the brackets refers to the test subjects, and b=boy and g=girl): *"I will say numbers"* (13b), *"The knowledge of numbers"* (18g), and *"Numbers will be needed "* (20g). The frequency in the next two next level were the same (25%). These levels are Counting of Numbers and Naming of Addition. The following children's responses were put onto the level of Counting: *"Mathematics is counting. Counting is 1,2,3,4,..."* (2g), and *"I can count from 1–100 all by myself"* (10g). In the children's responses, they should have used, at least, some calculation-word, in order to reach the Addition level. For example, *"Mathematics is calculation tasks"* (4b), *"Calculation is the adding of numbers"* (5g), and *"Mathematics is plus-calculations"* (7g).

Only two of the children mentioned subtraction or multiplication, e.g. *"It is calculation when one makes plus- and minus-calculations. One may calculate also by multiplying"* (8g). In addition, there was one child who gave very special answer: *"I can calculate with a pocket calculator"* (14b). No child has answered that he did not know what mathematics is.

Thus, we could expect that most of the school beginner are awaiting mathematics in school to be about the counting numbers and doing calculation tasks. Nobody mentioned anything concerning geometry.

2.2. What is a child's view on himself?

From the children's responses to Question (2), we will be able to receive an answer to research problem (B). The children's responses are grouped into three categories: good, average, poor (Table 2).

	categories	test subjects	frequency
1.	good	3,4,5,6,7,8,10,11,12,13,16,17,19,20	14
2.	rather good /average	1, 9 ,14 ,15	4
3.	poor	2, 18	2

Table 2. The frequency distribution of the responses to the question "What is a child's view on himself as a mathematician?"

Most of the children (70%) consider themselves to be good in mathematics. Typical responses here were the following (the number in the brackets refers to the test subjects, and b=boy and g=girl): *"I know all plus-calculations"* (7g), and *"I can do grade 1 tasks well"* (17g). Four of the children were not so sure how good they were. They used e.g. following wording: *"I don't know since I am not willing to calculate"* (1g), and *"Rather good"* (9b). Only two girls had a negative view on themselves as a mathematician. The following responses are given as examples: *"I am a poor one, since I have not yet done it much"* (2g), and *"A poor one"* (18g).

When we take these results into account we might be happy, since almost all children (90%) had a good or rather good view on themselves. However, one could ponder on what kind of a mathematics learner the girl (18g) will become who already considers herself to be poor in mathematics before her schooling has even started.

2.3. What is a child's attitude?

In the responses to questions (1)–(4), there were many with emotional responses. Therefore, I decided to try to find out a child's attitude toward mathematics, i.e. research problem (C). For this purpose, we will again use the children's responses to questions (1)–(4). The responses are grouped into five categories.

	categories	test subjects	frequency
1.	positive	4, 7	2
2.	rather positive	2, 3, 13	3
3.	neutral	5, 6, 8, 11, 12, 15, 16, 17, 18, 19	10
4.	rather negative	10, 14, 20	3
5.	negative	1, 9	2

Table 3. The frequency distribution of the responses to the question ”What is a child's attitude toward mathematics?”

The distribution of the responses was symmetric, almost similar to the Gaussian Bell Curve (Table 3). Half of the responses (50%) are in the middle, i.e. they were classified as neutral. From the responses which were classified as positive, the following examples are typical ones (the number in the brackets refers to the test subjects, and b=boy and g=girl): *”Calculating is nice”* (4b), and *”I like those [plus-calculations]”* (7g). An example of a rather positive answer is the following: *”It is a good thing”* (13b). The following examples *”It is stupid, too easy”* (1g), and *”Stupid”* (9b) stand for the negative answers, whereas the answers *”It is not so nice”* (14b), and *”I already can calculate enough”* (20g) were classified as being rather negative.

It is astonishing that many children react emotionally to mathematics, either liking or disliking it. Half of the children did not shown their emotions, while answering the questions in the interview. In addition, one could raise the question, how the boy (9b) who states that mathematics ”is stupid” will perform in school. His attitude is thus already negative, and perhaps also a bit arrogant – which is no good beginning for school.

2.4. A couple of childrens' responses discussed in detail

Let us consider the responses of some children in detail. The girl (18g) had a poor view on herself in calculations, and I raised the question how she will manage in school when she is considering herself to be a poor one in advance. If we look at her other responses, we could actually see in her answer to the next question (#3) that she is optimistic. She says *”Through practising I will learn to be a better one”*. Finally, in the last question, she is being realistic, and states: *”I will need somebody's help”*. I think she will manage with mathematics in school, if she only could develop a optimistic view on herself and if she gets a teacher who takes her special situation into account.

Now we will discuss the case of the boy (9b) who said that mathematics is stupid. In his other responses, one could read between lines that he has an arrogant attitude, at least, toward mathematics. With regard to calculations (question 2), he considers himself as *”a rather good one”*, and in question (3) he states: *”I can count as far as I like, even until a trillion.”* If he does not get rid of his arrogant attitude, he will experience difficulties at school. But on the other hand, a good teacher could, perhaps, help him by taking his personality into account.

3. Discussion

In summarizing the results, we could state the following: Most of the school beginners seem to be expecting that mathematics in school will be counting numbers and doing calculation tasks. Almost all children (90%) had a good or rather good view on themselves in mathematics. But astonishing many children seem to react emotionally to mathematics; mathematics seems to be from the very beginning a subject in school which will be either liked or disliked.

When trying to access how reliable the gathered information is, one should take the interview situation into account. Children will most probably exaggerate a little bit in such a situation, but in each case one might think that the results obtained will reflect some kind of upper limitation of childrens' knowledge of mathematics before starting school. The fact that only some of them were able to mention more than the counting of numbers and adding them, shows the limits of their knowledge. On the other hand, an unofficial questionnaire distributed among a group of preschool teachers by the author a couple of years ago, gave principally the same message: Children think that mathematics is, in the first place, the counting and adding of numbers. These beliefs might reflect the beliefs of their parents and other adults. On the other hand, one should note that instead of explaining the complicated word "mathematics" to children, the questionnaire used the term calculations as a synonym (cf. section 1.3). This could also be a reason why no geometry-related answers were given.

The results showed to us that there already exists a variety of beliefs on mathematics learning and teaching among preschool children. On the other hand, these expressed views should be taken into account at the very beginning of schooling if the teacher would like to educate children successfully. It looks as if many children would need special treatment from their teacher. Therefore, it might be fruitful if the teacher starts with finding out what kind of beliefs his pupils have about mathematics and the learning of mathematics. A structured interview might be a suitable tool for that.

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Monika Rahmann

Attitudes and Attitude Change of Mathematics Teachers at the 'Gymnasium'

It has been the aim of my study (Rahmann 1995) to, on the one hand, find out about teacher's attitudes while teaching. On the other hand the change of theses attitudes were to be examined because it is supposed that the development of teachers is a dynamic one. It was especially purposed to determine the conditions and the way to change teacher attitudes from the outside. Therefore eight 'Gymnasium' mathematics teachers, who have been in service for at least ten years, were interviewed.

It has been revealed that teacher attitudes are a very complex structure which has a great influence on several sectors. Attitudes help the teachers to structure the environment at school in order to enable themselves to act in numerous different situations. This organization is a subjective one because it is a generalization of attitude objects which isn't a realistic categorization or classification of the objects. Attitudes, adopted by teachers, have a great impact on their interpretation of the daily experiences at school.

The assumption was put forward that the teacher's view on mathematics plays a central role. That implies that it can heavily affect other attitudes. This correlation is shown in Figure 1 (Ernest 1989).

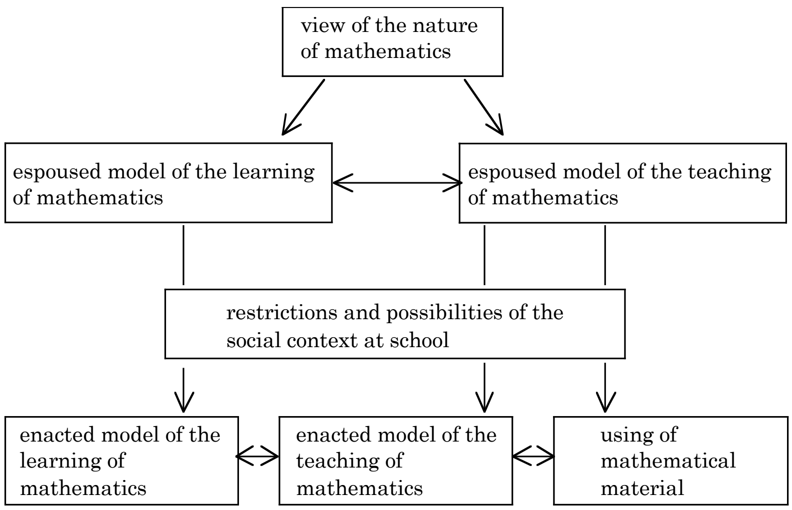

Figure 1.

Acccording to this model the thesis was established that the teacher's view on mathematics, especially at the beginning of his teaching career, determines his attitudes about teaching and learning of mathematics. There was distinguished between an espoused and an enacted model of learning, or respectively teaching. The reason for this discrepancy was attributed to the restrictions and possibilities within the social context at school. In contrast to this point of view the opinion of Dörfler & McLone 1986 was mentioned, namely that the experiences at school are the most important influences of the teacher's view on mathematics.

In order to solve this contradiction it was supposed that at the beginning of a teacher's career, the view on mathematics constitutes other attitudes – although with increasing professional experiences the practice at school becomes the most influencing factor. Simultaneously a specification occurs, this means that the teacher will make a distinction between mathematics and school mathematics.

Furthermore, connections between the mathematics teacher's different attitudes have been shown. Basically one can assume that all the examined components – view on mathematics, teaching, knowledge of subject and pupils – correlate and have influence on each other, which shows the complex and network-like character of attitudes. The following possible relationships have been worked out:

- teachers' view on mathematics – teaching practice and didactic – pupil's achievement and their view on mathematics;
- teaching practice – view on mathematics;
- kind of subject knowledge – teaching method and pupil's achievement;
- teacher's attitudes about pupils – teaching method resp. target and pupil's achievement.

It is not possible to mention certain relationships, because the degree of the component's consistence depends on different factors. The more conscious and the more critical the mathematics teacher handles with his attitudes, the more likely interactions between the components become, so that they don't include inconsistent information and evaluations. According to Ernest 1989 the following characterization of a mathematics teacher is ideally:

- consciousness concerning the takeover of attitudes about mathematics and their teaching and learning;
- the ability to justify these attitudes;
- self-awareness concerning the existence of practicable, workable alternatives;
- sensitivity regarding to the content in the selection and performance of appropriate strategies for teaching and learning, which match the teacher's attitudes;
- the ability to reflect, which allows the teacher to bring his attitudes to accord and to resolve contradictions.

In addition, the attitude change of eight mathematics teachers has been analyzed to get an idea, in which way such a change takes place. Table 2 shows the examined influence factors for attitude change.

change factor	number
experiences with pupils	7
acquisition of professional practice	7
discussions/conservation with colleagues	5
change in society	4
experiences with own children	4
change of school	3
experiences with other school forms	3
parents of pupils	3
individual experiences	3
experiences with further education	2
schoolbooks	2
organization of school	1

Table 2.

Additionally, the following changes of teacher attitudes have been discovered (The upper sector of Table 3 includes general changes, the lower part changes concerning mathematics):

change	number
development of sensitivity and understanding for pupils	5
education becomes more important than imparting the subject	4
more child-centered teaching	3
more teacher-centered teaching	3
more sovereignty in teaching	3
more acting of pupils	2
changed target of school	1
the attitude that pupils can't work independently	1
regarding reform approaches	1
changed view on proving	6
more creativity and imagination	2
more connections to other subjects	2
more applications	2
more exercise in lessons	1

Table 3.

According to these results it is possible to point out that one can distinguish between three kinds of change:

(A) changes which occur at the beginning of the teacher's career;

(B) changes which are the result of the changing of conditions with regard to schooling and teaching;

(C) changes which are aspired to without outside change and which occur consciously and from inside the teacher.

The first two changes may be understood as a general development of the teacher which could be regarded as a continuous process of integration and adaptation. Sector (C) posses a more individual character which presumably occurs with key experiences and which is like a dawning of consciousness and a critical analysis of one's own attitudes.

Looking at the examined change factors of the teacher interviews, one can establish that the two most important factors ñ *experiences with pupils* and *acquisition of professional experience* – have relevance to sectors (A) and (B). Additionally, the changes rather seem to take place concerning the aspects integration and adaptation into the environment at school.

The changes, which were mentioned by the interviewed teachers, primary concern general aspects, not aspects referring to mathematical or mathematics teaching. Although it was noted that subject instruction becomes less relevant than pupil's education, it seems to be independent from specific mathematical circumstances. Just one teacher reports that his view on mathematics changed during his studies at university, but this aspect doesn't occur in the explanations of the other teachers. Therefore the mentioned changes in the mathematics teaching are characterized as selective ones which have no influence on the teacher's view on mathematics.

Finally the subsequent assumptions have been worked out:

- Changes primary concern attitudes about pupils, school-mathematics, school objectives, teaching style, the way the teacher sees himself, and details in subject teaching.
- Changes can be primary understood as an adaptation resp. integration into the context at school and as an extension of the teacher's scope of acting.
- Intentional changes take place only if the teacher possesses the appropriate acting knowledge and if he evaluates the new acting as effective and useful.
- Almost all the changes don't refer to mathematics-specific aspects.
- The view on mathematics doesn't change without purposeful influence (from outside or by teacher's conscious analysis).
- The view on mathematics is in most cases unconscious and can therefore be regarded as a 'meta-attitude'.
- The teacher's process of development can be described as follows:
    1. The view on mathematics, which is influenced by his own schooldays and his studies, determines his attitudes about school mathematics and mathematics teaching from the very beginning.
    2. In the next step a distinction between mathematics as a discipline and as a subject in school takes place.
    3. Because of restrictions and possibilities of the social context at school the ideal/vision of teaching and learning school mathematics turns into an enacted model.
    4. Attitudes stabilize and (a) only change in small details, a consolidation takes place or (b) the teacher makes a conscious analyze of his vision and of his enacted model so that he tries to harmonize them if he recognizes inconsistencies.

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Hans-Joachim Sander

‘What is a Good Mathematics Teacher?’

Results of an Inquiry of Students at a Gymnasium[1] in North Rhine-Westphalia[2]

When I started to educate prospective mathematics teachers at the University of Vechta, I came to the idea to ask my former students, whom I had taught until then, what they thought to be ”a good mathematics teacher”. From this idea developed an inquiry of more than one hundred students about their views on mathematics, mathematics teaching, and the mathematics teacher.

The inquiry had to be officially approved by the school authorities of North Rhine-Westphalia. This approval was granted on several conditions, especially that the parents of the students had to agree, the inquiry had to be voluntary for the students, and the anonymity of all persons involved had to be maintained.

On this basis I was able to conduct an inquiry of four classes in April 1994 (see Table 1). In addition to that I tried to question former students of a *Leistungskurs* of mine (extended course of six periods of mathematics per week, forms 11 through 13), who had left school in 1993. This course consisted of 17 students. By the time of the inquiry they had thus already left school for one year, and I succeeded in reaching only 13 of them, 10 of whom sent me back the filled-in questionnaire.

Form (as at April 1994)	6	9	10	11	*Leistungskurs*
Taught by the author in forms	5	6 7 8	7 8	8 9 10	11.2 12 13
Number of students	31	32	20	24	17
Returned questionnaires	30	29	18	22	10 (out of 13)

Table 1.

The first page of the questionnaire contained a cover note, where I explained my reasons for the inquiry and referred to the conditions imposed by the school authorities. This cover

1 Highest-ranking high school in Germany with grades 5 (or 7) through 13 (or 12, depending on the federal state).

2 One of the 16 federal states of Germany.

note was followed by a likert-scale questionnaire consisting of 32 statements, which had to be (dis)approved of on a scale ranging from +3 (total agreement) to -3 (total disagreement). The third page of the questionnaire consisted of six free-response questions or open situations to be commented upon.

In detail the 32 statements of the likert-scale questionnaire were:

1. Mathematics is interesting.
2. Mathematics is boring.
3. You cannot understand mathematics.
4. Mathematics is useless (superfluous).
5. Mathematics should not be a subject taught in schools.
6. I think, mathematics is important in our society,
7. but I don't understand why.
8. The mathematical theorems and methods are not connected to each other,
9. (that is why) you can only learn them by heart,
10. (and that is why) I forget them very quickly.
11. I am afraid of mathematics.
12. It depends on the teacher whether I understand mathematics at school.
13. If mathematics lessons are fun, I learn more.
14. It is important to me to see in mathematics lessons, where this mathematics can be used in life.
15. In mathematics lessons it depends more than in other subjects on the teacher, whether I succeed in learning.
16. It is not so much the ability of the teacher to teach mathematics, which is important,
17. but the fact, whether he likes me
18. and that I like him.
19. He should be able – at least from time to time – to make and take jokes.
20. It is important that he has complete command of his subject (mathematics).
21. It is important that he likes his subject
22. and that he likes to teach.
23. I can show understanding that every now and then he makes a mistake.
24. In that case I would appreciate him admitting his mistake.
25. It is important to me to receive praise for a good answer or a good solution from the mathematics teacher
26. in front of the class
27. or alone.
28. It is important to me that in mathematics lessons I can discover things on my own and that the teacher guides me towards these discoveries.
29. I prefer to have mathematics explained to me without having to think for myself.
30. When I am at home I dip in the mathematics textbook
31. if I did not understand a topic
32. also just for fun or curiosity.

As can be seen e.g. from the statements 8, 9, and 10, not all items were independent of each other, but some formed a composite statement. Nevertheless the students were supposed to respond to each single statement. In retrospect, I realize this composition of complex items could cause problems. In item 8 a student may see the connection, but despite of that he forgets the theorems and methods very quickly. What should he then circle in item 10: + 3, since he forgets quickly, or -3, since it is not because of the lack of connection?

In my opinion this clearly points out the limitations of anonymous, likert-scale questionnaires. An alternative would have been to state the item ”I forget the mathematical theorems and methods quickly” and then to establish the reason for it in interviews.

In the second part of the questionnaire the students were asked to respond freely to the following questions (a space below the question was being left blank and extra-sheets could be used as well):

1. When I make a mistake or give a wrong answer, I expect the mathematics teacher to ...
2. It is important that the mathematics teacher ...
3. Besides that I expect him to ...
4. If you did not catch a certain topic in mathematics lessons, what do you do then (nothing; I ask my parents, class-mates, the teacher, ...)?
5. It is often said that a mathematics teacher must be able ”to explain well”.
What does that mean to you? Can you give an example?
6. What is most important to <u>you</u> to learn mathematics?

In this abstract I want to go into the details of only two items of page 1 of the questionnaire. Since I am a convinced supporter of discovery learning and of problem-oriented teaching, in my opinion the items 28 and 29 contained a very important question: What is the attitude of students towards discovery learning? This way of learning is often reagrded as being only suitable for good students or for a short time – for a show-lesson during an examination, but not for everyday year after year mathematics teaching.

28. It is important to me that in mathematics lessons I can discover things on my own and that the teacher guides me towards these discoveries.

		Frequencies							Range		Median	Mean
	Total	+3	+2	+1	0	-1	-2	-3	Max.	Min.		
Total	106	67	28	8	2	1	0	0	+3	-1	+3	2,5
Form 6	28	17	9	1	1	0	0	0	+3	0	+3	2,5
Form 9	28	17	9	1	1	0	0	0	+3	0	+3	2,5
Form 10	18	12	5	1	0	0	0	0	+3	+1	+3	2,6
(Form) 11	22	15	2	4	0	1	0	0	+3	-1	+3	2,4
Leistungsk.	10	6	3	1	0	0	0	0	+3	+1	+3	2,5

Table 2.

Table 2 shows on its left hand side the frequencies of how often the scale values from +3 to -3 have been circled. Since the class-representatives sent the filled-in questionnaires back to me, I was also able to see how a single class responded. The frequencies were then used to find the range (maximum and minimum of the circled values), the median, and the mean.

Although it was not a representative inquiry, since I questioned my former students, the results show clearly that it is possible not only to get the students interested in discovery learning, but to convince them of this way of learning: 67 out of 106 students, who responded to this item, agreed completely to it, another 28 still predominantly agreed. Only one student mildly disagreed with the statement, circling -1.

The rejection of item 29 (the opposite to item 28) was not as strong as the approval of item 28. It may be quite comfortable to have everything explained without having to think for one-self ... But still the overall median is -2, and the overall mean is -1.9. It is interesting that in item 28 the students of form 10 agreed most strongly of all forms, but disagreed with item 29 only by a mean of -0.9. This is another point where personal interviews would be helpful.

The reactions of the students on discovery learning where as satisfactory to me as their answers to the open situation ”When I make a mistake or give a wrong answer, I expect the mathematics teacher to ...” were depressing. In his fundamental book ”The Psychology of Learning Mathematics” (1993) in a section titled ”Insults to the intelligence” (p. 110) Richard Skemp demands: ”The teaching and learning of mathematics should thus be an interaction between intelligences, each respecting that of the other.” In the answers of the students, the respect required by Skemp was not very much in evidence. Here some examples (one from each class or course):

Form 6:	*that the mathematics teacher doesn’t blow me up or gets me down or that he says: ”You don’t understand maths. You are stupid. You don’t know a thing.”*
Form 9:	*On no conditions he should start to scold or become cross.*
Form 10:	*not blame me and show me up in front of the others,*
(Form) 11:	*not tick me off and give me a hell in front of the whole class,*
Leistungskurs:	*not misrepresent me personally as dull.*

In all only 4 out of 109 students did not respond to the first open situation on page 2 of the questionnaire. The answers of 68 students out of 105, which is about two thirds, who gave an answer, can be classified in the category: ”He should not run me down (etc.)”. It seems noteworthy to me that this category was not mentioned in the questionnaire.

In the wake of constructivism, a positive valuation of mistakes in the learning process becomes more and more important. In her lecture at the ”Klagenfurter Symposium zur Didaktik der Mathematik” in 1994 Anna Sfard, speaking about the ”Constructivist Call”, demanded: ”Look into students’ heads and take their thinking seriously. Novice conceptions make sense for the learner and are a necessary step in concept development.”

Although the students have most probably not heard anything about constructivism, some of them plead for corresponding activities by the teacher. It can be noticed that such calls did not appear at all in form 6. In form 9 there were hints, and finally in the former *Leistungskurs* they are precisely formulated. Here are the answers of a former student of this course to questions 1, 2, and 4 of page 2 of the questionnaire (inverted commas, underscoring etc. by the student):

When I make a mistake or give a wrong answer, I expect the mathematics teacher to ... *correct me, but he should try to understand my train of thought and ”investigate” the reasons for my ”wrong way”.*

It is important that the mathematics teacher ... *is committed (in the above-mentioned way): There must not only be the two extremes "completely right" and "completely wrong"*.

If you did not grasp a certain topic in mathematics lessons, what do you do then? ... *If I didn't understand something basically, I ask the teacher and explain to him, how I understood it and why I can't understand what is wrong with my thoughts.*

Sabine Weber

Social Beliefs about Mathematics - a Content Analysis of the German Press

The aims of the study

"Beliefs – to be interpreted as an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behaviour – will receive a telegraphic discussion. The discussion will take place in three parts: student beliefs, teacher beliefs, and general societal beliefs about doing mathematics. There is fairly extensive literature on the first, a moderate but growing literature on the second, and a small literature on the third." (Schoenfeld 1992, p. 35)

This investigation focuses on the mathematical beliefs that are being held by the society of the Federal Republic of Germany. This matter is of relevance to research dealing with the concept of mathematical beliefs, because it is being implied in available literature (on mathematics) that the society would hold an ambivalent attitude towards mathematics. On the one hand people think that mathematics is a very important subject, on the other hand they believe that this subject has no relevance for their own lives.

This idea is expressed for example by Jungwirth, Maaß and Schlöglmann 1993 (p. 43): "*Auf der einen Seite läßt sich eine autoritative Haltung rekonstruieren – der Mathematik wird mit (über-)großem Respekt begegnet, auf der anderen Seite gilt es in unserer Gesellschaft als Kavaliersdelikt, nur über geringe Mathematikkenntnisse zu verfügen bzw. im schulischen Mathematikunterricht schlecht abgeschnitten zu haben."*

The purpose of this investigation is to find out which beliefs about mathematics are really being held by the society. Therefore I investigated the German press by means of a content analysis. In my opinion this is a good method, because the investigation has no effect on the information released by the press. The articles exist independently and are not influenced by the interests of the research project.

The second aim would be to find out in which way mathematics is presented in the German press, because social beliefs are reflected and controlled by the press (see Merten 1983, p. 107-111). Therefore I also interviewed people within the German society to establish whether the views on mathematics that are mentioned in the press, do actually exist within the society. This paper will mainly focus on the results of the content analysis.

The methods of the study

At the beginning of the investigation of the press' view on mathematics, six hypotheses were formulated with the help of existing literature on mathematics and mathematics education:

1) In our society mathematics is regarded with great respect.
2) People think that it does not matter if you get low mathematics marks at school.
3) Only mathematical calculation is important to members of our society.
4) The society does not recognize that mathematics is essential in many parts of economic life and industry.
5) In the society the view exists that mathematics consists only of immutable truths – being strict and formal derivation from axioms.
6) mathematics is being perceived negatively in our society.

A content analysis was used to investigate these six hypotheses (see Merten,1983). For six weeks in September and October 1994, eight daily and six weekly newspapers were studied. The newspapers were selected with respect to different political point of views and different readers. I searched for all articles that contained words like ”mathematics”, ”mathematics teacher”, ”mathematical”, etc. .

In 316 newspapers, 92 articles containing words like ”mathematics” were found. 11 out of 92 articles dealt with mathematics as a main theme. These 11 articles were – for example – about the chance of winning in the lottery or about the application of mathematics in technology.

I formulated questions relevant to the above mentioned hypotheses in order to analyze the articles. For example with relevance to the first hypothesis: ”Is it mentioned in the article that mathematics is regarded with respect?” In addition to these questions I regularized the answers. For example: ”It is mentioned in the article that mathematics is regarded with respect if mathematics is described as being important or being a main subject at school etc.” Then the 92 articles were investigated seperately with regard to these questions. The questions were repeated with every article. Afterwards the outcomings of the answers to every question were counted.

The results of the study

The result of the press investigation is that only the first hypothesis could be confirmed. mathematics was described as being a main subject at school and as being important in 15 out of 25 articles that mentioned mathematics. mathematics was valued as being unimportant in two articles.

The second hypothesis (”People think that it does not matter if you get low mathematics marks at school.”) could not be proved during the investigation, because this matter received no attention in the articles.

Furthermore, it could not be confirmed that only mathematical calculation is of importance to people. The fourth hypothesis also proved to be incorrect. The articles in the newspapers emphasized that mathematics is essential in many parts of economic life and industry. 50% of the articles having mathematics as a main theme were about the applications of mathematics in computer sciences, in description of complex systems of nature and technics, in satellite programs on television etc.

The following question might arise here: We realize that when the press writes about mathematics, mainly applications of mathematics are being mentioned. Does that imply that people really have knowledge about the applications of mathematics? Or does this result imply that only articles about the application of mathematics are interesting for the press – whereas articles about purely mathematical research are not published, because nobody will read them? There are for example exactly two articles about purely mathematical research in the articles studied for this investigation.

Hypothesis 5 (”In society the view exists that mathematics consists only of immutable truths – being strict and formal derivation from stated axioms.”) proved to be incorrect as well. mathematics was described as formal and strict in 6 articles, whereas for example in 25 articles mathematics was seen as ”*a bag of tools”* (Thompson, 1992,p.132). This means that mathematics ”*is made up of an accumulation of facts, rules and skills to be used by the trained artisan skilfully in the pursuance of some external end.”* (Thompson,1992,p.132)

Hypothesis 6, that mathematics is being perceived negatively in the German society, could not be confirmed. In 12 articles mathematics was valued positively in the way that it was regarded as being important or being interesting. In contrast to this, mathematics was seen as unimportant and uninteresting in 6 articles.

The conclusion of this investigation is that in the articles I analysed, the attitude towards mathematics is not ambivalent. mathematics is mainly being presented as an important subject.

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